ATHABASCA UNIVERSITY

PERFORMANCE MEASUREMENT OF HEAVY EQUIPMENT RETAILING ORGANIZATIONS
A DATA ENVELOPMENT ANALYSIS (DEA) APPROACH

BY

CHANDRASHEKHAR YEGNARAMAN

A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE
DEGREE OF DOCTOR OF BUSINESS ADMINISTRATION

FACULTY OF BUSINESS

ATHABASCA UNIVERSITY
NOVEMBER, 2018

Approval Page

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PERFORMANCE MEASUREMENT OF HEAVY EQUIPMENT RETAILING ORGANIZATIONS: A DATA ENVELOPMENT ANALYSIS (DEA) APPROACH

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In partial fulfillment of the requirements for the degree of

Doctor of Business Administration

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DEDICATION

*In honor and memory of my father **Thiruvaiyaru Vaidyanathan Yegnaraman** and my grandparents for planting in me the seeds of scholarship**.***

Acknowledgement

I never thought that I would have the opportunity to pursue Doctorate studies, giving it up as a distant dream until I discovered Athabasca University's online program which enables working professionals to pursue research. I'd like to express my heartfelt gratitude to AU for giving me the chance to acquire the highest educational qualification and realizing my lifelong dream.

I'd also like to extend my sincerest thanks, respect and appreciation to the Doctoral committee for their tireless effort in reviewing my material in depth and providing invaluable guidance and feedback. I feel proud and honored that you have accepted to be on my committee. I am especially grateful and thankful to Dr. Rajbir Bhatti for taking me under his wing and guiding me through this journey with his knowledge on Data Envelopment Analysis. He spent hours guiding me through this research, but also gave me freedom to find my own path, offering support when needed and as I knocked on his door. I also want to take a moment to thank my Co-Supervisor Dr. Shaun McQuitty and committee member Dr. Saktinil Roy. Thank you both for investing time, providing insightful comments, valuable feedback and tough questions. The research was able to meet its objectives due to your thought-provoking questions and constant guidance. Dr. Shaun McQuitty has been a source of great inspiration for me as we share the same background before we embarked upon a research career. I could not have imagined a better committee for my thesis.

I am thankful to Prof. Kay Devine, former Director of the Doctoral program for her constant support and guidance. I also thank the staff in Centre for Innovative

Management, Athabasca University for their support over my many years as a student. My thanks to the staff in AU library for their help in getting me references when requested. I also thank all the Professors in the academic community who provided me with inputs, clarifications and approvals to use their material in the research whenever requested.

I owe a special debt of gratitude to the management of a heavy equipment distributor for providing me the data for my research, as without their support, it would have not been possible to complete this research.

Finally, I shall be falling short of vocabulary to thank my wife for all her unconditional support throughout this Doctoral journey. I could not have made this happen without her by my side. I also want to thank my lovely daughters Shruthi and Shwetha, as well as my son-in-law Rohit for the joy and happiness they bring into my life. Of course, I also want to thank my mother and my sisters for their constant support.

Lastly, a big thank you to GOD for giving me the energy and motivation to sustain and sail through the Doctoral program.

Abstract

Measuring and improving the performance has always been the center of attention of organizations. Organizations usually rely on different ratios to measure key performance. However, measuring performance by merely relying on ratios has its shortcomings. A more powerful tool in measuring relative performance is Data Envelopment Analysis (DEA). DEA is a mathematical programming technique for determining relative efficiencies of peer decision making units (DMU) and the technical efficiency of individual DMUs. It is a data-oriented approach for evaluating the performance of DMUs. DEA has been successfully used in both public sector and private sector. One of the industries that have been greatly overlooked is the heavy equipment industry and its retailing organizations.

The main objective of the thesis is to develop models using DEA for measuring performance of heavy equipment retailing organizations. In this research performance measurement of heavy equipment retailing organization is evaluated by treating each branch (DMU) as whole unit and by analyzing the internal structure of each DMU. The organization under study is a Canadian heavy equipment retailing organization(HERO).

The four DEA models used in the study measures efficiency from different perspectives. Such a measurement provides a comprehensive framework for measuring the performance of HERO. The study helps in benchmarking and locating best practices

that are not visible through other commonly used management methodologies in the heavy equipment industry.

The key findings of this research are: a) identification of branches that are efficient and inefficient b) Ranking of the branches based on super-efficiency scores that enable in benchmarking. d) The effect of environmental variables on the efficiency scores. f) Found that the efficiency of individual departments of the branch is less than the efficiency of the whole branch g) there is fluctuation in efficiency scores over a four-year period.

The contributions are a) facilitates in benchmarking b) enables inefficient branches to improve its efficiency levels c) identification of variables that affects efficiency scores. d) PEDMAS as a new tool to measure the performance of heavy equipment branches. e) Identification of factors that will assist in improving efficiency.

Keywords: Data Envelopment Analysis, HERO, Performance.

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List of Conferences

The following are the conferences where this study was presented as a research paper.

1) **Society of Business Research Conference, March 17-19, 2016. Orlando, Florida, USA.**

   "Performance measurement of Heavy Equipment Dealerships using Data Envelopment Analysis". Chandra Raman and Walid Belassi.

   (Paper presented for feedback prior to Candidacy examination)

2) **Administrative Sciences Association of Canada (ASAC) Conference May29 -June1 2017, Montreal, Canada.**

   "Performance Measurement of Heavy Equipment Dealerships using Data Envelopment Analysis (DEA)". Chandra Raman and Rajbir Bhatti.

   (Preliminary research with data sent to the conference for peer evaluation for feedback but not published in proceedings).

3) **15 th International conference on Data Envelopment Analysis. June 26-29,2017, Prague, Czech Republic.**

   "Performance measurement of heavy equipment dealership using Network Data Envelopment Analysis Approach(NDEA)". Chandra Raman and Rajbir Bhatti.

   (Paper presented at the conference for feedback on NDEA approach to measure performance of Heavy Equipment Retailing Organization).

Glossary of DEA Terminologies

DEA GLOSSARY		
Item#	**Terminology**	**Explanation**
1	**Aggregate Efficiency**	A term used to describe measure of efficiency in CCR model
2	**Allocative Efficiency**	Ratio of cost efficiency to technical efficiency.It isa measure of efficiency
		for a given input prices, the cost of production isminimized.
3	**BCC Model**	Banker Charnes, Cooper Model that addressess variable return to scale.,
4	**Categorical Variables**	These are used to indicate presscence or lack of a particular attribute.
5	**CCR Model**	Charnes,Cooper,Rhodes Model addressess constant returns to scale.
6	**Composite Unit**	This is a hypothetical unit and its inputs and outputs are determined by
		projecting onto the frontier.
7	**Constant Returns to Scale**	An increase in inputsleads to a proportionate increase in outputs.
8	**Controlled(discretionary)inputs**	This is an input where the management of the DMU has control.
9	**Convexity**	If two points are attainable then any point representing a weighted average
		of the two is also attainable.
10	**Correlation coeffecient**	It measures the strength of relation between two variables.
11	**Cost Efficiency**	Also knownas economic efficiency is the ratio of minimum cost to observed
		cost.
12	**Data Envelopment Analysis(DEA)**	A non-parametric technique used for performance measurement.
13	**Data Set**	The data set is set of inputs and outputs of a DMU, that are included in the
		analysis.
14	**Decision Making Unit(DMU)**	The unit whose performance is measured.
15	**Decreasing Returns to scale(DRS)**	This happems when an increase in a unit's input results in a less than proportionate
		increase in its outputs.
16	**Dual Model**	Primal model's alternative is dual model.
17	**Dual Weights(λ)**	It is weight associated to a DMU.
18	**Efficiency Frontier**	It is the frontier that has all the efficient units.
19	**Efficiency Score**	DEA analysis results in a score of 1 for the most efficient and 0 for the least efficient.
20	**Envelopment Form**	Formulation of DEAmodel that involves the concept of composite units.
21	**Epsilon (ε)**	Non Archimedean constant 1x10-6 that takes in to account all variables.
22	**Environmental Factor**	It is the attribute of the environment in which the units operate.
23	**Global Leader**	It is the most efficient unit that can act as a model for inefficient units.
24	**Homogeneous**	It refers to the degree of similarity between DMUs.
25	**Increasing Returns to Scale**	Happens when a unit's input results more than proportionate output.

26	**Inefficient Unit**	A unit that is not on the frontier.
27	**Inputs**	A resource used by a unit to produce outputs.
28	**Input Minimization**	When the analysis tries to minimize the amount of inputs used to produce specified outputs.
29	**Input Oriented**	It indicates that an inefficient unit may be made efficient by reducing the proportions of its
		inputs but keeping the output proportins constant.
30	**Input/Output mix**	It refers to the relative proportion of a unit's inputs and outputs.
	Most productive scale size(MPSS)	MPSS of an efficient unit refers to the point on the efficient frontier at which maximum
		average productivity is achieved for a given input/output mix.
31	**Multiplier Form**	The primal formulation is called the multiplier form.
32	**Ordinal Variable**	Aspecial type of categorical variable where the factor takes on a predefined set of values
		ranked in a specific order.
33	**Outlier**	It is unit whose input/output mix differs significantly from the other units in DEA analysis.
34	**Output**	The products that are produced from the processing of resources.
35	**Output Maximization**	DEA analysis that tries to maximize outputs for a fixed amount of inputs.
36	**Output Oriented**	An inefficient unit is made efficient by increasing proportion of its outputs while keeping
		the input proportion constant.
37	**Peer Group**	Another name for reference set.
38	**Primal Model(CCR)**	This is the original model published in the seminal paper by Charnes,Cooper Rhodes.It allows
		a set of optimal weights to be calculated for each variable to maximize a unit's efficiency score.
39	**Production Function**	It describes the optimal relationship between inputs and outputs.
40	**Productive Efficiency.(Efficiency)**	It is measure of unit's ability to produce outputs from a given set of inputs.
41	**Productivity**	It is the ratio of output to input of a unit.
42	**Radial Measure**	Unit's efficiency is measured by the ratio of the distance from the origin to the inefficient unit.
43	**Ratio models**	Both BCC and CCR models are called ratio modelsbecause they define efficiency as the ratio
		of weighted outputs divided by weighted inputs.
44	**Reference Set**	The reference set of an inefficient unit is the set of efficient units to which the inefficient unit
		has been most directly compared when calculating its efficiency rating.
45	**Results**	After analysis DEA model produces for each unit an efficiency score,virtual multipliers,intensity
		factors,dual weights and slacks.Virtual inputs,outputs,reference sets and improvement targets
		are calculated.
46	**Scale Efficiency**	A unit is calledscale efficient when its size of operation is optimal.Itis obtained by dividing the
		aggregate efficiency from CCR model by technical efficiency from BCC model.
47	**Slacks**	Slack represents under production of output or over use of input.It represents the improvements
		needed to make an inefficient unit to become efficient.

48	**Targets**	The value of inouts and outputs which would result in an inefficient unit becoming
		efficient unit.
49	**Technical Efficiency**	A unit is technically efficient if it maximizes output per unit of input used.
50	**Uncontrolled(exogeneously fixed)**	An uncontrollable unit is one over which the management does not have control.
	inputs/outputs.	
51	**Unit**	It is a short form for DMU.
52	**Variable**	Variables are the inputsand outputs identified as being of particular importance to operation of
		units.
53	**Variable Return to scale**	An increase in a unit's input does not result in a proportioonal change in its outputs, then the
		unit exhibits variable return to scale.
54	**Virtual Input/output**	Virtual inputs are calculated by multiplying the value of the input with the corresponding optimal
		weight for the unit as given by the solution.
55	**Virtual Multipliers**	Another term used to describe weights.
56	**Weights**	Weights are the unknown in DEA that are calculated by solving the linear program.
57	**Window Analysis**	It is analysis ofefficiency changes over time

Chapter I: Introduction

1.1. Introduction:

As we move into the twenty-first century business organizations are facing enormous challenges to succeed in a competitive market. There is a need for companies to be responsive to customers' needs. Due to increased competition companies now need to offer a greater number of customized products and more flexible processes with a lean supply chain to reduce costs. Management needs real-time business performance information that is accurate to proactively respond to these challenges (Bititci, Mendibil, Turner & Garengo, 2013). Managers also need predictive measures that would signal the outcomes of changing market conditions (Nudurupati, Bititci, Kumar & Chan, 2011).

Despite the advances, research in performance measurement systems that are properly integrated, dynamic, accurate, accessible and visible to improve the efficiency of businesses is still not available (Nudurupati et al., 2011). Some, of the performance measurement systems, are static and historic and therefore are not responsive to changes in the business environment (Marchand&Raymond,2008).In some cases, the MIS support is inadequate and this results in the delay of data collection and consequently reports (Marr& Neely, 2002) or there is no support from senior management for performance measurement systems(Davenport, Harris& Morison,2010). Thus, there is a lack of fit between the business environment, strategy and performance measurement (Melnyk, Bititici, Platts, Tobias & Anderson, 2014).

Performance measurement systems have also to meet the challenges of the volatile business environment. Many of the existing performance management measures were developed based on the assumption that organizations operate in stable environments and therefore performance measurement in turbulent and dynamic business environment has not been explored (Bititci, Bourne, Cross, Nudurpati&Sang,2015).

The drop in international prices of crude and its consequent effect on the economies of several countries and companies is an example of present-day volatility in business. The heavy equipment dealership business has been greatly affected by downturn and volatility in oil prices both in Canada and worldwide. For example, the total revenue of Caterpillar, the leader in heavy equipment industry fell from $66 billion in 2012 to $47 billion in 2015(Caterpillar Inc. 2016). Therefore, there is a need for performance measurement in heavy equipment retail organization's business that meets the challenges of a very volatile business environment.

1.2. Performance Measurement in Heavy Equipment Dealerships:

1.2.1. Characteristics of Heavy Equipment Industry:

Heavy equipment refers to heavy-duty machines specially designed for executing construction work, most frequently involving earthwork operations. The main characteristics of the industry are they are capital intensive, with low volumes of production, competitive, cyclical, customized, highly engineered and high tech (ISO, 2016). One size does not fit all and therefore the industry must produce many products to suit different applications based on functions into excavation, lifting, earthmoving, mining, roads, transportation, forestry, railroad, agricultural and others. The consequence of this is the low unit volume of models/products manufactured. This means the production of heavy equipment does not run into millions like automobiles but in a few thousands of units.

The leading manufacturers of heavy equipment in the world include Caterpillar and Terex of USA, Volvo of Sweden, Komatsu and Hitachi of Japan, Liebherr of Switzerland, and

SANY, Liugong and Zoomlion of China. The largest manufacturers operate globally and virtually in every country. Caterpillar is an industry leader in the heavy equipment business with an overall market share of 40%. In New York Stock Exchange, Caterpillar is one of the companies that determine the Dow Jones Industrial average. The financial performance of Caterpillar over a twelve-year period is given below.

Table 1.1. CAT Financials 2004-2015

CAT FINANCIALS	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Revenue(Billion USD)	30.25	36.33	41.51	44.95	51.32	32.39	42.58	60.13	65.87	55.65	55.18	47.01
Operating Income(billionUSD)	2.73	3.78	4.92	4.92	4.44	0.57	3.96	7.15	8.57	5.62	5.32	3.25
Operating Margin%	9	10	11.9	10.9	8.7	1.8	9.3	11.9	13	10.1	9.7	6.9
Earnings Per Share	2.88	4.04	5.17	5.37	5.66	1.43	4.15	7.4	8.48	5.75	5.88	3.5

The changed market conditions in the oil and gas industry have had an adverse effect on the earnings of Caterpillar financials as indicated in the above table. It is clear from the above table of financials, that the heavy equipment industry is a cyclical industry with fluctuating revenues.

Canada ranks among the world's top machinery manufacturing companies. There are over 9000 establishments and a labor force of more than 170,000 workers in Canada's machinery and equipment industry. It recorded sales of goods manufactured of nearly $45 billion and exports accounted for more than 60 percent of all sales in 2014 (Canadian Construction Association, 2014). Capital expenditures for machinery and equipment for agriculture, mining, oil and gas, construction, manufacturing and transportation and warehousing for the year 2014 alone accounted for $58.11 billion dollars. This has grown from $48.98 billion in 2010 a growth of 18.43%. Of this, the distribution of machinery and equipment account for $9.5billion in 2014.The Canadian services sector such as retail, transport, distribution, food services, professional services as well as other service-dominated businesses comprising of heavy equipment retailing organization is an important part of the Canadian Economy, representing seventy percent of Canada's gross domestic product (GDP) and employing three out of four Canadians (Canadian Chamber of Commerce, 2013**).** Therefore, heavy equipment retailing organization being part of the service industry is a substantial contributor to the Canadian service economy. The approximate size of the heavy equipment retailing industry in Canada would be in the range of 20-25 billion CAD and would be approximately 1.25% of the total Canadian GDP (Statistics Canada).

Having understood the heavy equipment, its characteristics, applications and the cyclical nature of the industry, will now examine how this heavy equipment is distributed.

1.2.2. Introduction to Heavy Equipment Retailing Organizations:

All the leading manufacturers of machinery and equipment sell and distribute their products in the market place through an intermediary called dealers or distributors. The dealers and distributors act as a bridge between the manufacturer and the consumer.

Heavy equipment dealerships are the real estate that the heavy equipment industry uses to sell its products. It is estimated that there are close to five hundred dealerships that are operating in Canada, as per the figures published by AED (Association of Equipment Distributors) the association of all heavy equipment dealers. (Association of equipment distributors. 2015).

Retail distribution of heavy equipment is done through a network of independent dealers. All these dealers operate with a business plan and recognize that the sale of equipment generates demand for auxiliary services such as leasing, financing, parts, and repairs. Dealers receive exclusive franchises for specific trade areas and act as a representative of the manufacturer to the equipment buying companies and individual operators (Carter, 2015).

The manufacturers and dealers are in effect partners and have a long-term business relationship on a principal to principal basis based on a legal contract (Lafontaine & Morton, 2010). Many dealerships market more than one brand. This helps them to cover the risk associated with changes in the market conditions and economy. Since there are always changes in the business environment, heavy equipment dealers offer products to suit different market segments that will help stabilize cash flows like investors who diversify their portfolios. Dealers seek this diversity either in a single location by selling the different brands under a single roof, or through opening multiple locations within the same market. All these dealerships have premises that are owned or leased based on the capital structure of the company (Carter, 2015). The departments of a heavy equipment dealership are based on their functions and each of these function acts as a profit center (Carter, 2015). The departments are New Equipment Sales, Used equipment sales, Finance and Insurance, Service operations and Parts operations.

1.2.3: Structure of Heavy Equipment Retailing Organizations:

New equipment sales are typically the primary as well as the most obvious function of the heavy equipment dealerships. In all the dealerships new equipment sales are viewed as the main source that generates and sustains demand for the other components or functions. New equipment sales contribute 62% of the total sales in a dealership in an average heavy equipment dealership (Association of equipment distributors, 2015).

Used equipment market is very large and is close to the size of the new equipment market (Carter, 2015). Used equipment has obvious appeal in economic downturns because of their lower prices and has a stabilizing influence on the income stream for the whole dealership (Association of equipment distributors, 2015).

Service operations offer after sales service to machines and look after both warranty obligations and after-sales service of machines. Service operations generate revenue from warranty work which is paid by the manufacturer, as well as the repair work and maintenance paid by the customer. In terms of revenue, the service operations may be less but account for a larger share of profit (Association of equipment distributors, 2015).

Parts are sold to upkeep the maintenance and repair of the machines. Parts are sold through the workshop and to customers across the counter. Some of the more sophisticated dealerships enable the purchase of parts through online by means of the web portal. The parts revenue may be less in relation to equipment sales but in terms of profit accounts for a sizeable share. Both parts and service operations are combined in some dealerships and called fixed operations. Parts operations contribute to approximately twenty-six percentage of total revenue (Association of equipment distributors, 2015).

Table 1.2. Department wise business and contribution in Heavy Equipment Dealership

Departments in a Dealership	**Size of total business in %**	**Gross Margin Contribution in %**
Sales	65-70%	0-8%
Service	8-10%	55-70%
Parts	20-25%	20-40%

There is an opportunity for the dealers to add revenue by offering finance options either through the finance arm of the manufacturer or through a tie-up with a third-party financier. Following are some of the major Heavy Equipment retailing organizations in Canada.

1.2.4. Current methods of Measurement of Performance in Heavy Equipment Retailing Organizations:

The current economic scenario in Canada has also affected the heavy equipment dealerships that support the infrastructure, oil and gas industry. Measuring performance and efficiency is important in a stagnant economy as it helps to survive and remain competitive. The need for greater efficiency that leads to profitable operations is one of the key issues for survival in the future.

Table 1.3. Major Heavy Equipment Retailing Organizations in Canada.

Major Heavy Equipment Dealerships in Canada		
Name of the Dealer	**Market Capitalization**	**Product Represented**
Cervus Equipment(www.cervusequipment.com)	$190 Million	John Deere,JCB
Finning International (www.finning.ca)	$4.4 Billion	Caterpillar
Ritchie Brothers (www.rbauction.com)	$2.1 Billion	Auctioneers
Rockymountain Equipment(www.rockymtn.com)	$156 Million	Case
Toromont Industries(www.toromont.com)	$2.2Billion	Caterpillar
Wajax Industries (www.wajax.com)	$597 Million	Hitachi,Hyster
Hewitt Equipment(www.hewitt.ca)	Not Available	Caterpillar
Strongco Corp(www.strongco.com	$125 Million	Volvo,Case

Measures of heavy equipment dealership performance use a certain form of output relative to input that quantifies various aspects of dealership operations. The Association of Equipment Distributors (AED), established in 1919 is an international trade association based in Schaumburg IL, USA and is the primary source of publishing performance indicators of participating AED firms annually in their survey.

The performance indicators are published annually through a publication called Cost of Doing Business. More than eight hundred dealership organizations are represented by the AED. AED serves the needs of dealers, manufacturers and service providers. It provides advocacy, benchmarking, networking and professional development to members across the industries. Cost of Doing Business report gives the most up-to-date comparative financial performance data and provides information that enables dealers to evaluate their operating results. The Cost of Doing Business Report /Profit Opportunity Report is done annually and serves as a reference for dealers to evaluate their own company's operating results and pinpoint strengths and weaknesses to identify areas of opportunities and acts as a representative sample of the way performance is measured in the heavy equipment dealerships.

The report has the following data.1) Comparison to evaluate operating ratios against the average of other distributors. 2) Balance Sheet and income statement performance. 3) Distributor performance by sales volume. 4) Employee performance measures. 5) Sales mix of high-performance dealers. 6) Gross margin for new and used equipment, rentals, parts and service departments.7) Operating ratios including debt to net worth. 8) Trend analysis year by year 9) Margin Management expenses. AED uses the above performance measures to find out the most successful firm.

Further, it is found that the key performance measures as reported by the largest Caterpillar dealer in the world, located in Canada, is based on the company's plans to build shareholder value by improving return on invested capital (ROIC). Management of the largest Caterpillar dealer has identified that customer and market leadership, supply chain optimization, service excellence and asset utilization as key to improving the performance of the organization. The management has also indicated that these operational priorities are directly linked to improving EBIT (earnings before interest and tax), performance and capital efficiency. The metrics earnings before interest and tax, invested capital, inventory, inventory turns, invested capital turnover, working capital to sales ratio, free cash flow, net debt to invested capital and net debt to EBITDA ratio are also used by the dealers to track company's progress in improving return on invested capital. Similar performance measures are adopted by other heavy equipment dealerships of John Deere, Hitachi, Kobelco, to name a few.

In all the above dealerships the revenues are tracked in each of the departments, new equipment sales, used equipment sales, parts, service, rentals, and finance at each branch level and collated at the national level resulting in financial measures that are used to measure performance. The critical profit variables that are measured are sales per employee, gross margin percentage, operating expenses percentage, inventory turnover (times), average collection period (days), rental fleet utilization and absorption factor. Absorption factor is the expenses covered by parts and service operations.

The sales department measures its performance by the number of new equipment sold, a number of attachments sold, and the number of used equipment sold, and the consequent margins earned by the respective sales. Similarly, in rentals, gross margin earned by number of rentals sold is measured. The parts department measures the performance by the gross margin generated by way of sales to customers, sales to internal customers, sales to warranty, freight and returns logistics. The service department measures service cost of sales by labor hours to customer pay sales, internal sales, and warranty sales, sublet repairs cost and service truck earnings.

Asset productivity is measured in the average collection period (days) and inventory turnover (times). Employee, productivity is measured in terms of sales per employee, gross margin per employee, salary per employee and payroll per employee. Parts employee productivity is measured in terms of parts sales per parts employee, service productivity is measured in terms of service sales per technician and similar productivity is measured for new equipment sales, used equipment sales and rental sales.

The profitability of heavy equipment dealership business has dropped steeply due to changed market conditions in the oil and gas industry and general economic conditions. To maintain profitability, the dealers need to constantly evaluate the performance of the individual operations within the organization. The above method of performance measure does not measure the relative efficiencies of real operating units that provide benchmarking opportunities to improve efficiency as they focus on only financial measures.

All the heavy equipment dealerships use an enterprise resource planning system (information technology) to manage their business. These systems sometimes are recommended by the manufacturer and sometimes the choice is made by the individual dealer management. They are normally referred to as Dealer Management System (DMS). The DMS has separate modules for each of the departments in the dealership, new equipment, used equipment, parts, service, rentals and finance operations.

All these DMS generate KPI (Key performance indicators) reports that measure the performance metrics of individual departments. Invariably all these metrics are focused on financial ratios. Some manufacturers like Caterpillar have specialized training for their dealers on performance measurement using their Dealer Management Simulation program (DMS). These programs focus on financial ratios like return on assets, increasing cash flow, managing receivables, inventory turnover, return on investments and internal rate of return (Caterpillar Inc, 2016). Some of the dealerships find that their DMS do not meet reporting requirements and add a bolt-on system that will enhance multidimensional reporting and enable the dealers to measure and analyze data.

There are also dealers who use a separate customer relationship management (CRM) system such as MS-CRM to deal with all customers facing business process and measure their performance. CRM measures such as performance measures as a percentage of deals in progress, deals that are cold, warm and hot and those that have potential to be converted to sales. CRM also helps to track the historical purchases of customers and helps you plan a strategy to improve sales. There are certain manufacturers that have a separate vendor managed inventory system to manage the parts inventory of the dealerships. Such systems work in parallel to the DMS and help in managing the parts inventory and help in measuring performance metrics related inventory such as obsolete stock, service level to customers, order fill rate to name a few.

The manufacturers of heavy equipment also measure the performance of the dealerships on a regular basis. This is because in the retailing business it is the dealer who adds value to the product of the manufacturer.

There is a manufacturer (not named for confidential reasons) who measures on a quarterly basis two metrics namely purchase score and ownership score of the dealers. Purchase score measures the customers' satisfaction with initial purchase experience and ownership report measures the customers' satisfaction with the overall product performance and service of their equipment. This is done by sending a questionnaire after the completion of each sale to the customer and the customer is asked to rate dealer attributes, delivery attributes, and comparison against the competition. Customer satisfaction with the dealer is determined with the above scores. Such scores are communicated to the dealer to indicate their performance. The dealers take this feedback and improve their performance where necessary.

There is a manufacturer in the heavy equipment industry (name withheld for confidentiality) who has a programme that identifies key Customer Support metrics and performance standards by which the manufacturer measures the Customer Support efforts of the dealer. The dealer will utilize these results as the basis for setting annual customer support business goals and actions. This programme also provides an opportunity for dealers to measure and improve their own performance against the goals set by the manufacturer. The programme is centered on eighteen criteria in three areas of customer satisfaction, customer support competency, and customer support business. Based on these performance scores the dealers are asked to review their business plan focussing on customer support goals, objectives and action plans and make an adjustment to the existing goals based on current business conditions.

There are also dealers who send a questionnaire to customers after each sale and after each major repair to measure customer satisfaction index (CSI). The dealers want to see a very high CSI (customer satisfaction index) score.

Manufacturers want their dealers to constantly improve their business year after year as the manufacturer's business fortunes are directly related to the performance of the dealer. In a recently held meeting between Caterpillar and its dealers named "Across the table", Caterpillar informed the dealers that their performance was way below their expectations and that there is a disconnect between the way they operate and the way they are expected to operate. Caterpillar said that this has resulted in losses running to several billions of dollars as many of them are operating at 40% of their efficiency. Caterpillar has observed that applying the practices of the best-performing dealers with lower performing dealers automatically improved the aftermarket share of parts by 6 to 8%. Although these changes are possible the dealers have no method of learning the process. Caterpillar wants their customers to have the same customer experience when they deal with different dealers across the globe and their new slogan is "One CAT, One Experience" (Trade Journal, 2016).

These are the various metrics used by the heavy equipment dealerships to measure performance that is operating in a cyclical and competitive industry. In the business, the demand by the manufacturer from the dealers is ever increasing as every business lost by the dealer is business lost by the manufacturer. Therefore, the fortunes of the manufacturer and dealer are tied together by the common thread of performance. The evaluation of performance will constantly keep on changing based on the demand of the manufacturer and the demand of the business environment. Therefore, there is a compelling reason to use contemporary performance measurement tools that can meet the demand of changing business environment.

1.3. Statement of the problem and Motivation:

As discussed above many dealerships focus on financial factors to measure their performance using their DMS. In addition, dealers also use systems either in parallel or as a bolt-on to measure metrics that cannot be measured by their DMS. The dealers are constantly monitored for their performance by the manufacturers to find out areas of improvement. Although there are several methods adapted to measure performance, there is a lack of integrated performance measure that can identify the best-performing dealers and the worst performing dealers as observed by Caterpillar.

Dealerships have multiple branches and customers interact with all these branches for business. The customer expects to have the same experience across all these branches and therefore all these branches should perform to the same expected level. These networks are homogeneous in nature each having multiple inputs and outputs and have environmental factors that have an impact on its performance. It is often necessary to compare the performances of these branches for example to 1) evaluate management performance 2) identify best practices 3) determine whether some loss-making branches can be made profitable. These branches have diverse types of inputs (number of employees, expenses, number of service bays) and outputs (sale of machines, the sale of parts, labor earnings) and therefore it is difficult to compare them. An analysis of the performance of the branches and comparing the performance with other branches leads us to understand the relative efficiency of the network of operating units. Such a study of relative efficiency will reveal the characteristics of the operations and methods that can be employed to improve the performance of inefficient units (Av Kiran, 2006).

In competitive markets such as heavy equipment, there is a need for efficient utilization of resources. Therefore, the study of relative efficiency is essential for identifying the inefficiencies in the utilization of scarce resources, determining the potential improvements and the long-run survival of the organizations that have multiple units in its network of operations.

In his study on productivity Farrell,1957 the pioneer in the measurement of productive efficiency observed *"The problem of measuring the productive efficiency of an industry is important to both the economic theorist and the economic policy maker. If the theoretical arguments as to the relative efficiency of different economic systems are to be subjected to empirical testing, it is essential to be able to make some actual measurements of efficiency." Equally, if economic planning is to concern itself with particular industries it is important to know how far a given industry can be expected to increase its output by simply increasing its efficiency, without absorbing further resources. This measure is quite general and applicable to any productive organization from a workshop to a whole economy" (Farrell, 1957, p253).*

With such multiple measures of performance involving a network of homogeneous units, there is a need for a different approach to take business decisions that involve relative efficiency. One of the advantages of relative efficiency analysis is that it helps to identify the efficient utilization of resources that can produce the desired outcomes. Effectively this efficient utilization of resources becomes a focus for benchmarking activities (Zhu, 2014).

The method that uses multiple inputs and outputs and measures relative efficiency is Data Envelopment Analysis (DEA). This method will fill the shortcomings in the efficiency measures that are currently used in the heavy equipment retailing organization. DEA is an optimization technique that is used to assess the relative efficiency of homogeneous organizational units called decision-making units(DMU). Since its first application to banking sector by Sherman & Gold, in 1985, DEA has also been widely used in measuring efficiency where there are many branch operations such as banks, hospitals, schools, agriculture and farm, and transportation.

In this research Data Envelopment Analysis (DEA) will be used as a tool to measure the performance of heavy equipment retailing organization as it has the ability (will be addressed in Ch3, theoretical framework) to offer a solution in terms of relative efficiency and benchmarking.

1.4. Purpose of the Study:

The purpose of the study is to develop models and methods that are more appropriate and suitable to measure the performance and relative efficiency of heavy equipment retailing organizations than the existing conventional methods of ratio analysis.

The motivation behind the study is to overcome the shortcomings in the approaches that are currently used to measure the performance of heavy equipment dealers. As discussed above the performance measures used were mostly financial in nature.

To assess performance, most of the heavy equipment dealers have used various ratio analyses as performance indicators. However, these methods only show the performance of the dealer as a single unit and do not give an indication about the performance of individual branches that comprise the dealership as an entity. In case of dealerships that have multiple branches, some of these branches may be more efficient than other and this cannot be identified purely based on financial ratios. Relative efficiency will help in identifying the efficient unit and utilize this as a benchmark for improving the performance of other inefficient units. Relative efficiency also helps us to understand the current performance and how it can be improved if it is inefficient. The difficulties of measuring and comparing performances under the above circumstances are faced by business in the real world. The simple ratios based on single input and single output give a limited picture of performance. Data Envelopment Analysis has advantages as it addresses some of the above shortcomings in performance measurement using single input and single output ratios (Harrisson, 2010; Singh, Motwani &Kumar, 2000).

1.5. Research Aim and Objectives:

The changing business environment that is uncertain and volatile is forcing organizations to take strategic decisions for survival and growth. A performance measurement system that will help organizations to know relative efficiency of its branches in the changing business environment and take appropriate decisions to improve profitability is the need of the hour. As observed above such systems are lacking in heavy equipment retailing organizations. Therefore, the following are the research objectives in view of the existing limitations in current methods of measuring performance in heavy equipment retailing organizations.

a) How can the efficiency of the departments in a heavy equipment retailing organization be measured?
b) How do you compare the efficiency of the branches of the heavy equipment retailing organization under study?
c) What conditions may account for the differences in the efficiency of the branches?
d) What factors or constraints create varying scores amongst inefficient branches?
e) Is there any change in efficiency over time and if so how it can be measured?
f) Can the efficiency of the branch have relation to the efficiency of the individual department?

The purpose of the study is to develop models and methods that are more appropriate and suitable to measure the performance and relative efficiency of heavy equipment dealers than the existing conventional methods. This method facilitates comparison of performance across branches in the same dealership and explains why some branches achieve superior efficiency in performance. This study has the potential to be extended to the manufacturers who can review the relative performance of their dealers. The important task would be to identify factors that relate to efficiency that are likely to be determinants and measure the extent to which they contribute to the existence of inefficiency.

Based on the shortcomings of the existing performance measurement system in heavy equipment retailing organizations, there is a need for measuring performance using a contemporary method. Therefore, the aim of the research is to develop a model using Data Envelopment analysis to measure performance in heavy equipment retailing organization. The strengths of DEA have the potential to meet the many shortcomings that are seen in the existing system.

The thesis will address the above research objectives and the research aim by way of research questions that will be discussed in detail in Chapter 4, methodology.

1.6 Significance of the Study:

The thesis tries to fill a gap in the literature of applications of DEA as a performance measurement tool by extending it to heavy equipment retailing organizations. The study also introduces the concept of relative efficiency using multiple inputs and multiple outputs to the heavy equipment retailing organizations. The study will be the outcome of the study of the efficiency of thirty-three business unit of a heavy equipment dealer in Canada. DEA has been applied to many industries such as healthcare, agriculture and farming, education, banking and transportation to mention a few but has not been applied to heavy equipment retailing organization despite its crucial impact on the Canadian economy.

The overall objective of the research is to provide practitioners with a new, practical and robust methodology to measure performance in heavy equipment dealerships using DEA that will also help in measuring performance variations over time. The insights thus gained can be used to enhance the performance of the organization and improve its sustainability and profitability in a fast-changing business environment. The benefits of the study are:

- Compares service units and identify the most efficient units.
- It identifies the amount of savings that can be achieved to make inefficient units efficient.
- Identification of specific changes in inefficient units.
- Helps in receiving information on management of efficient units.

The study can also be used by manufacturers to assess the relative efficiency of its dealerships within a country and even across countries and help the manufacturers to use as a benchmarking tool in measuring efficiency. The application can also be extended to automotive dealerships where there is a chain of dealerships under a single ownership to measure relative efficiency. The study can also be used by an industry body like Association of Equipment Distributors (AED) as a tool to measure performance and relative efficiency.

Finally, the main findings can integrate with the existing ERP systems used by dealerships and yield relative efficiency scores on demand. In other words, this tool has potential to be used as a KPI in the management and control of the organization. Most importantly this study introduces the concept of using multiple inputs and outputs to measure efficiency in the heavy equipment dealerships and thereby improve profitability in a highly competitive industry and cyclical industry.

Two approaches to study performance measures will be used in this thesis. One approach used is the black box approach where the internal structure of the DMU (decision-making unit) is not considered and another approach is by using the extended DEA model called Network DEA (NDEA) proposed by Fare & Grosskopf, 2000, where the internal structure is also considered. The NDEA, approach helps in finding the efficiency of the internal structure of the DMU. The DMU (business unit) in turn has sub DMUs by way of three divisions, sales, service and parts operations each of them operating independently with a parallel structure. Such an in-depth analysis of the structure of DMU reveals the elements of inefficiency in the sub-DMUs and helps in setting targets to improve efficiency.

The study also provides an in-depth understanding of technical efficiency, pure technical efficiency and scale efficiency. The study provides a method to rank the efficient units that help in benchmarking. The window analysis and Malmquist productivity index help in understanding the efficiency change over time. Measurement of efficiency using Network approach will help in comparison of efficiencies under both Black box and Network approach.

The findings of this thesis are of excellent value to practitioners at the management level, policymakers at the manufacturer level and the community of dealers and to the academic community.

1.7: Thesis Structure:

The thesis is organized into seven chapters. **Chapter 1** provides a background information to heavy equipment industry, current performance measurement in heavy equipment retailing organizations, statement of the problem, the purpose of the study, research aim and objectives and significance of the study. **Chapter 2** provides a literature review of the DEA applications in various fields with a focus on retail organizations (automotive and retail sector) and identifies the inputs and outputs used in such studies that can be used in the research. **Chapter 3** gives the details of the theoretical framework of DEA that is used in the research with specific reference to the various extended models to be used in the research.

Chapter 4 is about the research methodology. In this chapter, the research questions that address the research objectives and aim are restated. All the DEA models that are needed are listed in the architecture of the study. The chapter also covers data collection, selection of DMUs and selection of factors for the research from the various available factors both for black box and network DEA approach. This chapter also lists the models that would be used in the research. **Chapter 5** is about developing four DEA models to measure the efficiency of heavy equipment retailing organizations and analyze the four models in greater detail. **Chapter 6** is on discussing the results, limitations and future course of research. **Chapter 7** is on conclusions and recommendations describing model PEDMAS as a performance measurement system for heavy equipment retailing organizations along with summary of findings.

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Chapter II: Literature Review

2.0: Introduction:

The basic nature of the current research is one of an application of an existing theory and model to measure the performance of heavy equipment retailing organizations. As such the literature review will not analyze and critique the latest developments in the theoretical aspects of Data Envelopment Analysis. However, in the application of DEA, there have been improvements and extensions that have been made to the original methodology of the technique and this will be analyzed. DEA as a tool has been used to measure efficiency in healthcare, banking, transportation, education, agriculture, hotels and numerous other applications. However, DEA has not been used to measure performance in heavy equipment retailing business. The closest application of DEA is to automotive dealerships and therefore application to the automotive industry will be critically reviewed after establishing similarity between automotive and heavy equipment dealerships. DEA has also been used to analyze efficiency in other retail sectors such as grocery stores, supermarkets, apparels etc. A literature review of such applications will also be made to broaden the review of the literature on efficiency measurement in the retailing industry as there is no literature on efficiency measurement in heavy equipment retailing organizations.

Therefore, I would be reviewing the literature from two perspectives. The first perspective will be to review the recent research papers on the application of DEA in the automotive industry as it has similarity to the heavy equipment industry. The second perspective will be to review research papers on efficiency measurement in other retail industry such as grocery stores, supermarkets, apparels, wine stores etc. to understand the methodology used and the various inputs and outputs used for the study. While reviewing the papers special emphasis will be on reviewing the current research papers in the automotive industry and retail industry post year 2010 to make the literature review most current.

2.1: Applications of DEA:

Since the advent of DEA in 1978, numerous papers have been published on extending the basic methodology of DEA and DEA has found applications both in public and private sector. In every application, DEA evaluates the relative efficiency of a decision -making unit within a peer group and sets targets for improving the efficiency of inefficient units. The application domain covers a wide array of industries like the banking industry, healthcare industry, agriculture industry, transportation industry, educational institutions to mention a few. It is found that these industries use DEA for multiple reasons such as to identify sources of inefficiency, rank the DMUs, evaluate the management, (supply chain management, human resource management, technology management etc.), evaluate the effectiveness of programs or policies and create data for reutilizing resources. DEA has found very wide applications in the real world. According to Gattoufi et al., (2004) of the total papers published 67% of the papers presented were on real-world applications. According to Emrouznejad et al., (2008) banking, education (including higher education), healthcare and hospital efficiency were the areas where DEA found maximum use.

According to Gattoufi, Oral, Kumar & Reisman (2004), research in DEA is basically of three types. The first is purely methodological, second is application centered and the third is a mix of theory together with empirical data. The first type focuses on mathematics and models but does not relate to empirical data (although simulated data is used to test theory occasionally). The second type is application oriented, where DEA is applied to real-world problems and the focus is mainly on the application. In between the above two lay the third type that is a mix of theory together with empirical data. This type proposes a methodological innovation and then validates or tests the proposed method with a set of empirical data. Most of the published papers fall into one of the above three categories.

As of 2013, of the total 4936 research papers published on DEA, purely methodological papers account for 36.5%(1802) and 3134 application embedded papers account for 63.5% (Liu, Lu, Lu &Lin, 2013). In other words, approximately one-third of the paper is purely-methodological whereas two third of the paper is application oriented. Please note that application oriented are those that use real-world data whereas purely methodological do not use real-world data. It is interesting to note that during the first 20 years since the arrival of DEA, the number of methodological papers published were more than application-oriented papers. It is only after 1999 the number of application-oriented papers exceeded the purely methodological papers (Liu et al., 2013). In short, DEA has gained acceptance as an application tool for efficiency measurement in the real world.

The table 2.1 below lists in order the application embedded papers. As you can see from the list banking ranks number one in application followed by healthcare, agriculture, and farm, transportation and education. Although these five applications accounts, for 41% of all application-oriented papers, DEA has been widely applied across many industries except heavy equipment retailing business. In their study Liu et al., (2013) suggest that need for performance measurement, data accessibility and support from application journals as the reason for using DEA in the above industries that account for most of applications. The table also indicates the wide array of industries where DEA has found applications. The last two columns in the table below indicate the number of papers published as a percentage for the top 10 applications. The five most researched applications of DEA in 2015 and 2016 are agriculture, banking, supply chain, transportation and public policy (Emrouznejad & Yang, 2017).

Table 2.1. Number of papers published in DEA applications (Source Liu et al., 2013)

	Number of Papers Published in DEA Applications				
Item #	Real World Applications	Total Number of Papers	Percentage	Number of papers 2005 to2009	Fraction of total papers (%)
1	Banking	323	10.31	147	45.5
2	Health Care	271	8.65	107	39.5
3	Agriculture and Farm	258	8.23	140	54.3
4	Transportation	249	7.95	131	52.6
5	Education	184	5.87	75	40.8
6	Power	156	4.98	87	55.8
7	Manufacturing	146	4.66	75	51.4
8	Energy& Environment	109	3.48	75	68.8
9	Communication	70	2.23	28	40
10	Finance	51	1.63	33	6.47
11	Insurance	44	1.4		
12	Tourism	42	1.34		
13	Petroleum	41	1.31		
14	Fishery	39	1.24		
15	Sports	31	0.99		
16	Construction	29	0.93		
17	Automobile	28	0.89		
18	Retailing	28	0.89		
19	Forestry	27	0.86		
20	Water	27	0.86		
21	Real Estate	25	0.8		
22	Software	25	0.8		
23	E-Business	22	0.7		
24	Mining	22	0.7		
25	Miscellaneous	351	11.2		
26	Disciplines	536	17.1		
26	Disciplines	536	17.1		

From 1978 since the finding of DEA as a tool to measure efficiency till today spanning forty years, DEA has expanded not only as a tool for educational research for which it was originally intended, but has found application in various fields of economics, social sciences, engineering and different types of industries covering both public and private sector, but never used in the heavy equipment industry and heavy equipment dealerships. By using DEA as a tool to measure efficiency the heavy equipment industry and dealerships will be benefitted as much as other industries have benefitted by using DEA as a performance measurement tool.

A cross examination of various approaches reveals that researchers start with basic DEA models and find efficiency of the whole unit and this approach is termed as black box approach. Once these approaches are explored researchers graduate to the extended models of DEA and go up to three stage analysis. The efficiency study where the internal structure of DMU is analyzed is called Network DEA (Fare &Grosskopf,2000). Network DEA model is being widely used to understand the internal structure of DMU by opening the black box and to understand how the internal structure affects efficiency.

The next section provides a detailed literature review of application of Data Envelopment Analysis to heavy equipment retailing organization.

2.2: Application of DEA to Heavy Equipment Retailing Organizations:

A search in various electronic research database such as EBSCO, Elsevier's Science Direct, European journal of operations research, DEA conference journals, Omega to mention a few, for research papers over the last ten years with key words on performance measurement in heavy equipment dealerships/ retail organizations, using DEA yielded the lone paper "Construction machinery dealer's benchmarking for efficiency measurement using data envelopment analysis" by Edmar De Paula, Paulo Henrique Oliveira under the supervision of Prof. Ana Lucia Miranda Lopez of CPEAD,UFMG Brazil. It was found that this research was never carried out. This leaves with virtually no literature on performance measurement in heavy equipment dealerships using DEA.

However, DEA has been used to measure performance in automobile dealerships and automobile industry. There are a lot of similarities between the functioning of automobile dealerships and heavy equipment dealerships and hence an attempt is made to critically analyze the various research carried out in performance measurement of automobile dealerships and automotive industry using DEA.

2.3: Similarity between Automobile dealers and Heavy Equipment Dealers:

In the following paragraph the similarities and dissimilarities between automobile and heavy equipment dealerships are explored.

Automobiles are classified as consumer durable goods and heavy equipment are classified as industrial goods. Consumer durable goods are owned and purchased for personal needs whereas industrial goods purchases are for commercial use to make profit. However, both these goods are marketed through an intermediary distribution channel called dealers or distributors. An industrial distributor performs a variety of marketing channel functions, including selling, stocking, delivering a full product assortment, financing and post sales service much like the same way as the dealer of consumer durable goods. In many ways industrial distributors are like dealers in consumer durable goods channels (Kerin & Hartley, 2015). When this interpretation is applied to the real world there is similarity between automobile (consumer durable goods) dealers and heavy equipment (industrial goods) dealers.

Major automobile manufacturers like GM, Ford, Toyota sell their cars through intermediaries called dealers who sell new cars, used cars, sell parts for the new cars, render services for maintenance of the cars and extend credit for purchase of both new cars and used cars to consumers (Halweg, Luo, & Oliver, 2009).

Similarly, large heavy equipment manufacturers like Caterpillar, Komatsu, John Deere, CASE Holland, Volvo sell their equipment through a network of dealers who are independently owned businesses with exclusive geographical territories. Dealers provide sales, parts, maintenance and repair services, rental equipment, used equipment and financing (Matsumoto, 2011).

Thus, the dealership operations in both automobiles (consumer goods) and heavy equipment (industrial goods) are very similar and therefore performance measurements methodology used in automobile dealerships using DEA can be used in heavy equipment dealerships. We will now critically review how DEA has been applied to measure performance in the automotive industry.

2.4 DEA Applications in Automotive Industry:

Table 2.2 lists twenty-six different studies carried out in automobile industry and dealerships using Data Envelopment Analysis.

A critical review of thirteen recent research papers in the automotive industry and another thirteen papers in the automotive dealership environment reveals that DEA has been applied to measure efficiency in such diverse automotive industry segments such as in car manufacturing, ancillary manufacturers, parts manufacturers and automobile dealerships. In these segments DEA has been used in car marketing to study branding and product positioning, to find strategic partner in auto industry, study of automotive stocks, to find out efficiency in automotive ancillary manufacturing that led to automation, measure relationship between market share and efficiency of dealership and measure efficiency of service networks. Such wide application of DEA in the automotive industry has greatly benefitted it in improving efficiency. Similar application of DEA to heavy equipment industry will greatly benefit the various segments of heavy equipment industry such as heavy equipment dealerships, its ancillaries and the industry itself much like the same way auto industry benefitted.

2.5 DEA Applications in other Retail Sectors:

There have been many studies conducted by researchers using DEA in retail sectors such as departmental stores, grocery stores, restaurant chain, fast food chain, super market to mention a few. A review of these studies also has been made to understand the methodology of DEA application and the factors used in such studies and to compare if they have any similarity to studies in automotive industry. Therefore, these studies can be used as a reference to study the efficiency in heavy equipment retailing organizations.

One of the earliest studies on efficiency of retail stores using DEA was by Athanassapoulos in 1995, who studied the efficiency of 31 restaurants in U.K. In 1998, Donthu and Yoo, have analyzed the efficiency of 24 outlets of fast food restaurant chain in USA using DEA. Some of these studies by Donthu & Yoo,1998; Thomas et al.,1998, Keh & Chu,2003; Barros & Alves,2003; have evaluated technical efficiency and studies by Keh & Chu,2003; and Barros & Alves,2003, have studied scale efficiency. Most of these studies by the above researchers have adopted a static perspective whereas Barros and Alves,2004; Sellers-Rubio and Mas-Ruiz (2006) studied change in efficiency over time using Malmquist Productivity index(MPI) to know the dynamic aspects of efficiency. For example, Sellers-Rubio and Mas-Ruiz (2006) have used DEA to study the efficiency change over time using MPI on a sample of 96 super market chains operating in Spain in the period 1995-2003.They estimated total productivity change in these retail organizations and decomposed them in to efficiency change and technical change. Various other studies in the retail sector are listed in table 2.3 below.

2.6: Network DEA Applications:

Network DEA concepts, where the internal structure of black box is studied has been used in varied applications ranging from sports to education and the details are listed in table

2.2. In all the Network DEA application it is found that by knowing the internal structure of DMU, more is known about the transformation process. Therefore, by applying the principles of network DEA to heavy equipment dealerships more will be known about the factors that contributes to the efficiency measurement of its internal structure: sales, service and parts operations of a heavy equipment dealership.

2.7: Selection of input and Output variables in Retail Sector and Automobile Industry:

The successful application of DEA largely depends on the choice of input and output variables (Wu &Ramanathan,2008). It was stated by Donthu & Yoo (1998) that objectives of the sales organization should be known from the choice of input and output variables. Table 2.3 summarizes the various inputs and outputs criteria that were used in the examination of retail efficiency and productivity as found in the literature. From the table it can be seen that some authors (Thomas et al., 1998, ; Donthu and Yoo, 1998; Keh and Chu,2003) have used measures of output in monetary units like sales revenue, profit volume and value added. Some authors (Donthu and Yoo, 1998; Keh and Chu,2003) have used non-monetary units such as customer store loyalty, customer satisfaction and service quality.

Two kinds of inputs are found in the literature on productivity measurement in retail sector. One is the controllable inputs and the other is non-controllable inputs based on whether the organization considers them in its management action plans or not (Donthu and Yoo, 1998, Sellers-Rubio and Mas-Ruiz,2006). Organizations, have control over the controllable inputs to achieve competitive advantage and therefore it is a widespread practice to use them as inputs in efficiency measurement. Some of the controllable inputs are number of employees (Sellers-Rubio and Mas-Ruiz,2006, Thomas et al.,1998) area of facility (Pilling et al.,1995, Lusch and Serpkenci,1990) and number of outlets in supermarket chain (Sellers-Rubio and Mas-

Ruiz,2006), current total assets(Doutt,1984). At the same time non-controllable inputs are considered as environmental variables as they are beyond the control of companies but can influence the efficiency of the companies. Some of the non-controllable variables are location (Donthu and Yoo, 1998,), national economic development (Pilling et al.,1995), demographics of clientele in the area (Donthu and Yoo, 1998), Number of competition stores (Ko et al.,2017), square of number of competition stores (Ko et al.,2017). While calculating productivity non-controllable factors are ignored (Donthu and Yoo, 1998).

Similarly in the literature on DEA applications in automotive industry, that is listed in table2.2 some of the inputs used are cost of goods sold and selling and general expenses (Narasimhan et al.,2005, Wang and Wang,2016), Number of employees(Chen2011,Tran and Ngo,2014),Fixed assets(Wang and Wang,2016).Similarly the outputs found from the literature are gross income(Saranga,2009,net income(Narasimhan et al.,2005),sales revenue(Hour Ali,2009, Biondi et al., 2013) and Total number of customers(Lin, Lee and Chang,2011).

Network DEA(NDEA) approach has been used in such varied applications such as sports, education, manufacturing to mention a few. Rayeni and Saljooghi (2010) used NDEA to study efficiency and productivity in Universities in Iran. Monafred and Safi (2012) used NDEA to study efficiency of public universities in Iran. Moreno and Lozano (2012) used NDEA to study the team efficiency in NBA. Lei et al, (2014) used a parallel DEA approach to study Olympic achievements. Zadmirazei et al, (2015) used NDEA to study efficiency of paper mill in Iran.

In summary DEA black box approach has been used to study efficiency in both automotive retail industry as well other retail sectors such as grocery, super markets, restaurant chain etc. NDEA has also found use in finding the efficiency of internal structure of a DMU in universities, paper mill, NBA and in Olympics.

2.8: Research Gap Analysis:

Table 2.2, lists twenty-six research papers of DEA applications in the automotive industry that were found in leading journals since 1998 till August 2016 and table 2.3 lists twenty research papers with DEA applications in the retail sector. The application of DEA to automotive industry covers a wide spectrum from manufacturing, brand positioning, evaluating stock of automotive companies, dealership operations etc. Similarly, the application of DEA to retail sector covers a wide range such as grocery stores, supermarket chains, apparel stores, wine stores etc.

However, there are no published papers of DEA applications to heavy equipment industry and its retailing organizations as described in section 2.2 where a search was conducted for such a study in various journals, either using either black box approach or network DEA approach. By using DEA to study efficiency in the heavy equipment industry, the industry will greatly be benefitted in the same way as an automotive industry in the areas of manufacturing, brand positioning and benchmarking dealership operations that have a network of branches. Therefore, there is a need to study the efficiency of heavy equipment dealerships using DEA. This study will be first of its kind to use both black box approach and network DEA approach to study efficiency in heavy equipment retailing organizations.

2.9 Conclusion:

As described above the two approaches to DEA applications are the black box approach and Network DEA approach. It has been found from a critical review of DEA applications using both the black box approach and Network DEA approach in both the automotive industry and retail sectors that similar factors and similar models have been employed in both industries to study efficiency. This helps in concluding that similar models and factors can be used to study the efficiency of heavy equipment retailing organizations.

In the next chapter 3, we will be describing the basic concepts of efficiency, theoretical aspects of DEA, extended models used in DEA, Bootstrap DEA, variations in efficiency over time, the effect of contextual variables on DEA scores and detection of outliers using DEA scores. These theoretical aspects will be dealt in detail to lay a foundation for the research methodology in Chapter 4 and apply these models in Chapter 5 as per research methodology to find out the efficiency scores of the retailing organization under study.

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Table 2.2. Summary of Literature review of DEA application in automotive retail applications.

Item Srl#	Analysis Levels	Studies	Year	Country	Inputs	Outputs
1	Passenger Car dealer efficiency	Trilochan Sastry, Arindam Mukherjee	1998	India	Dealer Strategy	Performance Parameters.
2	Relative efficiency and quality of					
	Global Auto Companies.	Narasimhan,Graham and Wang.	2005	USA	COGS,SG&A	Revenue,Net Income,Consumer Satisfaction
3	Product Performance Evaluation:					
	A superefficiency model	Staat&Hammerschmidt	2005	Germany	Price,Runningcost	Engine Power(HP),Comfort,Safety Features
4	Determinants of brand advertising	Buschken	2007	Germany	Media Budgets,	Brand Familiarity,Sympathy,Brand Consideration,Brand Purchase Intention
	efficiency					
5	Indian Autocomponent Industry					
	Estimation of Operational eff using DEA.	Saranga	2009	India	Raw material,Labour,Capital,Sundry Expenses	Gross Income
6	Performance Assessment and Optimization	Hour Ali,Montazeri,Saberi	2009	Iran	Warranty Cost	Automotive Sales Income,Parts Sales
	of after-sales networks					
7	Performance Management of Auto dealers	Lin,Lee,&Chang	2010	Taiwan	Number of Salesperson,Training Expense	Revenue from Sales of vehicles,Service Sales,Total Number of customers.
8	Measuring Operational eff of Car dealer	Lu He	2011	Taiwan	Cost, Volume,Time	ROA,ROI,
9	Efficiency of Ford Car dealer using DEA	Hsiao	2011	Taiwan	Number of Techs, Number of repair Orders	Service sales,Expenditure
10	Productivity of Auto Industries using MalmIndex	Yao Chen	2011	USA	Number of employees	Revenues,Assets, Equity (Labour efficiency index)
					Employees,Assets, Equity	Revenues(Resource Utilization Index)
11	Mathematical model for Product positioning using DEA	Eshlaghi,Jamalou	2011	Iran	Engine size,Price,	Power,Top speed,Fuel Consumption,Driving comfort,Passenger comfort.
12	Performance of Vietnamese Auto Industry using DEA	Tran,Ngo	2014	Vietnam	Labour,Capital,	Production Value,Turnover

Item Srl#	Analysis Levels	Studies	Year	Country	Inputs	Outputs
13	Evaluation of auto companies using DEA	Alvandi,Masoumi,Rezaei	2012	Iran	Humn Resources,Fixed Capital,Raw Material	Financial Index,Customer Index,Internal Business Process Index,Learning &Growth Index
14	Performance of Auto Industry in Taiwan using DEA	Alex, Chich-Jen	2013	Vietnam	Number of employees,Operating Cost,Gross Asset	Operating Income
15	New approach for assessing dealership performance in	Biondi,Calabrese,Capece,Costa, Pillo	2013	Italy	Number of salespeople,number of outlets,number	
	Auto Industry				of competitors,Number of days since sale closed	Revenue, Quality of Service
16	Product efficiency in Spanish Auto market	Gonzalez,Ventura,Carcaba	2013	Spain	Discounted Price	Ecology,Fuelconsumption,HP,Max Speed,Volume,Boot Space,Safety,Accelaration and Equipment
17	Selecting a fuel efficient vehicle using DEA	Partovi,Kim	2013	USA	Annualized MSRP,Annual Cost, Fuel Annualized	Annual carbon footprint,Range,Power,Speed,Size
18	Most efficient auto vendor using DEA	Toloo,Ertay	2014	Czech Rep	Number of branch office,total number of vehicle exhibited	Number of sold vehicles,customer satisfaction,satisfaction index of vendors,Parts availability,
					bited,total number of test vehicle,sales people,	Availability of vehicle loans
					number of employees of vendor,	
19	Analyze TFP in Auto industry	Darijani,Taboli	2014	Iran	Materials,Energy,Capital, Labour	Production
20	Classification of Iran Auto and Parts Manufacturer Stock	Elahi,Afshar,Hooshangi	2014	Iran	Price to earning ratio,Beta,Sigma	EPS, 1 Year,2 year and 3 Year return
21	How car delers adjust prices in Spanish Market-DEA approac	Gonzalez,Ventura,Carcaba	2015	Spain	Car Features	Ecology,Fuelconsumption,HP,Max Speed,Volume,Boot Space,Safety,Accelaration and Equipment
22	Auto Industry Strategic partner selection using DEA	Wang,Nguyen,Wang	2016	Taiwan	Fixed Assets,COGS,OP Expenses,Longterm Investment	Revenues, Equity,Net Income
23	Service Performance evaluation in Auto Industry using DEA	Tan,Zhang,Khodaverdi	2016	China	Physical Aspects,Reliability,Customer relationship,Problen	Profit,Order Processing time,Number of customer serviced per day,order processng time,
					Solving,Policy	Complaints handled.
24	Product Modularization and effects on efficiency-An analysis	Piran et al.	2016	Brazil	Commercial lead time,Engg lead time,Number of parts,	Number of projects developed
	a bus manufacturer using DEA.				Number of items purchased,Number of reported tech	
					problems,Number of items with customer complaints	

Table 2.3. Literature review of DEA application to retail sector.

Item Srl#	Analysis Levels	Retail Outlet Study Using DEA	Number of Outlets	Country	Year	Inputs	Outputs
1	Efficiecny of Restaurants using DEA.	Athanassopoulos	31	UK	1995	Capital,Occupancy,Utilities,Maintenance and Gen expenditure	Sale of food,Sale of drinks.
						of the area of the stores,Drinking Area,Number of covers,	
						marlet's size,number of restaurants in 1 mile radius,	
						number of restaurants in 3 mile radius,	
2	DEA models of fast food restaurant chain	Donthu&Yoo	24	USA	1998	Labour Hours,Literes of Inventory Depletion,Store size,Manager	Sales, Customer Satisfaction
						tenure,Stroe Location,Promotion/Give away expenses	
3	DEA of multistore,multimarket retailer	Thomas	552	USA	1998	Labour,Employees,Wages,experience,employees,store manager,	Sales, Profit
						Location related costs,occupancy,operating expenses,internal	
						processess,inventory, transactions.	
4	Influence of IT investment.	Dasgupta	162	USA	1999	Information Tech Budget,Employees in IT	Net Income
5	DEA BCC model of 13 retail stores	Keh and Chu	13	USA	2003	Labour, Floor Staff,Management wagws abd benefits.Number of	Distribution servcies,Accessibility,Assortment,Assurance of product
						hours worked.	delivery,availability of information,ambience, sales revenue.
6	DEA of leading supermarket chain in	Barros and Alves	47	Portugal	2003	Fulltime employees,Part time employees,cost of labour,	Sales,Operational results
	Portugal.					absenteeism,Areas of outlets,number of points of sale,age of the	
						outlet,inventory, other costs	
7	DEA-Malmquist Productivity Index of	Sellers-Rubio and Mas-Ruiz	100	Spain	2006	Employees,Capital,Outlets	Sales, Profit
	supermarket						
8	Productivity of European retailers	Moreno		Spain	2006	Spending,Fixed Assets, Number of employees	Sales
9	Optimal paths in dynamic DEA in Chilean	Mateo et al	35	Chile	2006	Salesperson Labour, Cashier Labour,Sales and admin expenses,	Gross Sales
	stores.					marketing expenses,Store floor space	
10	DEA Operational efficiency of UK grocery	Yu and Ramanathan	41	UK	2008	Total Assets, Shareholders funds,Number of employees	Turnover, Profit before tax
	store					Tobitt Regression: Head office location,types of ownership,years	

Item Srl#	Analysis Levels	Retail Outlet Study Using DEA	Number of Outlets	Country	Year	Inputs	Outputs
11	Technical efficiency of French retailers.	Perrigot and Barros	11	France	2008	Number of employees,Capital ,Total Cost	Turnover Value, Profits
						Tobit Regression: Trend,Square of trend,Mergers and Acquisitions	
						Group, International	
10	US Speciality retailers and food stores	Mostafa	45	USA	2009	Number of employees,Assets	Revenues, Market share,Earnings per share
11	Efficiecny of coffee stores	Joo et al	8	USA	2009	Cost of sales,Wages and Benefits,Other expenses,Occupancy	Revenues,
						expenses	
12	DEA -Restaurant chain	Banker et al	12	USA	2009	Total selling hours,Store size,Average inventory,Support activities	Store Sales
						Regression model: Sales index,Holiday season,household	
						income,age,family size,college education,population,rural area,	
						competition.	
13	Benchmarking large US retailers	Malhotra et al	7	USA	2010	Average collection period,Debt Ratio	Profit Margin,ROA,Quick ratio,Inventory and Asset turnover.
14	Retailing efficiency using NDEA	Vaz et al	78	Portugal	2010	Area in sq metre,stock, number of references,products perished	Sales
15	Retail productivity of Food and Grocery	Gupta &Mittal	43	India	2010	Number of employees,Cost of Labour,Number of hours worked,Are	Sales,Customer conversio ratio
	store.					of outlet,Number of POS machines, Number of SKU	
16	A return on Asset perspective.	Joe et al	14	USA	2011	Current Assets,Fixed Asset,Other Assets,Cash, Receiveables,	Revenue
						Inventory,Cost of goods sold,selling general and admin expenses,	
						Depreciation and Amortization	
17	Efficiency of Indian retailers	Gandhi&Shankar	18	India	2014	Cost of labour,Capital employed	Sales,Profit
18	Operational Efficiency of Wine Stores	Barth	8	Canada	2007	Labour measured in hours,Wine Sold(Inv Depleted)	Retail Sales
19	Efficiency of Italian wine producers	Urso,Timpanaro,Caracciolo,Cembalo	623	Italy	2018	Value of land Capital,Labour,Working Capital	Production
20	Efficiency of wine making	Goncharuk and Figurek	33	Ukraine	2017	Material Cost,Number of employees,Fixed Assets	Net Sales

Chapter III: Data Envelopment Analysis (Theoretical Framework)

3.1: Introduction:

In this chapter the theoretical aspects of Data Envelopment analysis (DEA) will be covered comprehensively as the main research technique used in the current study. The first part of the chapter will cover the efficiency concepts as in production economics. In this section, concept of efficiency, input oriented and output-oriented measures and scale efficiency will be discussed. In the context of DEA, the second part will outline the background, terminology, the theoretical aspects of DEA and the various mathematical formulations of DEA. This will be followed by review of extended models of DEA, such as super efficiency, cross efficiency and weight restrictions that increases the discriminative power of basic DEA analysis. Bootstrap DEA, that helps in identifying the bias in efficiency scores will be discussed. This will be followed by how the effect of environmental variables on efficiency scores can be assessed using OLS regression and Tobit regression in the second stage of the analysis. The concepts of variation in efficiency change over time will be discussed and how it can be measured with window analysis and Malmquist productivity index. Network DEA (NDEA) that analyzes the efficiency of a DMU ‘s internal structure will also be discussed followed by methods to detect outliers in DEA.

3.2: Introduction to Efficiency Concepts:

3.2.1 Productivity, Efficiency and Effectiveness:

Productivity and its importance has been articulated in many ways at many times based on the context. Efficiency in production is productivity, how much output is obtained from a given number of inputs (Syverson, 2011).

Figure 3.1. Transformation processes of inputs to outputs (Constructed by author).

Similar is the concept of efficiency and effectiveness but not equal. Many authors do not differentiate between efficiency and productivity in the literature. Both productivity and efficiency are defined as the ratio between output and input by Cooper, Seiford & Tone (2007). There is a component called efficiency that contributes to changes in productivity as productivity varies based on production technology, process used in production and variations in environmental factors in which production occurs (Porcelli,2009).

Many researchers concur that efficiency is utilization of resources and mainly deals with the input of the productivity ratio. In other words efficiency is the minimum resource that is needed theoretically to run the operations in a given system in relation to how much resources are actually used (Tangen, 2004).The term efficiency and productivity are used interchangeably but with a caveat with the use of these terms(Sherman &Zhu,2006).The term efficiency is more effective as it is used to interpret the value judgement of a manager's performance whereas productivity is less sensitive as it is less used as a value judgement term. As per Sherman & Zhu (2006) efficiency has a narrower meaning as compared to productivity.

The difference between productivity and efficiency can be best understood by using the following figure 3.2, from Coelli et al., (2005). Consider a simple production process in which a single input(**x**) is used to produce a single output (**y**). The curve OF represents a production frontier that defines the relationship between input and output and represents the maximum output achievable from each input level and therefore represents the current technology used in the industry. Technically efficient firms operate on the frontier and inefficient firms operate below the frontier. In the figure3.2 below, A represents an inefficient firm and efficient firms are represented by points B and C (Coelli et al.,2005; Kokinou,2012).

Productivity at a point can be measured by using a ray passing through the origin. The slope of the ray is output/input (**y/x)** and therefore productivity can be measured using the slope. The slope of this ray would be higher if the point A moves to point B on the efficient frontier, indicating a higher productivity at B. Let us assume that the firm moves to point C, where it falls on the ray from the origin that is tangent to the production frontier. Then this point becomes the point of maximum possible productivity. The movement of the firm from B to C is an illustration of exploiting scale economies (Coelli et al.,2005, Kokinou,2012).

Since the point C has maximum productivity it is the point of having a technically optimal scale of productivity and operation at any other point will have lower productivity. Given the same input OD, productivity can be enhanced by moving from point A to point B. BD/OD is the new productivity due to the above movement. The efficiency of firm A can be measured by the ratio of productivity at point A and productivity at point B. This will be $\frac{AD/OD}{BD/OD}$**.** This is equal to AD/BD and is termed Technical Efficiency (Coelli et al.,2005, Kokinou,2012). There are output and input oriented technical efficiencies. Given the same input, a firm can improve output (output

-oriented moving from point A to B) or reduce the input given the same output (input-oriented,

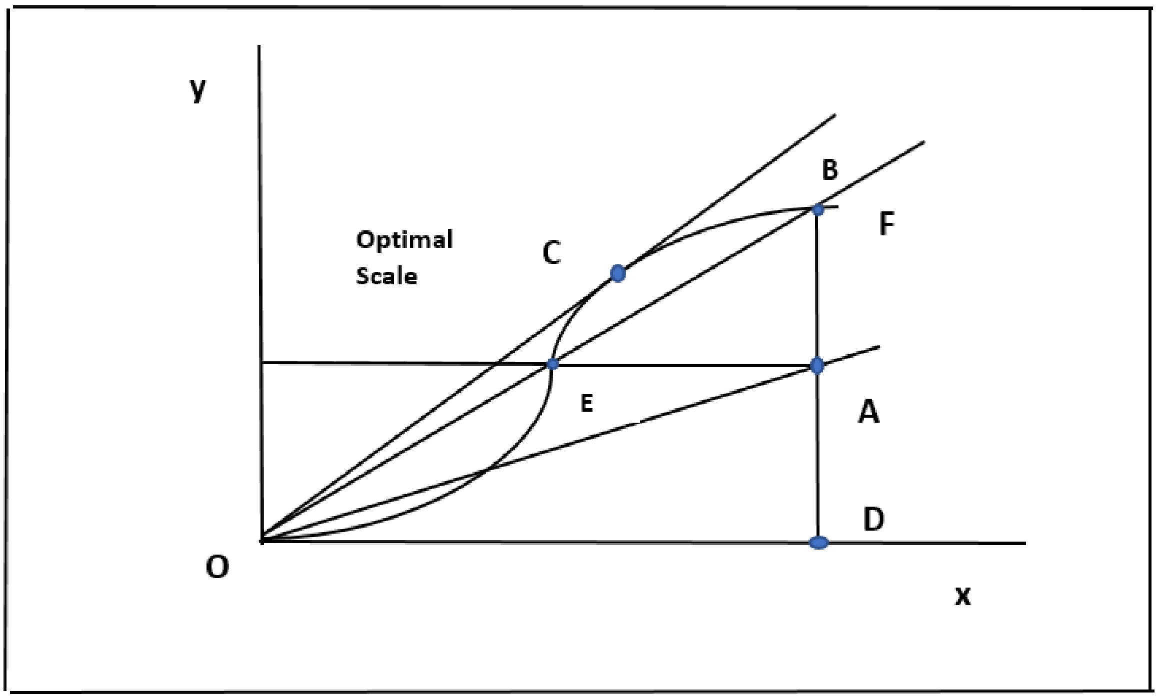

Figure 3.2. Productivity and Efficiency (Created by the author, Source Tim Coelli et al., 2005).

moving from point A to E). This will be discussed in detail in the next section. Therefore, efficiency is the relationship between what a firm produces and what it could feasibly produce (Coelli et al.,2005). In other words, efficiency of a production unit represents a comparison between observed and optimal values of its output and input.

Effectiveness is the extent to which the stated objectives are met and is linked to the creation of value for the customer and products and deals with the output of the productivity ratio (Tangen, 2004). Studying effectiveness will generate information that will help in identifying the potential for productivity improvements. The ability of an organization to set and achieve its goals and objectives is effectiveness, to do the right job whereas efficiency is the ability to produce outputs with minimum resources, to do the job right (Sherman &Zhu, 2006). In short, efficiency is doing things right and effectiveness is doing the right things. The following diagram shows the difference between efficiency and effectiveness.

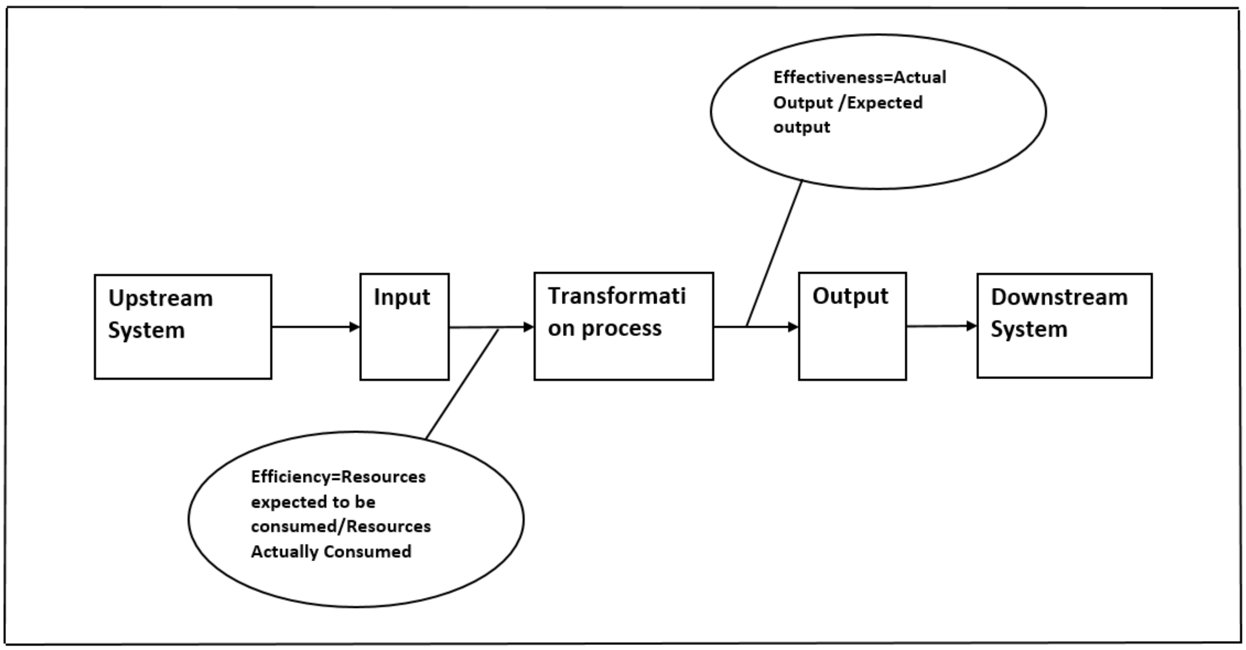

Figure 3.3. Efficiency and Effectiveness (Created by the author. Source: Sink and Tuttle, 1989).

With these concepts of productivity, efficiency, and effectiveness that are fundamental to performance measurement, we will now discuss in detail about efficiency, as DEA measures relative efficiency.

3.2.2 Efficiency Concepts and measures:

The efficiency of a production unit is defined in terms of a comparison between observed and optimal values of its output and input (Fried, Lovell & Schmidt, 2008).

$$\text{Efficiency} = \frac{\textbf{Output}}{\textbf{Input}} \quad \ldots\ldots\ldots\ldots \quad \textbf{(3.1)}$$

The drawbacks with the above measure of efficiency are: 1) The above model cannot incorporate multiple inputs and outputs,2) Other process dimensions such as quality cannot be accommodated easily in to the above equation,3) Similarly , contextual factors that has an influence in the process under study cannot be modelled easily,4) When there are multiple inputs and outputs, varying units of these inputs and outputs cannot be handled by the above equation.

Farrell (1957) introduced a new measure of technical efficiency that considered all inputs and outputs, considering the above-mentioned drawbacks and showed how it can be computed in practice. This measure determined for each firm/industry *would know how far a given industry can be expected to increase its output by simply increasing its efficiency without absorbing further resources* (Farrell, 1957, p254).

The new measure compares the observed performance of a firm with some *postulated standard of perfect efficiency* (Farrell,1957). Farrell's methods of measuring technical efficiency of a firm consist in comparing it with a hypothetical firm that uses factors in the same proportion. This hypothetical firm is constructed as a weighted average of two observed firms. The measure has a score for each firm and the firm is analyzed within a group of comparable firms and is evaluated by comparing it with some ideally performing firm. This ideally performing firm is found either theoretically or empirically (Farrell, 1957).

1) **Theoretical**: This is represented as a theoretical production function as specified by engineers where perfect efficiency is attainable theoretically, providing an ideally

performing firm. However, it is difficult to specify a theoretically efficient production function for a very complex process. For example, it is difficult to estimate a plant's need for indirect labor in advance. The more complex the process the less accurate will be the theoretical production function.

2) **Empirical:** Here the efficient production function is estimated from observations of inputs and outputs of many firms. The hypothetical firm is constructed by a weighted average of an appropriate number of observed firms. In other words, the performance of a firm is determined by comparing it to a relative production combination that can be achieved in practice.

Farrell (1957) treated the definition of technical efficiency as a relative idea, an idea that is relative to the best observed in practice in the reference set or comparison group. This provides a method to differentiate between efficient and inefficient production units. Farrell (1957) demonstrated that overall efficiency can be decomposed into allocative efficiency and technical efficiency. Allocative or price efficiency refers to the ability of a firm to use the inputs in optimal proportions so that the resource cost is minimized. When a producer is technically efficient, the maximum output is produced from a given level of inputs. Similarly, an allocatively efficient producer would produce the outputs that use the lowest combination of cost inputs. In other words, technical efficiency demonstrates a comparison of actual output and the maximum output whereas allocative efficiency deals with the relationship between minimum cost and the actual cost of bundles of inputs. Price efficiency is the efficiency of the organization to purchase the input that meets the quality standards at the lowest price. Farrell proposed the concept of scale efficiency at an industry level. Scale efficiency measures whether an organization is operating at its optimal size. When goods or services are produced greater or less than the optimal level, there are added costs due to volume and size(Farrell,1957).

According to Koopmans (1951) *"a producer is technically efficient if an increase in an output requires a reduction in at least one other output or an increase in at least one input and if a reduction in any input requires an increase in at "least one other input or a reduction in at least one output."* Koopmans thus offered a definition and characterization of technical efficiency. However, it was Debreau (1951) who first provided a measure or an index of the degree of technical efficiency with his coefficient of resource utilization. A production unit is said to be technically efficient with score one when there is no such feasible reduction. In any other case production unit is characterized as inefficient and has a technical efficiency score of less than one. Both Koopmans (1951) and Debreau (1951) were concerned mainly with the measurement of efficiency and although they produced a careful measurement of some or all the inputs and outputs used in production process, they did not succeed in combining these measurements into any satisfactory estimate of efficiency. According to Kalirajan and Shand (1999), while the interest to measure technical efficiency is continuing, the concept of technical efficiency is as old as neoclassical economics. Therefore, the characteristics that affect the way productivity can be measured can be summed up as a) Environment Complexity, b) Output complexity, and c) Input complexity. These characteristics lead to a spectrum of productivity management techniques and the most appropriate technique that suits the heavy equipment industry should be identified. The diagram below gives a framework of performance measurement.

Farrell's original idea on technical efficiency leads to two important measures known as input-oriented measures and output-oriented measures (Tim Coelli et al.,2005). Both these measures are discussed below.

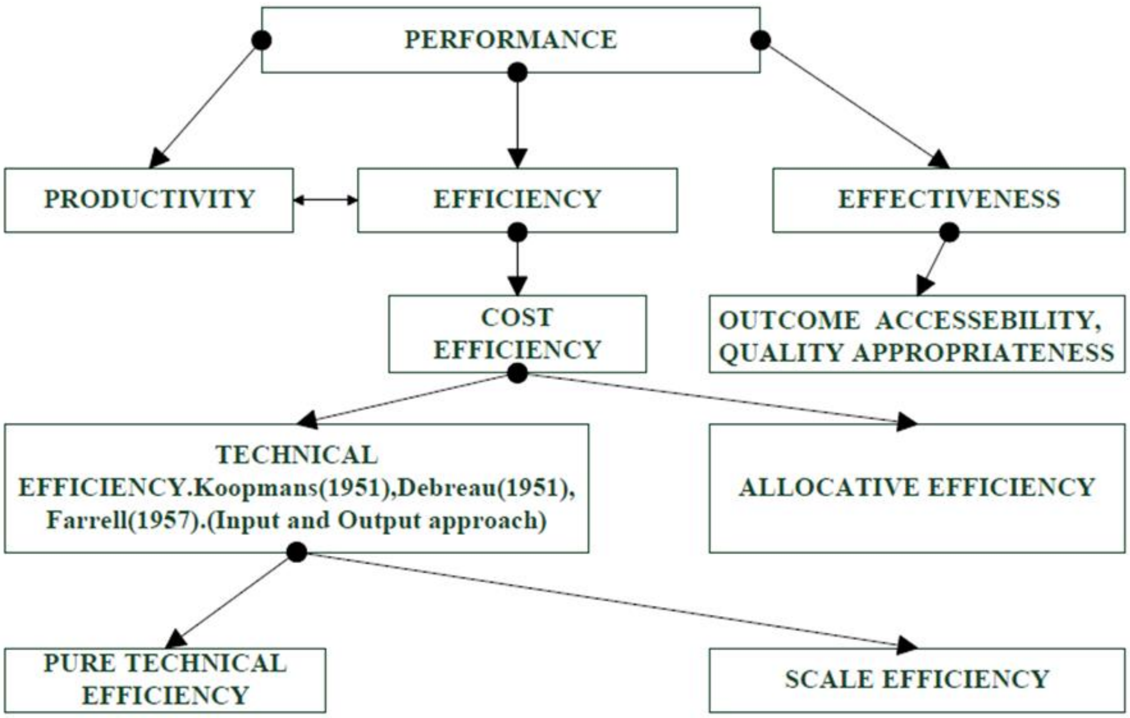

Figure 3.4. Framework of Performance Measurement (Created by the author).

3.2.3 Input - Oriented Measures:

The graph below represents a production function with two inputs X_1 and X_2 and one output Y under the assumption of constant returns to scale. In economics, production function is a relation between physical outputs and physical inputs of a production process. In the following graph SS' is an ***isoquant*** and represents an efficient production function. An **isoquant** is a contour line drawn through the set of points of all possible combinations of inputs that produce the same amount of outputs. Isoquant SS' is convex to the origin as it shows the lower limits/bounds on the inputs. The output Y is held at a fixed quantity to be one (Tim Coelli et al.,2005)

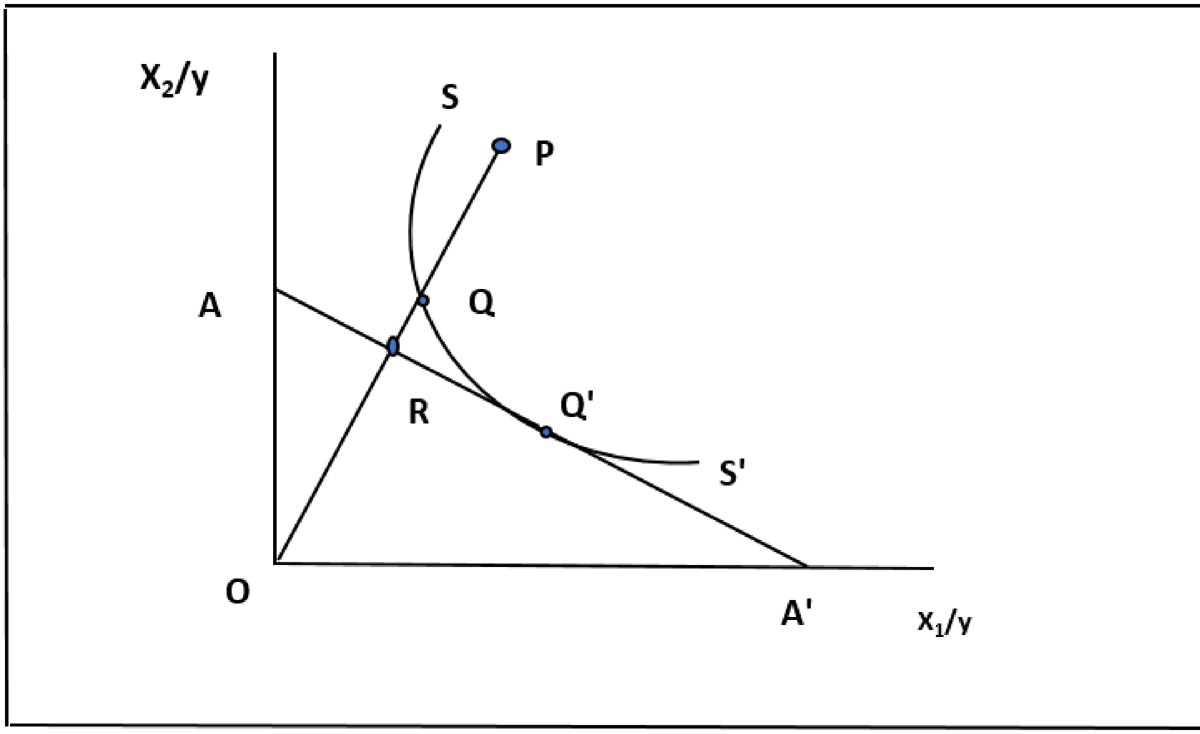

Figure 3.5. Input Oriented Measures (Created by author, Source Tim Coelli, 2005)

Let us assume P is a firm that utilizes quantities of inputs defined by P and produces a unit quantity of output.

The **technical efficiency** (TE) of the firm measured by the ratio $TE_1 = \frac{OQ}{OP} = 1 - \frac{QP}{OP}$

This takes a value between zero and one and hence provides an indicator of the technical efficiency of the firm. A value of one indicates the firm is fully efficient and zero indicates the firm is fully inefficient (Tim Coelli et al.,2005). For example, the point Q is technically efficient as it falls on the efficient isoquant.

Line AA' represents the input price ratio and if it is known, then allocative efficiency (AE)of the firm can be found.

Allocative efficiency (AE) of firm P is $\frac{OR}{OQ}$. If production were to occur at Q' that is technically and allocatively efficient, then RQ represents a reduction in production cost (Tim Coelli et al.,2005).

Total Economic Efficiency (EE) = $\boldsymbol{\frac{OR}{OP}}$

According to Farrell 1957, the total economic efficiency of the firm, $\boldsymbol{\frac{OR}{OP}}$

TE x AE = (OQ/OP) x (OR/OQ) = (OR/OP)

Total economic efficiency = Allocative Efficiency X Technical efficiency

All the above three efficiency measures have an upper limit of one and a lower limit of zero. It is assumed that the production function of the fully efficient firm is known while estimating the above three efficiency measures (Tim Coelli et al.,2005). However, in real life situation, this does not happen, and sample data must be used to estimate the production function and efficient isoquant. The production function may be too difficult to be determined or may not be known at all. Farrell (1957) suggested the use of a non-parametric piecewise-linear convex isoquant constructed such that no firm lies either to the left or to the bottom of the isoquant as depicted in the figure3.6 below. Such a function envelops all the data points as shown in the figure3.6 below.

The question that is addressed in the input-oriented measure is by how much input quantities be proportionately reduced without changing the output quantities produced?

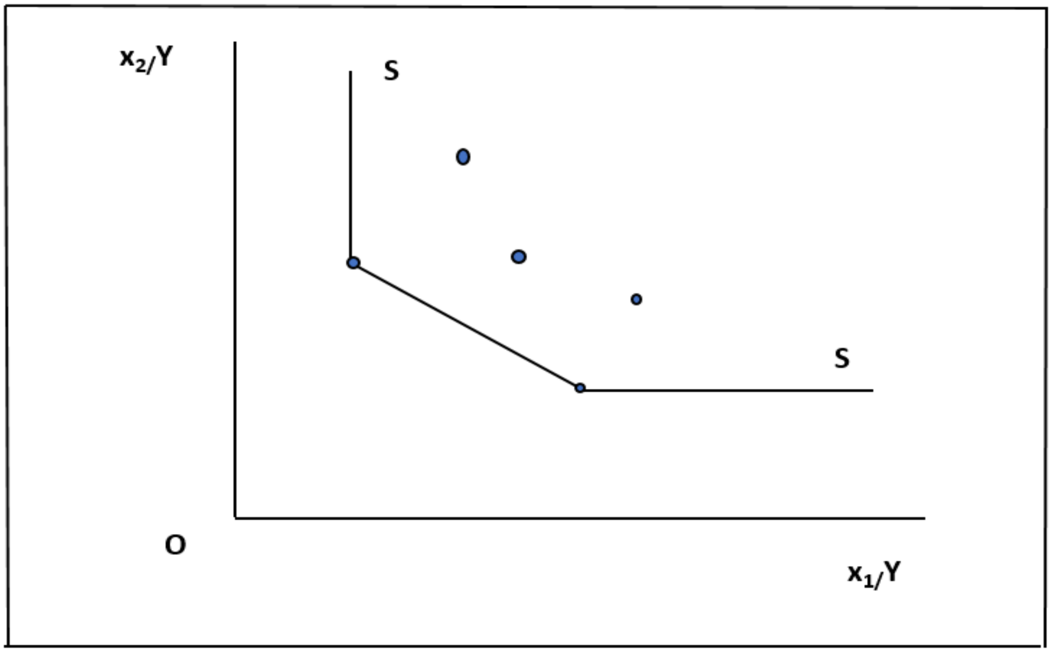

Figure 3.6. Piecewise linear function. (Created by author, Source Coelli et al., 2005)

3.2.4 Output Oriented Measure:

Output oriented measure addresses the question of how output quantities are proportionately increased without changing the inputs. This is illustrated in the figure 3.7 below. Here the production involves single input x_1 and two outputs y_1 and y_2 . The production possibility curve ZZ' exhibits a constant quantity of input that is used to produce varying combinations of the two outputs y_1 and y_2. ZZ' represents the upper limit of production possibilities and is concave to the origin and all the firms lie to the left and bottom of ZZ' (Coelli et al., 2005).

One such firm is A and lies below the curve as it is inefficient and because ZZ' represents the upper bound of production possibilities. Point B is the projection of firm A on to isoquant ZZ' The Farrell output-oriented efficiency can be expressed as follows (Tim Coelli, 2005).

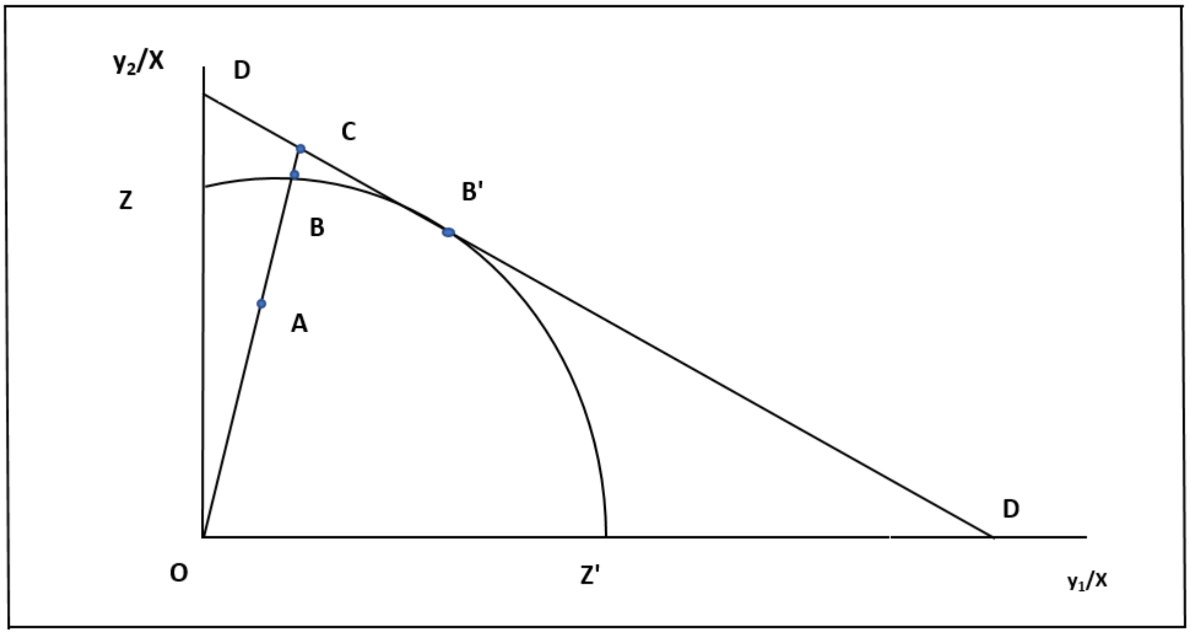

Figure 3.7. Output Oriented Measure (Created by author, Source Coelli et al., 2005)

The distance AB determines the technical efficiency. This is the amount by which there could be expansion of outputs without needing extra inputs. Therefore, a measure of output-oriented technical efficiency is the ratio $TE_0 = \frac{OA}{OB}$

The Iso-Revenue line DD' can be drawn if the prices of outputs are known.

(All combinations on the Iso-Revenue Line yield the same revenue)

The allocative efficiency= $AE_0 = \frac{OB}{OC}$

This can be attributed to increase in revenue as compared to an explanation of cost reduction in the input-oriented case.

The overall efficiency $EE_0 = \frac{OA}{OC} = \frac{OA}{OB} \times \frac{OB}{OC} = TE_0 \times AE_0$

Again, all these measures are bound by zero and one.

Both the input oriented and output-oriented measures are measured along a line from the origin to the observed point of production. Therefore, the relative proportion of inputs or outputs is held constant. One of the advantages of these radial measures is that they are units invariant. This means changing the units of measurement will not alter the value of the efficiency measure.

3.2.5 Returns to Scale:

The above input oriented and output-oriented measures of efficiency are based on the assumption of **constant returns to scale** (CRS). This indicates that the production process under consideration is such that an increase in all inputs and outputs will take place by the same proportion. There are instances where there is a disproportionate change (increase or decrease) in the outputs. This is termed as **variable returns to scale.** The different types of returns to scale are depicted in the following figure3.8. The diagram represents the difference between input-reducing and output-increasing situations.

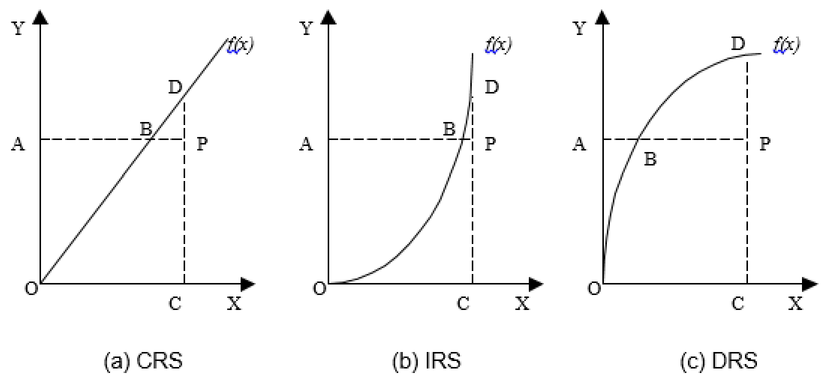

Figure 3.8. Returns to scale (Source: Pasupathy, 2002)

The diagram represents the production process involving one output from one input. In figure (a) the production function f(x) is a straight line and has a constant slope indicating that for every unit increase in input there is a proportionate increase in output and represents constant returns to scale.

Fig (b) represents a function with a slope that is increasing indicating that for every unit of input the output increases by more than the input and hence depicts an increasing return to scale (IRS). In Fig (c) the function has a slope that is decreasing indicating that as the input increases the output decreases, and this is called a decreasing return to scale (DRS). In the figure P is a firm that lies below the efficient frontier. P could be projected to the frontier in each of these cases either by reducing input or increasing output method. B and D are two points that are projected on to the frontier (Pasupathy,2002).

Input reducing efficiency $\frac{AB}{AP}$ =, Output increasing efficiency = $\frac{CP}{CD}$

Triangle OAB and triangle DCO are similar under CRS. This implies $\frac{AB}{AP} = \frac{CP}{CD}$ .Therefore under CRS, the same technical efficiency is obtained either by reducing inputs or by increasing the output. Both these measures give different efficiency scores under variable returns to scale. The distances AB or CD tends to increase under the increasing returns to scale assumption. Therefore, under IRS, an input -reducing efficiency score is higher than the output increasing efficiency score. In the case of decreasing returns to scale assumption the converse is true as either AB or CD gets reduced and hence the output-increasing efficiency score is higher (Pasupathy, 2002).

We have now established concepts of productivity, efficiency, effectiveness, the drawbacks of measuring efficiency, input-oriented measures, output-oriented measures and return to scale. We will now review various productivity management techniques.

3.2.6. Productivity Management Techniques:

There are several approaches that have evolved to evaluate and manage productivity. However. there is no one single approach that is adequate and sufficiently comprehensive that can be used as a single technique. Depending on variations in environment, availability of resources, leadership style and organization culture, tools used for measuring productivity may vary from amongst organizations.

The following are some of the productivity measurement techniques 1) Standard cost system 2) Comparative efficiency analysis 3) Ratio Analysis 4) Profit and return on investment measures 5) Zero-base budgeting 6) Program Budgeting 7) Best Practice Analysis 8) Peer reviews 9) Management Reviews 10) Activity analysis 11) Process analysis 12) Functional cost analysis 13) Staffing models and 14) Data Envelopment Analysis (Sherman & Zhu, 2006). We will discuss in detail Data Envelopment Analysis and for other methods please refer to Sherman & Zhu, (2006).

The next section provides a detailed review of theoretical aspects of Data Envelopment Analysis, the tool that will be used to measure the performance of heavy equipment retailing organizations that have multiple branches/ decision-making units and has multiple inputs and outputs.

3.3 Theory of Data Envelopment Analysis:

3.3.1 Introduction:

Data Envelopment Analysis (DEA) is a linear programming quantitative technique to estimate best practice production frontiers and evaluates the relative efficiencies of different entities called decision-making units(Bogetoft,2012). It is a powerful benchmarking tool and is widely used to measure productivity in service organizations (Sherman and Zhu,2006). DEA permits the use of multiple inputs and outputs in a linear programming model that develops a single score of efficiency(Ozcan,2014). DEA is ultimately a method of performance evaluation and benchmarking against best practice (Cook, Tone and Zhu,2014). DEA is a non-parametric approach meaning that it does not need a prior functional form for the frontier (Paradi et al.,2017). DEA can be adapted to improve service productivity and every organization can benefit from DEA in many ways (Sherman and Zhu,2006). DEA is used to compare the performances of multiple operating units in service organizations such as bank branches, hospitals, and schools.

The following chart indicates various performance measurement techniques. Data Envelopment Analysis (DEA) is a non-parametric performance measurement methodology originally developed by Charnes, Cooper, and Rhodes (1978) to measure the relative efficiency of organizational units. The parametric methods assume a functional form for the production set of inputs and outputs whereas the non-parametric method does not assume any functional form for the production set (Thrall& Seiford,1990).

Organizations have found it a challenge to improve productivity and therefore efficiency measurement has been a subject of great interest. Farrell (1957) stated the reason for this focus in his seminal paper on the measurement of productive efficiency.

"The problem of measuring the productive efficiency of an industry is important to both the economic theorist and the economic policy maker. If the theoretical arguments as to the relative efficiency of different economic systems are to be subjected to empirical testing, it is essential to be able to make some actual measurements of efficiency. Equally, if economic planning is to concern itself with particular industries, it is important to know how far a given industry can be expected to increase its output by simply increasing its efficiency, without absorbing further resources." (Farrell, 1957, p253).

It was further stated by Farrell that the inability to combine the measurements of the multiple inputs and outputs into any satisfactory measure of efficiency was the main reason in the unsuccessful attempts to solve the problem. The approaches that were used was measuring average productivity for a single input not considering other inputs and constructing an index of efficiency in which a weighted average of inputs is compared with the output (Cook & Seiford,2009).

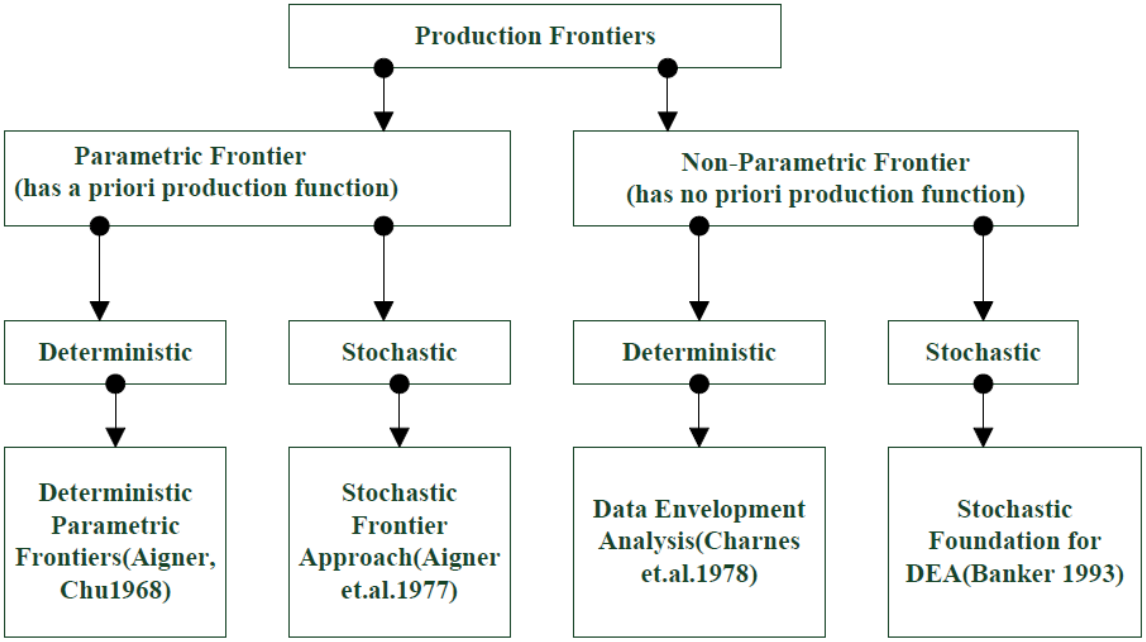

Figure 3.9. Classification of Production Frontiers (constructed by the author).

Farrell (1957) addressed the problem by providing a satisfactory measure of the efficiency that can be computed in practice that takes in to account all inputs and avoids problems posed by index numbers by estimating a relevant production function. In Farrell ‘s own words such a measure could be used to any productive organization *from a workshop to a whole economy*. Efficiency measurement in the presence of multiple inputs and outputs was addressed by Farrell by assigning weights to the inputs and outputs. The efficiency score is a ratio of the weighted sum of the outputs to the weighted sum of inputs and is represented as below (Farrell &Fieldhouse, 1962).

$$\text{Efficiency} = \frac{\textbf{Weighted Sum of Outputs}}{\textbf{Weighted sum of inputs}} \quad \ldots\ldots\ldots\ldots\ldots \quad \textbf{(3.2)}$$

The above equation considers multiple inputs and outputs in the efficiency measurement process. The definition requires a set of weights to be assigned and this may be difficult especially if a common set of weights has to be assigned for multiple organizational units.

Let us consider **n** firms with **m** inputs and **s** outputs. Let x_{i_o} be the inputs and y_{r_0} be the outputs of the observed DMU. The mathematical representation of the above model would be as below assuming controllable inputs and constant returns to scale(Ramanathan,2003).

$$h_o = \frac{\sum_{r=1}^{s} u_r y_{r_o}}{\sum_{i=1}^{m} v_i x_{i_o}} \qquad \text{..................... } (\mathbf{3.3})$$

$s = number\ of\ outputs$

$u_r = weight\ of\ output\ r$

$y_{r_0} = amount\ of\ output\ r\ produced\ by\ the\ observed\ DMU$

$m = number\ of\ inputs$

$n = number\ of\ outputs$

$v_i = weight\ of\ input\ i$

$x_{i_0} = amount\ of\ input\ i\ used\ by\ the\ observed\ DMU$

In the above equation inputs and outputs can be measured and entered. The next problem is how to determine the weight reflecting the relative importance of inputs and outputs. One method is to adopt the fixed weight of each input and output through subjective forms such as expert consultation or panel of experts or discussion. This is a tricky issue as there is no unique set of weights(Ramanathan,2003). For example, a school that has a good reputation for teaching sciences would like to attach higher weights to its sciences' output. A university that has a higher percentage of socially weaker sections in its students would like to emphasize this fact, assigning a greater weight to this input category. If a common set of weights are assigned, the individual firms do not have the freedom to choose their own set of weights for their inputs and outputs. Therefore, the efficiencies of the firm are determined under a predetermined set of weights without the option to choose weights as per the needs of the firm. By assigning weights that are most favorable to the firm, there is no possibility of increasing the efficiency score of the firm as all firms may appear efficient. The relaxation given to a firm to choose its own weights led to the introduction of Data Envelopment Analysis (Ramanathan,2003)

In 1978, twenty years after Farrell's seminal work, Charnes et al., (1978) introduced a powerful methodology called Data Envelopment Analysis (DEA) that could assess the relative efficiency of multiple–input and multiple output production units (Cook and Seiford,2009). Originally DEA was to provide a methodology to identify best performing units from a set of comparable decision -making units (DMUs), that forms an efficient frontier. However, the methodology also enables as a benchmarking tool to measure the level of inefficiency of non-frontier units by comparing it with peer efficient units (Cook and Seiford,2009). There have been tremendous advancements both in theoretical developments and application of DEA to real life situations since the advent of DEA in 1978.

3.3.2 CCR (Charnes, Cooper, and Rhodes 1978) Model:

In their study Charnes, Cooper and Rhodes (1978) overcame the drawbacks of assigning weights by assigning a unique set of weights for each DMU by proposing a mathematical programming model(Ramanathan,2003). The DMU for which the efficiency is maximized is normally termed as focal production unit or reference or base DMU and is designated as the decision-making unit (DMU). The objective was to allow the DMU to select the most favorable weights (or multipliers) $\mathbf{u_r}$ and $\mathbf{v_i}$ that computes the highest possible efficiency ratio of outputs to inputs (score) for the service unit under evaluation (Sherman and Zhu,2006). The only restriction is that the productivity ratio of all DMUs calculated by the weights selected by the DMU must be less than or equal to one(Kao,2017). This ratio that has a value between zero and one is a measure of efficiency as it is the ratio between the actual output (the maximum output) that can be produced with the same amount of input of this DMU(Kao,2017). This is called as CCR model (Kao,2017). The mathematical formulation of the ratio form of DEA for the **input-oriented model** is provided below.

$$E_0 = \max \frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{i=1}^{m} v_i x_{i_0}} \qquad \textbf{(3.4)}$$

Subject to

$$\frac{\sum_{r=1}^{s} u_r y_{r_0}}{\sum_{i=1}^{m} v_i x_{i_0}} \leq 1 \;\; j = 1, 2, 3 \ldots n$$

$$u_r, v_i \geq \varepsilon \quad r = 1, 2 \ldots s, \;\; i = 1, 2 \ldots m$$

The multipliers u_r and v_i should be greater than a small positive number ε .This is to avoid assigning zero values to multipliers as it may avoid some unfavorable factors (Charnes et al., 1979). This small number is called a non-Archimedean number (Charnes and Cooper,1984). If E_0=1, then this DMU is said to be in a state of Pareto optimality, also called Pareto efficiency (Koopmans,1951). Koopmans (1951) extended Pareto efficiency to productive efficiency and therefore called as Pareto-Koopmans Efficiency (Charnes and Cooper,1961). The model 3.4 is called ratio model and is a linear fractional program. This is converted to a linear program by Charnes-Cooper transformation as below(Kao,2017).

$$E_0 = Max\sum_{r=1}^{s} u_r y_{r_0} \ \ for\ DMU_0$$
$$subject\ to \sum_{i=1}^{m} v_i x_{i_0} = 1,$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0,\ j = 1,2,3....n \ \text{............} \ \textbf{(3.5)}$$
$$v_i > \varepsilon \ \ i = 1,2,3......m$$
$$u_r > \varepsilon \ \ r = 1,2,3......s$$

This model 3.5 is called a **multiplier model**. This model uses a straight line passing through the origin and passing through all DMUs, as production frontier when there is one input and one output. Since the production frontier passes through the origin, this indicates that a proportional change in inputs leads to a proportional change in outputs and this is called constant returns to scale (Kao,2017).

Model (3.5) has a dual and that can be formulated as below.

$$E_0 = \min \theta - \varepsilon \left(\sum_{r=1}^{s} S_r^{+} - \sum_{i=1}^{m} S_i^{-} \right)$$

$$\begin{aligned}
&\textit{subject to } \theta X_{i_0} = \sum_{j=1}^{n} X_{ij}\lambda_j + S_i^{-} \quad i = 1,2,3...m \\
&Y_{r_0} = \sum_{j=1}^{n} Y_{rj}\lambda_j - S_r^{+} \qquad r = 1,2,3.....s \\
&\lambda_j \geq 0 \quad j = 1,2,3........n \qquad \ldots. \ \textbf{(3.6)} \\
&S_r^{+} \geq 0 \quad r = 1,2,3.......s \\
&S_i^{-} \geq 0 \quad i = 1,2,3........m \\
&\theta_0 \ \textit{is unrestricted in sign}
\end{aligned}$$

As $S_r^{+}, S_i^{-} \geq 0$, the first two constraints indicate that $\sum_{j=1}^{n} X_{ij}\lambda_j \leq \theta X_{i_0}$ and $\sum_{j=1}^{n} Y_{rj}\lambda_j \geq Y_{r_0}$

This implies that all the observations have a larger amount of inputs and a smaller amount of outputs than the point ($\sum_{j=1}^{n} X_{ij}\lambda_j, \sum_{j=1}^{n} Y_{rj}\lambda_j$) on the production frontier. In other words, observations are enveloped by the production frontier and hence called **the Envelopment Model** (Kao, 2017)

The second constraint indicates that the DMU fixes its outputs at the current level Y_{r_0} or to an adjusted amount $Y_{r_0} + S_r^{+}$ where S_r^{+} is an excess of output slack and finds by how much the amount of inputs can be reduced using the reduction ratio θ. Input excesses S_i^{-} and output shortfalls S_r^{+} are known as input and output slack variables (Kao, 2017).

Therefore, both models 3.5 and 3. 6 are input oriented models.

Output Models:

Efficiency can also be measured from the output side. The CCR efficiency in the output model is reciprocal of E_0 and therefore is $1/E_0$ (Kao,2017)

The ratio form of output model is

$$\frac{1}{E_O} = \min \frac{\sum_{i=1}^{m} v_i x_{i_0}}{\sum_{r=1}^{s} u_r Y_{r_0}} \leq 1 \;\; j = 1,2,3...n$$
$$Subject\,to \qquad\qquad (3.7)$$
$$\frac{\sum_{i=1}^{m} v_i x_{ij}}{\sum_{r=1}^{s} u_r Y_{rj}} \geq 1, \quad j = 1,2,3.....n$$
$$u_r, v_i \geq \varepsilon \quad r = 1,2...s, \quad i = 1,2...m$$

This model is same as Model 3.4 except that the objective function is represented in the reciprocal form. This is again transformed to an LP problem using Charnes Cooper transformation

in the **Multiplier Output Model** as below (Kao,2017).

$$\frac{1}{E_0} = \min \sum_{i=1}^{m} v_i X_{i_0}$$
$$subject\,to,$$
$$\sum_{r=1}^{s} u_r Y_{r_0} = 1$$
$$\sum_{i=1}^{m} v_i X_{ij} - \sum_{r=1}^{s} u_r Y_{rj} \geq 0 \quad j = 1,2....n \qquad .... \, \mathbf{(3.8)}$$
$$u_r, v_i \geq \varepsilon \quad r = 1,2...s, \quad i = 1,2......m$$

The constraints of the model 3.9 indicate that the inputs are fixed at the current level X_{io} or the adjusted amount ($X_{i_0} - S_i^-$) to be precise. This finds the largest value of ϕ by which outputs can be expanded. The slack variables s_r^+, s_i^- are introduced to convert the constraints from inequalities to equalities. s_r^+ is the output slack and the positive sign indicates augmentation of outputs and s_i^- is the input slack and the negative sign indicates reduction in inputs (Ozcan, 2014). The constraint space of equation 3.9 defines the production possibility set (Cook and Seiford,2009).

The dual of the above model is the **Envelopment output CCR Model** and it is as below.

$$\frac{1}{E_0} = \max\ \phi + \varepsilon\left(\sum_{r=1}^{s} S_r^+ + \sum_{i=1}^{m} S_i^-\right)$$

$$subject\,to\ \ X_{i_0} = \sum_{j=1}^{n} X_{ij}\lambda_j + S_i^- = X_{i_0} \qquad i = 1,2,3...m$$

$$\sum_{j=1}^{n} Y_{rj}\lambda_j - S_r^+ = \phi Y_{r_0} \qquad r = 1,2,3.....s$$

$$\lambda_j \geq 0 \quad j = 1,2,3........n$$

$$S_r^+ \geq 0 \quad r = 1,2,3.......s \qquad \textbf{(3.9)}$$

$$S_i^- \geq 0 \quad i = 1,2,3........m$$

$$\phi\ is\,unrestricted\ \ in\,sign$$

DEA identifies the most efficient units and indicates the inefficient units in which real efficiency improvement is possible. The amount of savings in resources or service improvement that is required to make the same unit efficient is identified and can be used as a benchmark for management. The CCR model assumes constant returns to scale *i.e., if all inputs are increased proportionally by a certain amount then the outputs will also increase proportionately* by the same amount and allows both input reducing and output increasing orientations(Kao,2017).

3.3.3: BCC (Banker, Charnes, and Cooper, 1984) Model:

When comparing DMUs differing significantly in size it was found by Banker, Charnes, and Cooper (1984), that the constant returns to scale assumption skewed the results. Therefore, in such cases it is important to know how the scale of operation of a DMU affects its efficiency or inefficiency (Kao,2017). Banker *et al. (1984)* developed a new formulation of Data Envelopment Analysis that is commonly known as BCC model. The BCC model is used to compute efficiency under the assumption of variable returns to scale *i.e. an increase in inputs need not necessarily yield a proportional increase in outputs.* The envelopment form of the BCC model would be the same as the CCR model but with an additional constraint sum of $\boldsymbol{\lambda_j = 1}$. ($\boldsymbol{\lambda_j}$,is the weight associated with the DMU and $\sum \lambda_j = 1$). The BCC model is shown below.

Banker et al., (1984) extended CCR model to allow variable returns to scale referred to as BCC model. Conceptually, the BCC model allows the production frontier to move away from the origin by introducing a constant that aggregates either the inputs or outputs (Kao,2017). This model also has two versions namely input and output oriented.

Firms operating at different scales are recognized as efficient with the introduction of this constraint**.** The introduction of this constraint enables formation of an envelope formed by the multiple convex linear combination of best practice DMU (Ramanathan, 2003). The effect of this is that it removes the constraint in the CCR model that the DMUs must be scale efficient. If $\sum_{j=1}^{n} \lambda_j \geq 1$, then the model is referred to as Non-Decreasing return to scale (NDRS) and $\sum_{j=1}^{n} \lambda_j \leq 1$, then the model is referred to as Non-Increasing return to scale(NIRS) (Bhatti, Bhanot and Singh,2014).

Input Model:

In the input model, the outputs are kept constant to calculate the expected minimum virtual input. The input efficiency is the ratio of the minimum virtual input to the actual virtual input aggregated from the multiple inputs (Kao,2017). The BCC model developed by Banker et.al (1984) to measure efficiency from the input side is as below(Kao,2017).

$$E_0 = \max \frac{\sum_{r=1}^{s} u_r Y_{r0} - u_0}{\sum_{i=1}^{m} v_i X_{i0}}$$

Subject to,

$$\frac{\sum_{r=1}^{s} u_r Y_{rj} - u_0}{\sum_{i=1}^{m} v_i X_{ij}} \leq 1 \quad j = 1, 2, 3....n \quad \text{.....................................} \quad \textbf{(3.10)}$$

$$u_r, v_i \geq \varepsilon, \quad r = 1, 2...s \quad i = 1, 2, 3......m$$

$$u_0 \text{ unrestricted in sign.}$$

The difference between this model and the model 3.4 under constant returns to scale is the presence of the intercept u_0 **.** The above fractional can be transformed to a LP problem using Charnes-Cooper transformation as below by assigning the denominator to one and leaving the numerator as the objective function. The Multiplier BCC input model is (Kao, 2017)

$$E_0 = \max \sum_{r=1}^{s} u_r Y_{r0} - u_0$$

Subject to,

$$\sum_{i=1}^{m} v_i X_{i0} = 1$$

$$\sum_{r=1}^{s} u_r Y_{rj} - u_0 - \sum_{i=1}^{m} v_i X_{ij} \leq 0 \quad j = 1, 2, \ldots . n \text{ ....} \mathbf{(3.11)}$$

$$u_r, v_i \geq \varepsilon, \quad r = 1, 2 \ldots . s, \quad i = 1, 2 \ldots . m$$

$$u_0 \text{ unrestricted in sign}$$

The envelopment BCC input model is the same as the envelopment CCR model with the inclusion of convexity constraint $\sum_{j=1}^{n} \lambda_j = 1$ (Kao,2017).

$$E_0 = \min \theta - \varepsilon \left(\sum_{r=1}^{s} S_r^{+} - \sum_{i=1}^{m} S_i^{-} \right)$$

$$\text{subject to } \theta X_{i_0} = \sum_{j=1}^{n} X_{ij} \lambda_j + S_i^{-} \quad i = 1, 2, 3 \ldots m$$

$$Y_{r_0} = \sum_{j=1}^{n} Y_{rj} \lambda_j - S_r^{+} \qquad r = 1, 2, 3 \ldots . . s$$

$$\lambda_j \geq 0 \quad j = 1, 2, 3 \ldots \ldots . n \qquad \text{...........} \mathbf{(3.12)}$$

$$S_r^{+} \geq 0 \quad r = 1, 2, 3 \ldots \ldots . s$$

$$S_i^{-} \geq 0 \quad i = 1, 2, 3 \ldots \ldots . m$$

$$\theta_0 \text{ is unrestricted in sign}$$

$$\sum_{j=1}^{n} \lambda_j = 1. \qquad \lambda_j \geq 0$$

BCC Output Model:

The output model looks for the maximum amount of outputs that can be produced from the given amount of inputs to measure efficiency. The output BCC fractional model for measuring efficiency of a DMU is as below (Kao,2017).

$$\frac{1}{E_0} = \min \frac{\sum_{i=1}^{m} v_i X_{io} + v_0}{\sum_{r=1}^{s} u_r Y_{r0}}$$

$$\begin{aligned} & \textit{subject to} \\ & \frac{\sum_{i=1}^{m} v_i X_{ij} + v_0}{\sum_{r=1}^{s} u_r Y_{rj}} \geq 1 \qquad j = 1, 2, ...n \\ & u_r, v_i \geq \varepsilon, \quad r = 1, 2, ...s, \quad i = 1, 2.......m \end{aligned} \qquad \text{..................... } \mathbf{(3.13)}$$

The fractional program is converted to LP problem using Charnes-Cooper transformation

and the **Multiplier BCC Output model** is as below(Kao,2017).

$$\begin{aligned} & \frac{1}{E_0} = \min \sum_{i=1}^{m} v_i X_{io} + v_0 \\ & \textit{subject to,} \\ & \sum_{r=1}^{s} u_r Y_{r0} = 1 \\ & \sum_{i=1}^{m} v_i X_{ij} + v_0 - \sum_{r=1}^{s} u_r Y_{rj} \geq 0 \qquad j = 1, 2...n \\ & u_r, v_i \geq \varepsilon, \quad r = 1, 2, ...s, \quad i = 1, 2.......m \\ & v_0 \ \textit{unrestricted in sign} \end{aligned} \qquad \text{...........} \mathbf{(3.14)}$$

The corresponding **Envelopment BCC Output model** which is dual of the above multiplier model is as below (Kao,2017).

$$\frac{1}{E_0} = \max\ \phi + \varepsilon\left(\sum_{r=1}^{s} S_r^{\ +} + \sum_{i=1}^{m} S_i^{\ -}\right)$$

$$subject\,to\ \ X_{i_0} = \sum_{j=1}^{n} X_{ij}\lambda_j + S_i^{-} = X_{i_0} \qquad i = 1,2,3...m$$

$$\sum_{j=1}^{n} Y_{rj}\lambda_j - S_r^{\ +} = \phi Y_{r_0} \qquad r = 1,2,3.....s$$

$$\lambda_j \geq 0 \quad j = 1,2,3........n$$

$$S_r^{\ +} \geq 0 \quad r = 1,2,3.......s \qquad ....\ \mathbf{(3.15)}$$

$$S_i^{-} \geq 0 \quad i = 1,2,3........m$$

$$\phi\ is\,unrestricted\ \ in\,sign$$

$\sum_{j=1}^{n}\lambda_j = 1$. $\lambda_j \geq 0$.This model is the same as the output CCR Envelopment model except that there is inclusion of convexity constraint $\sum_{j=1}^{n}\lambda_j = 1$.(Kao,2017)

A summary of various DEA models is given in the following table. In the table I stands for input-oriented model and O stands for output-oriented model. Efficiency score for all models ranges from 0 to 1. All models are unit variant meaning that inputs and outputs can be in any units. Semi-p means semi positive. This means non- negative with at least one positive element. Free permits data with negative, positive and can be zero.

Table 3.1. Summary of DEA Models (Constructed by author)

Model	MULTIPLIER MODEL				ENVELOPMENT MODEL			
	CCR- I	CCR-O	BCC-I	BCC-O	CCR- I	CCR-O	BCC-I	BCC-O
Data Input	Semi-p	Semi-p	Semi-p	Free	Semi-p	Semi-p	Semi-p	Free
Data Output	Free	Free	Free	Semi-p	Free	Free	Free	Semi-p
Unit Invariance	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Efficiency	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)
Returns to Scale	CRS	CRS	VRS	VRS	CRS	CRS	VRS	VRS

3.3.4 Multiplier Models:

In multiplier models $u_r, v_i \geq \varepsilon$ and ε is a small positive number ensures that all the variables are taken into account without being ignored by assigning zero to the corresponding multipliers while the efficiency score is determined (Charnes et al.,1979).This small number is called non-Archimedean number(Charnes and Cooper,1984). A DMU that is inefficient may end up as efficient in the absence of above constraint as one or more of the inputs or outputs could assign a zero weight to these variables. By restricting these weights greater than ε, the decision maker ensures that none of these variables are neglected by any of the DMUs. In multiplier models, the second constraint indicates that the output cannot exceed the input for any DMU. This makes the LP bounded as in the absence of the second constraint value of objective function will increase with increasing weights to outputs. However, this is restricted by the second constraint. As the weights of the inputs are determined, the second constraint restricts the assigning of weights to outputs so that the difference between the weighted sum of outputs and weighted sum of inputs should be less than zero (0) (Norman and Stoker,1991).

The above multiplier formulation also connects with economics. The objective is to maximize virtual output subject to unit virtual input (value of weights that gives optimal value) while maintaining the condition that virtual output cannot be more than virtual input for any DMU. This indicates that the conditions for Pareto optimality are met as any further increases in the maximal value can be attained only if some of the input values x_{ij} are increased or some of the output value y_{rj} are decreased(Kao,2017).

The multiplier models 3.5, 3.8, 3.11, 3.14 have $m+s$ variables and $1+n+m+s$ constraints. Since models' equation is a linear programming problem, it has a dual and it would have $1+n+m+s$ variables and $m+s$ constraints(Ramanathan,2003). In general, n is quite large as compared to $m+s$ and hence primal has large number of constraints as compared to dual. With every linear programming problem (maximization or minimization) there always exist another linear programming problem that is based on the same data and having the same solution. The original problem is called the **primal** problem while the associated one is called the **dual** problem. It is important to note that the two linear programming problems can be treated as the primal and the other as its dual(Ramanathan,2003). Due to less constraints in dual, dual formulation is computationally more efficient than primal(Ramanathan,2003).

3.3.5: Envelopment Form:

Therefore, primal is more difficult to solve with many constraints (Boussofiane et al., 1991). As the number of DMUs is generally larger than the total number of inputs and outputs, it is easier to solve the dual model as it reduces the burden of computation (Bhanot, Singh &Bhatti, 2014). The dual of the linear program is obtained by assigning a variable to each constraint and transforming the constraints and is called the **envelopment model.**

Any choice of λ_j provides an upper limit for the outputs and lower limit for the inputs of DMU_0 and against these limits E_0 is tightened with $\lambda_j, s_{i,}^{-} s_r^{+} \geq 0$ representing optimal choices associated with minimizing E_0. The collection of such solutions then provides an upper bound which envelops all the observations and hence leads to the name Data Envelopment Analysis(Kao,2017).

3.3.6 Production Possibility Set:

The BCC model allows variable returns to scale and measures only the pure technical efficiency of each DMU. In other words, if a DMU is to be considered as CCR efficient, it must be both scale and technically efficient. If the DMU needs to be considered BCC efficient then it needs pure technical efficiency (Bowlin, 1998). The production possibility set of both CCR and BCC models are shown graphically below (Cooper, Seiford and Tone,2007).

The CCR model addresses the aggregate (technical and scale), efficiency whereas the BCC model decomposes the overall aggregate efficiency of a unit into its *pure technical* and its *scale efficiency*.

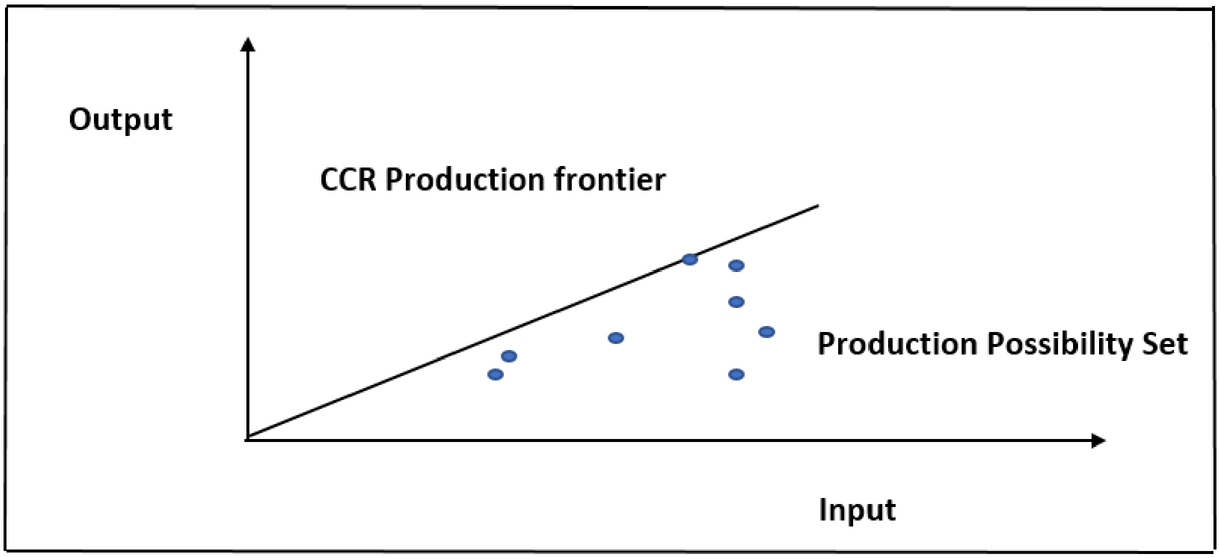

Figure 3.10. Production Frontier-CCR Model (constructed by author)

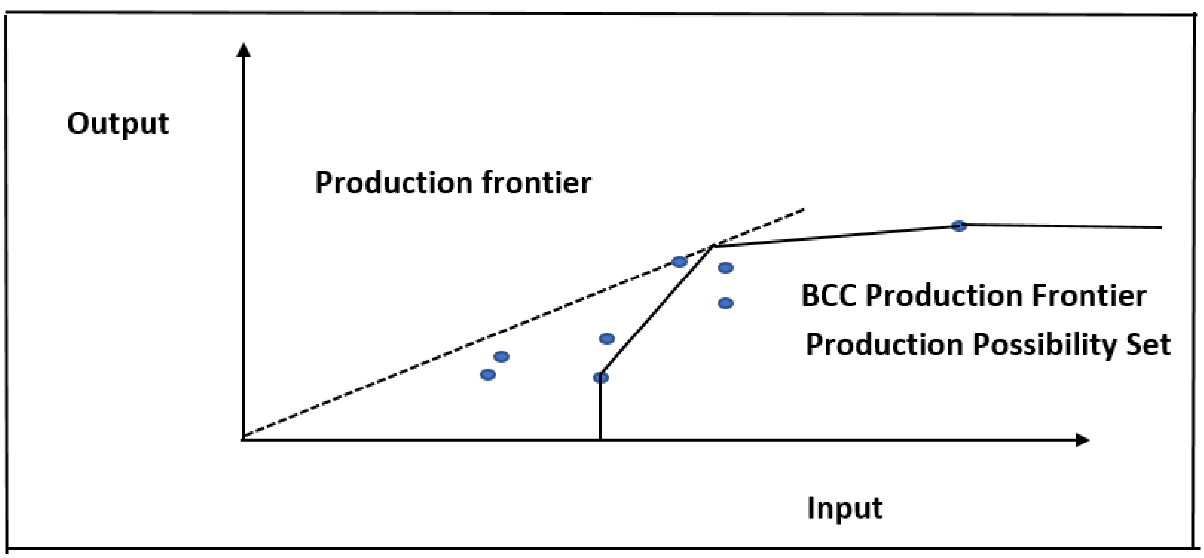

Figure 3.11. Production Frontier BCC Model (constructed by author)

3.3.7 Scale Efficiency:

The figure shows production possibility set for the input **x** and output **y.** The CRS and VRS frontiers are also shown for this input output mix (x, y). The input oriented technical inefficiency of the firm P under CRS is PP_C and under VRS the technical inefficiency would be

PP_V. The difference between the technical efficiency under CRS and VRS is P_CP_V. (CRS is constant returns to scale and VRS is variable returns to scale (Tim Coelli,2005).

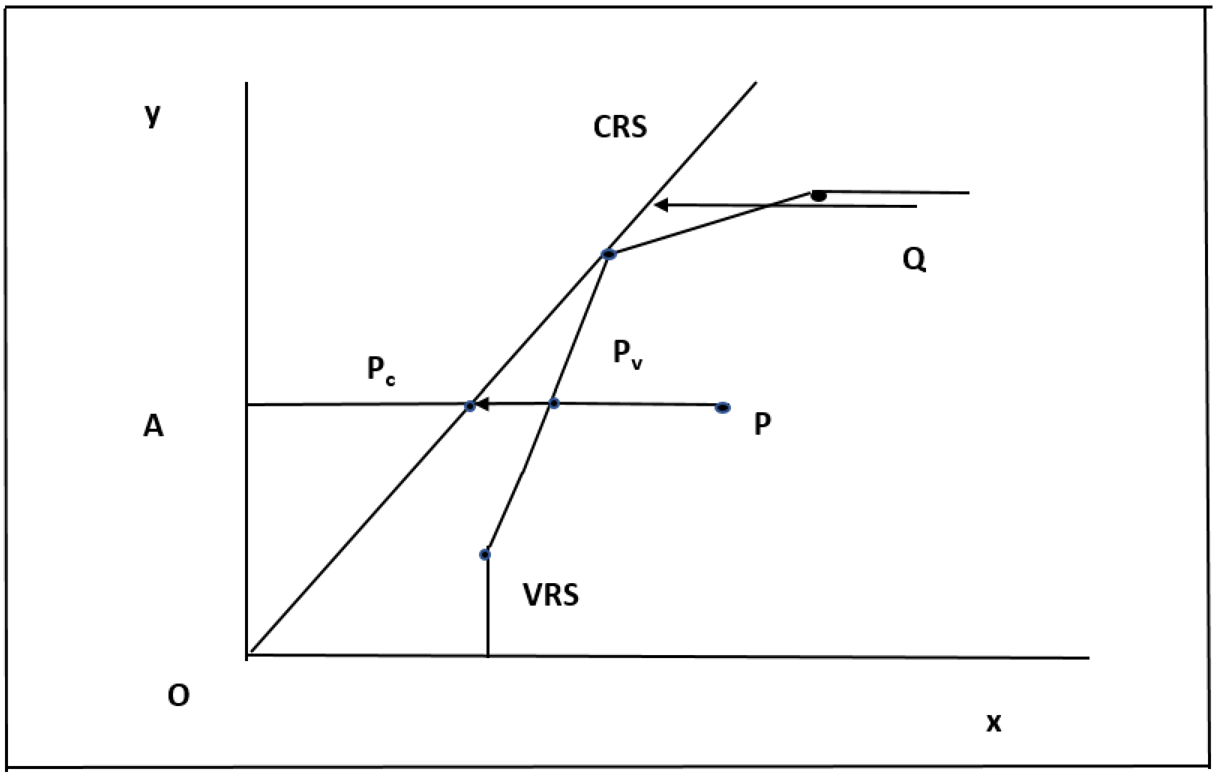

Figure 3.12.Technical and Scale efficiencies (constructed by author, Source: Coelli et al., 2005).

The technical efficiencies can also be expressed in ratios.

Technical Efficiency under CRS, $TE_{1CRS} = \frac{AP_C}{AP}$ TE_{1CRS}

Technical Efficiency under VRS, $TE_{1VRS} = \frac{AP_V}{AP}$ TE_{1VRS}

$$Scale\ Efficiency, SE_1 = \frac{AP_C}{AP_V}$$

$$\sin ce \frac{AP_C}{AP} = \frac{AP_V}{AP} \times \frac{AP_C}{AP_V}$$

$$TE_{1CRS} = TE_{1VRS} \times SE_1$$

The above equation proves that the CRS measure of technical efficiency is decomposed in to technical efficiency and scale efficiency (Tim Coelli,2005).

3.3.8 Peers of Firms:

DEA assumes convexity as in economic theory. Simply put this means that if two points are feasible in practice then a weighted average of the two points is also feasible. In other words, a convex combination is also feasible. If two observed DMUs are on the frontier, it can be proved that their convex combination is also feasible and lies on the frontier. The weighted combination of actual firms is called virtual firms or hypothetical firm, or composite firms and DEA compares the actual firms to composite(virtual) firms based on the assumption of convexity. Peers are the firms that are on the frontier or on the best performing practice frontier. The peers are used as the reference for benchmarking and to compare inefficient firms (Cooper, Seiford, Tone,2007).

In figure 3.13 firms A, B and G lie outside the frontier and are therefore inefficient. Firm A is projected on the frontier and falls on point D and D is an actual firm on the frontier. Therefore, firm D is a peer of the firm A with respect to efficiency measurement. In this instance inefficient firm A is compared to actual firm D. However, when firm B is projected on to the frontier, we have firm B'. Firm B' is on the efficient frontier and falls between firms, D and E. Hence both firms, D, and E, are peers of firm B and B are compared to the weighted combination of D and E (Cooper, Seiford, Tone,2007).

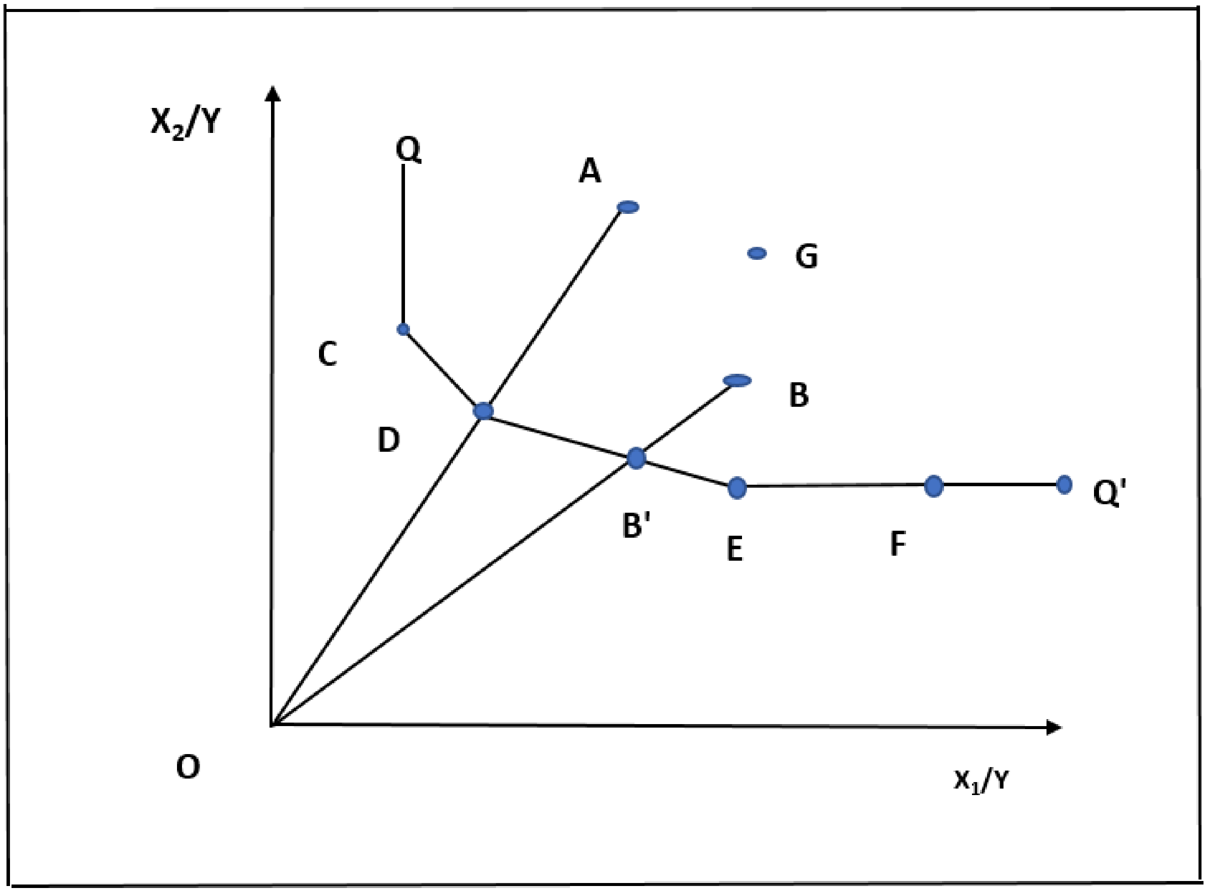

Figure 3.13. Peers of firms' (constructed by the author, source: Cooper et al., 2007)

3.3.9 Efficiency Measurement and Slacks:

There are instances where the piecewise linear form of efficient frontier in DEA can cause a few difficulties in measurement of efficiency. The problem arises when sections of piecewise linear frontier run parallel to one of the axes. Efficiency measurement of DMUs that lie on such parallel axes needs to be considered (Tim Coelli,2005).

In the figure below 3.14 let us consider the input combination of firms, C, and D that are efficient and are on the frontier. Firms A and B are inefficient as they lie outside the frontier. As per Farrell (1957), technical efficiency of firm A and B are $\frac{OA'}{OA}$ and $\frac{OB'}{OB}$

respectively. However, it is not quite true if firm A' is an efficient firm since one could reduce the amount of input X_2 by the amount CA' and still produce the same output. This is the excess of input X_2 used by firm A'. This is called as *input slack*. Hence the slack associated with firm A' has to be considered while calculating its efficiency score (Tim Coelli,2005). A similar situation will also occur for outputs, resulting in *output slacks* with respect to the outputs due to a shortfall in production (Tim Coelli,2005).

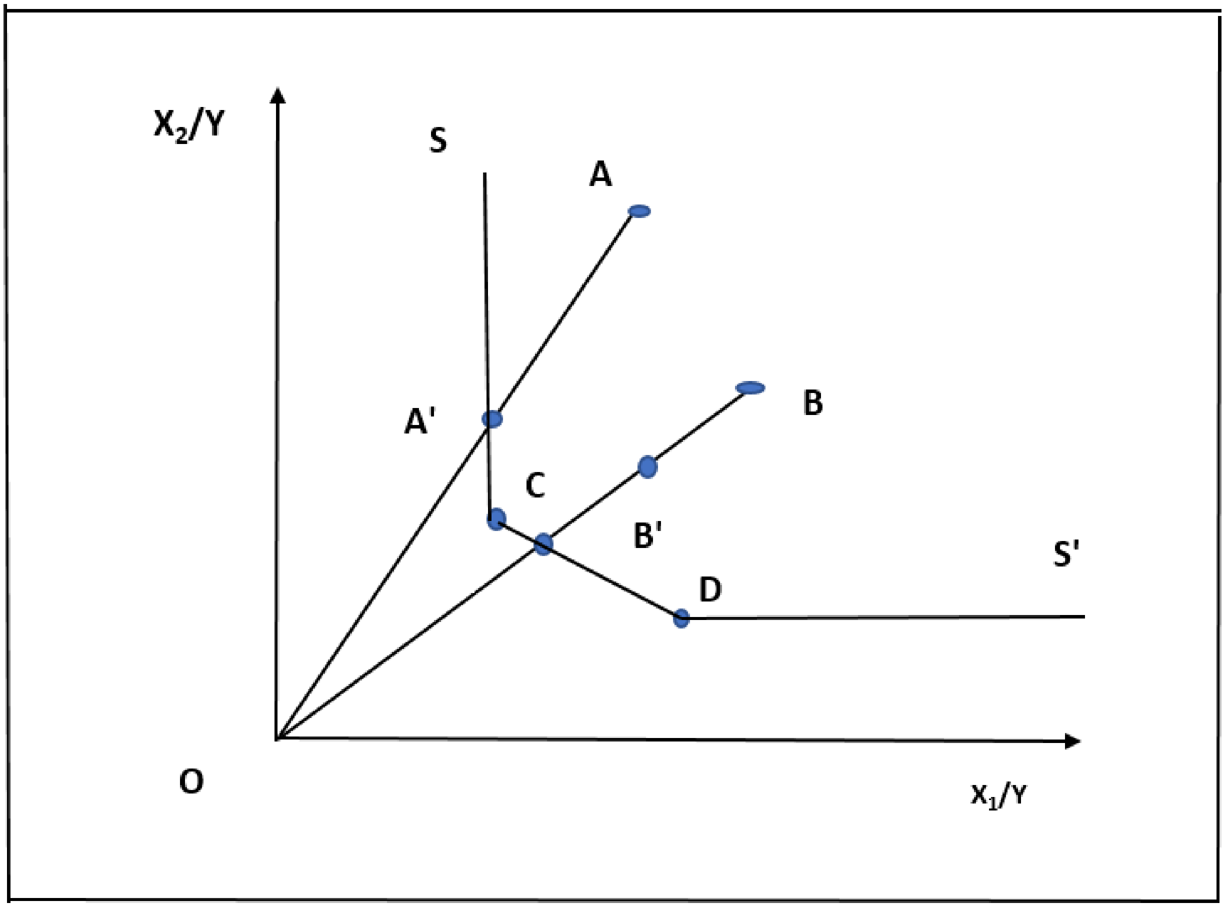

Figure 3.14. Input Slacks (constructed by author Source: Coelli et al., 2005)

3.3.10 Additive Model:

Cooper et al. (2007) computed a different type of non-radial measure by using

Additive Model. In CCR and BCC models, the distinction is required between input and output orientations whereas additive model combines both orientations**.** This is shown in the following figure.

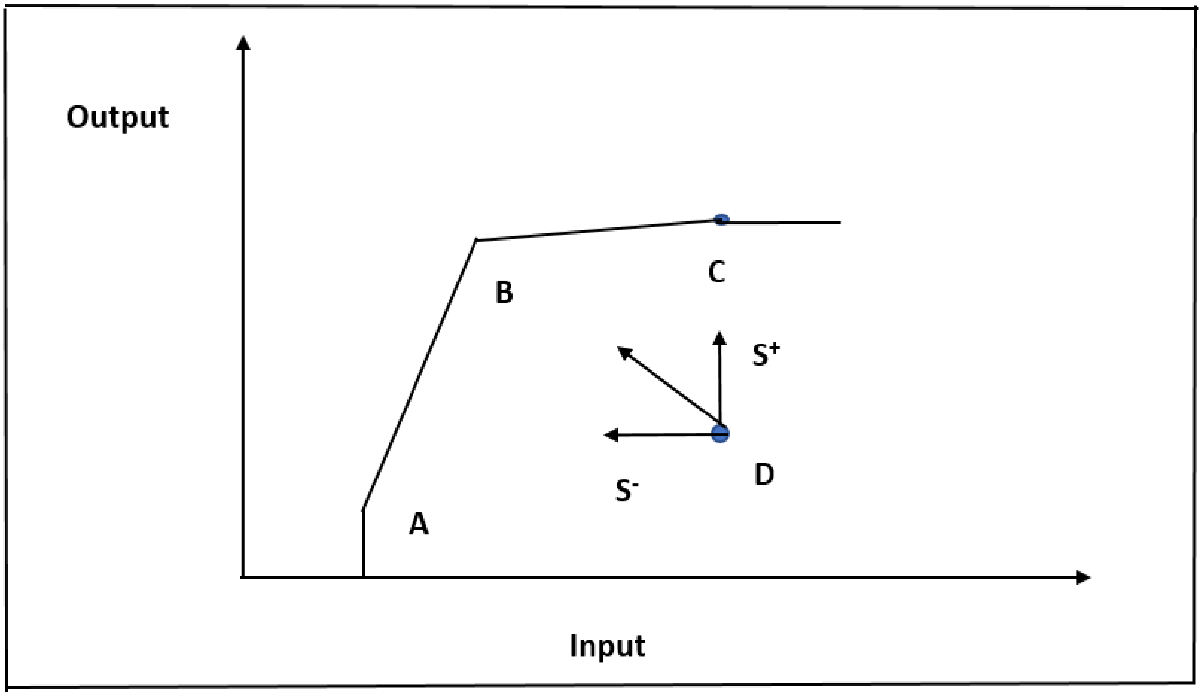

Figure 3.15. Additive Model (constructed by the author, Source: Cooper et al., 2007)

The output-oriented measure and the input-oriented measures are combined to determine a score of technical efficiency. It has the same envelopment surface as the BCC model that has variable returns to scale. However, it projects each DMU on to the envelopment surface in a direction that tends to increase the output and also reducing the input simultaneously. In the additive model the sum of all the slacks associated with each of the inputs and outputs is maximized. The difference is both CCR and BCC model requires a distinction between input and output orientation whereas the additive model combines both orientations.

3.3.11 Multiplicative Model:

The models previously discussed were CCR, BCC, and additive models. Charnes et al., (1982) developed a multiplicative model for efficiency analysis. The primary difference between this model and other model is that the virtual inputs and outputs are formed multiplicatively instead of additively. The frontier has a piecewise log-linear frontier rather than piecewise linear as is the case for CCR, BCC, and other models. The multiplicative model identifies inefficiencies through slack values like an additive model.

3.3.12 Slack Based Model (SBM):

The Slack Based Model is an extension of the additive model and was introduced by Tone (2001). The main difference between ADD and SBM model is that the former is an absolute measure (a summation of slacks) whereas SBM calculates an efficiency score based on the ratio of average relative input consumption to average relative output production. The measure is invariant with units of measurement of each input and output item (Cooper et al., 2007).

3.4: Ranking Methods in DEA:

The basic DEA models groups the two DMUs into two sets those that are efficient and define the Pareto frontier and those that are inefficient. Under these conditions in order to rank all the DMUs alternative approach or modification is required (Adler et al.,2002). To refine the evaluation of the units beyond the dual classification decision makers need a method that can completely rank all these units. Lack of discrimination in DEA applications is one of the problems that has been frequently discussed in the literature when DMUs do not meet the degree of freedom. That is when there are insufficient DMUs or the number of inputs and outputs is too high relative to the number of DMUs (Adler et al.,2002). Therefore, this is an additional reason for the complete ranking of units. The decision makers are interested in complete ranking and therefore techniques in ranking help in marketing the DEA approach. Such techniques to rank the DMUs should be considered post analysis as they do not replace the use of standard DEA models but provide added value to the analysis (Adler et al.,2002).

There are several techniques to rank the DMUs but in this section only the two most widely used techniques of Super Efficiency and Cross efficiency will be discussed in the following sections.

3.4.1: Super-Efficiency Models

While the efficiency scores have been calculated using the basic DEA models, one of the problems is to differentiate these efficient units as all of the efficient DMUs are found to have a score of unity. Andersen and Petersen (1993), provided an approach called Super-Efficiency model. In the super-efficiency model, the efficiency is evaluated using the standard DEA models (CCR or BCC) but under the assumption that the DMU being evaluated is excluded from the reference set. In the output-oriented case, the model provides a measure of the proportional increase in the outputs for a DMU that could take place without disturbing the efficient status of that DMU in relation to the frontier created by the remaining DMUs.

The super efficiency model helped in ranking the DMUs that are extremely efficient. The inefficient DMUs have an efficiency score of $0<\theta_j <1$ and hence have a natural ranking method based on the efficiency scores. However, all the efficient DMUs are on the boundary of the production possibility set and have a score of 1. This tie of all DMUs having a score of 1, has to be broken if they all have to be ranked. Anderson and Peterson (1993) suggested that by using super-efficiency model the extreme efficient DMUs can be ranked as their efficiency score in the super-efficiency model is no more bound by the upper limit of 1 but now more than one in Super efficiency model. The linear programs related to super-efficiency model are given after the graphical explanation of the model below.

Table 3.2. Sample data of inputs and outputs

DMU	Input 1, I_1	Input 2, I_2	Output 1 O_1
DMU1	4	18	1
DMU2	8	12	1
DMU3	12	7	1
DMU4	20	5	1
DMU5	24	5	1
DMU6	5	26	1
DMU7	18	10	1
DMU8	10	13	1
DMU9	16	6	1

Let us consider a hypothetical example of 9 DMUs having two inputs I_1 and I_2 producing one output O_1(Parthasarathy,2010). The following graph is drawn to scale with the above two inputs producing the same output for nine DMUs.

In the Figure 3.16 below, DMUs 1,2,3,9 4 and 5 have a technical efficiency of 1 as they are on the frontier. The other units not on the frontier can be ranked according to inefficiency scores as they are not unity. Let us consider an efficient unit DMU3 and let us try ranking them on SE score. The line connecting DMU1,2,9 and 4 and the vertical line above DMU1 parallel to the y-axis and the horizontal line beyond DMU 4 parallel to y-axis form the production possibility set when unit DMU3 is evaluated using SE score by using super efficiency model super-LP1. As DMU3 is extremely efficient it lies outside the PPS of DMU1,2,9 and 4 and hence super efficiency score $\theta_k > 1$. The score can be given geometrically as $\frac{ODMU3'}{ODMU3} = \frac{15.9511}{13.8924} = 1.1482$ = 114.82%. This indicates that unit DMU3 can increase its input usage by 1.1482 times and remain technically efficient. Similarly, the super efficiency scores of DMU1, DMU2, DMU9 and DMU 4 can be computed and they will be greater than 1 and therefore the DMUs can be ranked based on these scores.

The super-efficiency model was introduced by Banker and Gifford in 1988 and 1989 (Yao Chen 2005). Charnes, Haag, Jeska, and Semple (1992) used a super efficiency model to study the sensitivity of the efficiency classifications. Zhu (1996) and Seiford and Zhu (1999) developed a number of super-efficiency models to determine the efficiency of stability regions. Anderson and Peterson (1993) used the CRS super-efficiency model to rank efficient DMUs. Wilson (1995) used the super-efficiency model in detecting influential observations and Thrall (1996) used the super-efficiency model in identifying the extreme efficient DMUs. Banker and Chang (2006) have used super-efficiency models to identify outliers in the data and argued that super-efficiency should not be used for ranking extreme efficient units.

Anderson and Peterson (1993) failed to notice that as DMU k is compared with everyone else except itself, the solution to the LP problem can become infeasible. This was noted by Banker and Gifford (1989) and reported by Thrall (1996) and Banker and Chang (2006). Thrall (1996) shows that the super-efficiency CRS model can be infeasible. However, Thrall (1996) fails to recognize that the output-oriented CRS super-efficiency model is always feasible for the trivial solution which has all the variables set equal to zero. Zhu (1996) showed that the input-oriented CRS super-efficiency model and output-oriented VRS super-efficiency models is infeasible if and only if a certain pattern of zero data occurs in the inputs and outputs. Authors have suggested several methods to overcome infeasibility. However, the FPA approach proposed by Cheng and Zervopoulos (2011) is used in this study due to its availability in the software used for other analysis.

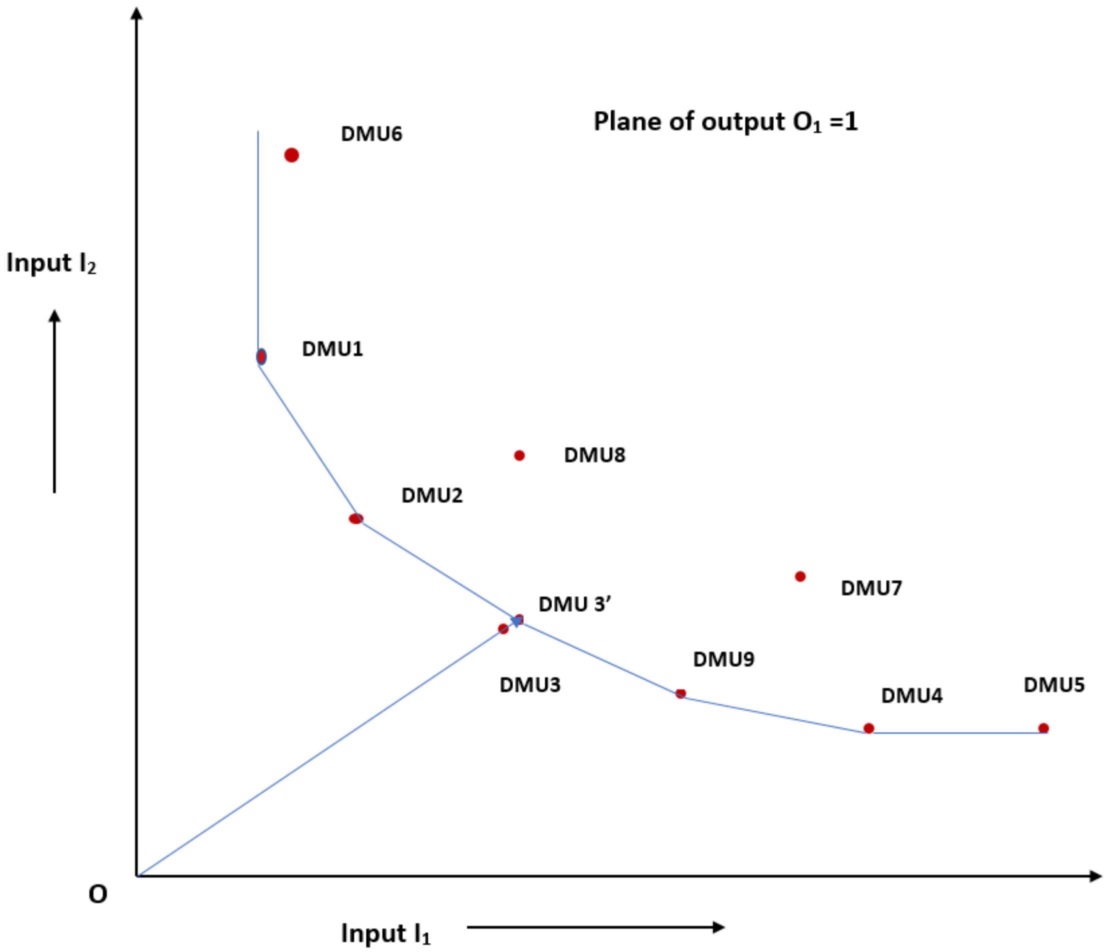

Figure 3.16. Graphical illustration of evaluating super-efficiency of DMU 3 (Created by the author. Source, Parthasarathy,2010)

In FPA approach a proxy unit for the efficient DMU is found on the frontier for the efficient DMU and this is projected on to the frontier constructed by other DMUs (Cheng and Zervopoulos,2011). The infeasibility issue remains still a researched topic in DEA.

The standard envelopment form or multiplier form of the super-efficiency model resembles the CCR and BCC models described in section 3.3.2 and 3.3.3 above except that the

DMU under evaluation DMU_e is not included in the reference set. In other words, when evaluating DMU_e, it is compared with all other DMUs and their convex combinations except itself.

The standard multiplier form of the super efficiency CRS model when the orientation is input minimization is shown below **Super- LP1(3.16) (**Anderson-Peterson,1993**)**.

$$h_k = \text{Max} \sum_{r=1}^{s} u_r y_{rk} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \textbf{(3.16)}$$

subject to

$$\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \geq 0 \; for j = 1 \ldots. n, \; j \neq k$$

$$\sum_{i=1}^{m} v_i x_{ik}$$

$$u_r \geq \varepsilon \; for \;\; r = 1, \ldots. s$$

$$v_i \geq \varepsilon \;\; for \;\; i = \;\; 1 \ldots\ldots m$$

The methodology enables an extremely efficient unit k to achieve an efficiency score greater than 1, by deleting the k th constraint in the primal formulation as shown above. The difference between the basic CCR model and Super efficiency model is the exclusion of DMU in the constraint set where j≠k. This enables outputs to be maximized without restriction and in turn, makes it possible to rank efficient units.

The envelopment form of the input-oriented super-efficient model as shown below computes the distance between the Pareto frontier evaluated without unit k and the unit itself i.e. for j=1,2,3…n and j≠k, and shown below **Super-LP2 (3.17) (**Zhu,2003**).**

$$
\begin{aligned}
& Minimise\, \theta_k \\
& Subject\ to \\
& \theta_k X_k - \sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j X_J \geq 0 \qquad \text{............... } \textbf{(3.17)} \\
& 0 + \sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j Y_J \geq Y_j \\
& \theta_k\ \ free\, \lambda_j \geq 0,\ j = 1....n,\ j \neq k
\end{aligned}
$$

The standard envelopment form for the output-oriented super efficiency CRS model is shown below **Super-LP3 (**3.18**) (**Zhu,2003**)**.

$$
\begin{aligned}
& Maximise\, \phi_k \\
& Subject\ to, \\
& \sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j X_j \leq X_k \qquad \text{.................... } \textbf{(3.18)} \\
& \phi_k Y_k - \sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j Y_j \leq 0 \\
& \phi_k\ \ free, \lambda_j \geq 0,\ j = 1...n,\ j \neq k
\end{aligned}
$$

The VRS counterparts of the above three SE model equations have the additional convexity constraint $\sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j = 1$ to the input-oriented model and $\sum_{\substack{j=1 \\ j\neq k}}^{n} \mu_j = 1$ for the output-oriented model (Anderson and Peterson,1993).

In the input-oriented model Super-LP2, the objective function value θ_k gives the savings in the input that a super- efficient DMU exhibits when compared to other DMUs. The higher the value of $\theta_{k,}$ than 1 for a super-efficient unit the greater the input saving in the unit. The super-efficient unit can increase its current input usage by (θ_k-1) x100% proportionately and remain efficient. Therefore, only super-efficient units will have θ_k values greater than 1 while using CRS or VRS super efficiency models and hence can be ranked using S-E score. The efficiency scores of the non, super-efficient units will remain the same irrespective of whether you use the normal model or the super-efficient model and will not affect the contour of the production possibility set (Charnes et al.,1991).

Super-efficiency models can be used for the following purposes (Lovell& Rouse,2003, Bhatti et al.,2014); a) Ranking of efficient DMUs b) Classification of DMUs in to extreme- efficient and non-efficient groups, c) Sensitivity of efficiency classifications, d) Two-person ratio efficiency games, e) Identifying outliers in the data, f) Overcoming truncation problems in second stage regressions intended to explain variation in efficiency, g) Calculating and decomposing a Malmquist productivity index.

In the formulation of the super-efficiency model, the DMU_0 under evaluation is excluded from the reference set as shown in the Figure 3.16 above, thereby producing a super-efficiency score for each DMU. However, under certain conditions, this procedure can lead to infeasibility. According to Charnes, Cooper, and Thrall (1991), the DMUs can be partitioned into four categories E, E', F and N. DMUs E are the set of extreme efficient DMUs, E' are DMUs that are not extreme points. DMUs E' can be expressed as a linear combination of the DMUs in set E. The third class of DMUs F, are those that are with non- zero slacks and lie on the edge of the frontier and not on the frontier itself. These are usually called weakly efficient. The fourth set of DMUs N, are inefficient DMUs. When DMU_0 belongs to E', F or N, DEA models are always feasible and equivalent to the original DEA models. We must consider the case of extreme efficient unit E. Zhu (1996) showed that input-based SE-CCR model is infeasible if a certain pattern of zero data occurs in the inputs and outputs and similar infeasibility occurs in the output-oriented VRS model.

3.4.2 Cross –Efficiency Models:

Although DEA has proved to be an effective approach in identifying the best practice frontier, its ability to choose weights independently and its nature of self -evaluation has been criticized. The cross-efficiency method was developed as an extension to DEA to rank DMUs (Sexton et.al., 1986) with the idea of doing peer evaluation of DMUs rather than allowing it to operate purely in a self-evaluation mode. Doyle and Green (1994), further investigated Cross-Efficiency. The two main advantages of the cross-evaluation method are one, it provides an ordering among DMUs and second, it eliminates unrealistic weight schemes without requiring experts from application area to endorse the weight restrictions (Anderson et al.,2002).

Cross-efficiency has been used in various applications. Sexton et al.,1986 used it in efficiency evaluation of nursing homes, Oral et al.,1991 used it in R&D project selection and Greene et al., 1996 used in preference voting and project ranking using DEA and cross-evaluation.

According to Doyle and Greene (1994), the usefulness of cross efficiency is reduced by the non-uniqueness of DEA optimal weights/multipliers. The cross-efficiency scores obtained from the original DEA methodology are generally not unique. Therefore, it may be possible to improve DMU's cross efficiency performance rating based on the choice of the alternate optimal solutions to the linear program (Cook and Zhu,2005). But this can be done only by worsening the rating of the others. Sexton et al., (1986) and Doyle and Greene (1994) proposed a secondary goal to deal with the non- unique DEA solutions. They developed aggressive and benevolent model formulation to identify optimal weights that not only maximizes the efficiency of a DMU under evaluation but also minimize or maximize the average efficiency of other DMUs.

The concept of cross-efficiency is illustrated by adopting the cross-efficiency matrix from Doyle and Greene (1994) in Table 3.3 as below.

There are six DMUs and E_{dj}, is the cross efficiency of DMU_j based upon a set of DEA weights calculated for DMU_d. The DMU weights calculated as above gives the best efficiency score for DMU_d under evaluation by a DEA model. The cross efficiency for a given DMU_j is defined as the arithmetic average down column j given by $\bar{\mathbf{E}}_{\mathbf{j}}$.

Let us say there are n, DMUs and each DMU_j has s different outputs and m different inputs. The i_{th} input and r_{th} output of DMU_j (j=1,2, 3, ... n) as x_{ij} (i=1,2,3,..m) and y_{rj} (r=1,2,3…s) respectively. Cross- efficiency is calculated in two phases.

In phase 1, DEA scores are calculated using the CRS model of Charnes et al. (1978). In phase 2, the multipliers arising from phase 1 are applied to all peer DMUs to arrive at the cross -evaluation score for each of the DMUs (Cook &Zhu, 2015).

Table 3.3. Cross-Efficiency Matrix (Constructed by author, Source: Doyle and Greene,1994)

Rated DMU

Rating DMU	**1**	**2**	**3**	**4**	**5**	**6**	**Averaged Appraisal of peers**
1	E_{11}	E_{12}	E_{13}	E_{14}	E_{15}	E_{16}	A_1
2	E_{21}	E_{22}	E_{23}	E_{24}	E_{25}	E_{26}	A_2
3	E_{31}	E_{32}	E_{33}	E_{34}	E_{35}	E_{36}	A_3
4	E_{41}	E_{42}	E_{43}	E_{44}	E_{45}	E_{46}	A_4
5	E_{51}	E_{52}	E_{53}	E_{54}	E_{55}	E_{56}	A_5
6	E_{61}	E_{62}	E_{63}	E_{64}	E_{65}	E_{66}	A_6
	$\bar{E}_1$	$\bar{E}_2$	$\bar{E}_3$	$\bar{E}_4$	$\bar{E}_5$	$\bar{E}_6$	

Average Appraisal by Peers(Peer Appraisal) -> Cross Efficiency

The output-oriented CRS multiplier CE model is given below.

$$\begin{aligned}
& \boldsymbol{Min} \sum_{r=1}^{s} v_{id} x_{id} \\
& \boldsymbol{subject\ to,} \sum_{i=1}^{m} v_{id} x_{ij} - \sum_{r=1}^{s} u_{rd} y_{rj} \geq 0, j = 1,2,3...n \\
& \sum_{i=1}^{n} u_{rd} y_{rd} = 1 \qquad \text{..........................}\textbf{(3.19)} \\
& v_{id} \geq 0, i = 1,2...m \\
& u_{rd} \geq 0, r = 1,2.....s
\end{aligned}$$

And the input-oriented CRS multiplier model is given below. …….

$$\begin{aligned}
& \max E_{dd} = \sum_{r=1}^{s} u_{rd} y_{rd} \\
& \boldsymbol{Subject\ to,} \\
& \sum_{i=1}^{m} v_{id} x_{id} = 1 \qquad \text{……….}\ \textbf{(3.20)} \\
& \sum_{r=1}^{s} u_{rd} y_{rj} - \sum_{i=1}^{m} v_{id} x_{ij} \leq 0, j = 1...n \\
& u_{rd}, v_{id} \geq 0
\end{aligned}$$

With the addition of the convexity constraint, the above models will become VRS output and input-oriented Cross-Efficiency models.

The cross efficiency goes beyond self -evaluation that is inherent in the regular DEA analysis and combines this with another (n-1) scores obtained using peer multipliers. This approach was originally proposed by Sexton et al., (1994), and was further investigated by Doyle and Greene (1994). An ordering among DMUs to differentiate between good and bad is provided by Cross efficiency thereby eliminating the need for additional weight restrictions on multipliers (Anderson et al., 2002).

As per Doyle and Greene (1994), cross –efficiency is a complement to simple efficiency. Cross efficiency can be used to overcome the problem of maverick DMU. Cross efficiency can be used to distinguish among 100%efficiecnt DMUs thus establishing meaningful ranking within that set. Mavericks are those DMUs that enjoy the greatest relative increment when shifting from peer efficiency to simple efficiency. When we say a DMU is maverick it means that this DMU operates far from rest of the DMUs.

Maverick Index is calculated by using the formula (Doyle &Greene, 1994)

$$\mathbf{M_k = (E_k - e_k)/\ e_k} \quad \text{.........}\mathbf{(3.21)}$$

Where $\mathbf{E_k}$ = Simple efficiency from the conventional DEA model.

and $\mathbf{e_k}$ = Mean cross efficiency score.

A DMU would be classified as a maverick if its simple efficiency is very high and its peer efficiency is very low leading to a low peer efficiency and a high maverick indicator.

3.4.3 Weight Restrictions in DEA

In the above analysis, in basic DEA models that are used to evaluate efficiency, no judgment has been made about the importance of one input versus another and also assumed that all the outputs had the same importance. However, in real life, the importance of various inputs and outputs varies.

Weight restrictions were first used by Dyson and Thanassoulis (1988) to utilize top corporate objectives on the relative importance of inputs and outputs used in the assessment of

London Boroughs and Metropolitan District rates department. Charnes et al., (1990) developed a model that can impose constraints on the weights to control how much a DMU can freely use the weights to become more efficient. However, Thannasoulis, Dyson, and Foster (1987) found the preselection of DMUs may not be a good approach in their study of tax rates department (rate-collection function of London Boroughs and Metropolitan District Councils.). Thompson et al., (1990) developed assurance region models (AR) or cone ratio models that is a generalized version developed by Charnes et al., (1990). This means that lower bounds and upper bounds can be established on the ratio of weights of a given pair of inputs or outputs, in such a way that DMU cannot freely choose weights to become efficient through using excessive outputs or insufficient inputs. In other words, the DMUs will use their inputs and outputs within a range that are as per policy or managerial requirements. This approach was called Assurance Region (AR1). Thomson et al., (1986) introduced ARII concept where restrictions are imposed on the ratio of input and output weights. Podinovski and Chameeva (2015, 2016) used consistent weight restrictions with production trade-offs in their weight restriction models.

A review of various weight restriction methodologies and their origin is presented in Allen et al., (1997) and Thannasoulis (2004) and these authors have discussed the advantages and limitations of different approaches. The weight restrictions that are included in DEA models can be classified as direct restrictions to weights and restrictions to virtual weights.

Under direct restrictions to weights there are three types of weight restrictions viz; Absolute weight restrictions, Assurance Regions Type 1(AR1) and Assurance Regions Type II (ARII). The restrictions to virtual outputs or inputs are called virtual weight restrictions and were originally proposed by Wong and Beasley (1990).

The following Figure 3.17, illustrates the concept of assurance region for the three styles of inputs in the heavy equipment dealership. By drawing lines from the origin, these styles can be identified by choosing the extreme use of either input. For example, the line drawn from the origin passing through DMU25 represents a situation that uses more service sales and less of parts sales proportionately. On the other hand, the line passing through DMU15 represents a high usage of parts sales with a proportionately less use of service sales. The other two lines going through DMU1 and DMU 18 shows a more balanced usage of either input. The three styles of usage pattern can thus be identified. Each of these styles can be shown in a cone in which the tip of the cone is the origin and hence the name cone ratio. If the manager of the branch decides that style one and style three are not an acceptable practice, then the manager can impose restrictions as per style two. For all practical purposes style two, can be termed as assurance region where the DMU operates efficiently.

The following ratios with upper and lower bound restrictions to impose restrictions on input or output weights can be written as (Ozacan,2014)

$$L_{ik} \leq \frac{v_i}{v_k} \leq U_{ik} \;\; where\, i = 1, 2, 3....m \text{ .... } \mathbf{(3.22)}$$

v_i and v_k represent the weights for two different inputs and L_{ik} and $U_{i,k}$ denotes the lower and upper bound on this ratio respectively. This indicates that there are possibilities to calculate many such ratios by establishing their lower and upper bounds. There are three outputs as in the present research, there can be 3! (n!) ratios, six ratios can be calculated(Ozcan,2014). However, for the manager the proper selection of ratios should be influenced by industry requirements so that policy and managerial implications can be tested appropriately. Finally, weight restrictions can also be used between input and outputs.

In the current research parts, sales and service sales contribute to more profitability than the total sales of equipment. Similarly, gross margin and sales revenue from equipment, service, and parts is a very important measure of profitability in all corporate organizations. These weight restrictions will be used in the analysis in Chapter 5.DEA in its original form allows total flexibility in the selection of weights such that each DMU will achieve maximum efficiency rating feasible for its input and output levels.

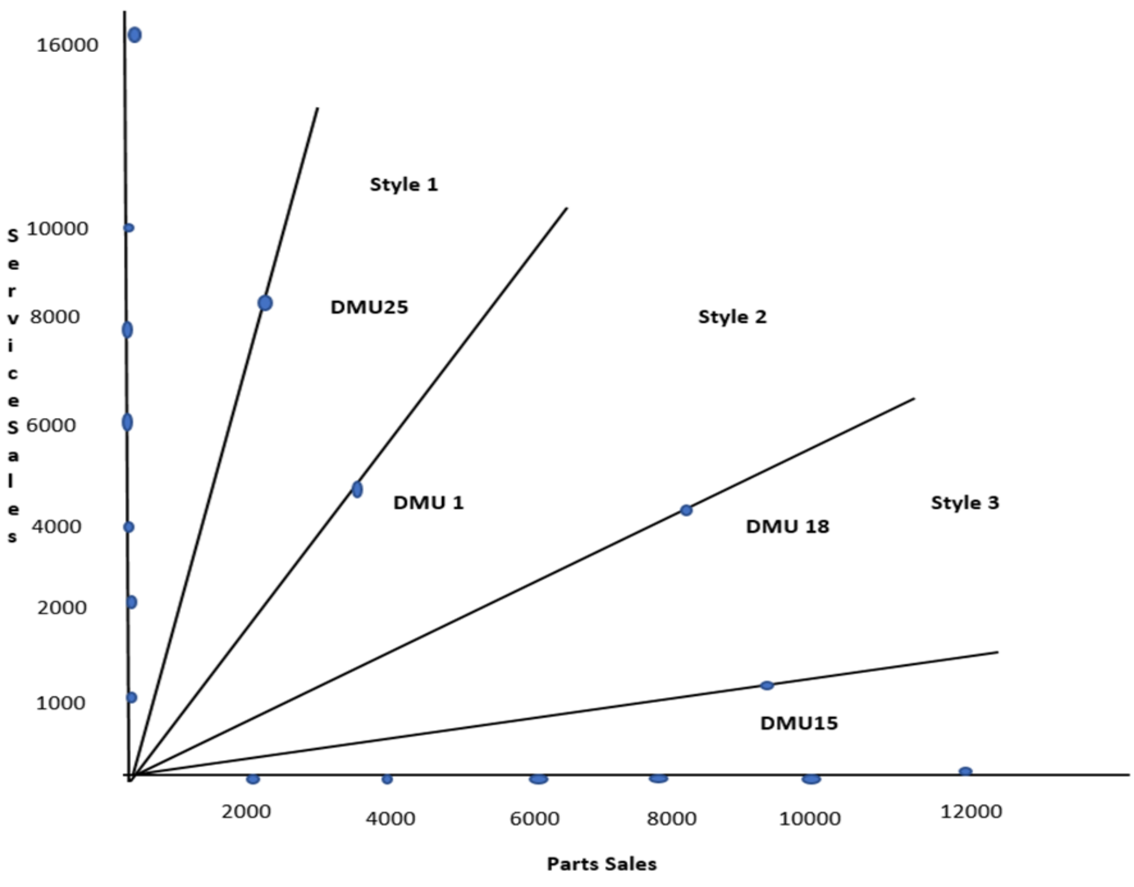

Figure 3.17. Conceptualization of Assurance region for inputs (Constructed by the author)

The prior knowledge of the relative values of inputs and outputs and then incorporated into the DEA model is called weight restrictions or value judgment. A value judgment is considered as a logical construct incorporated within an efficiency assessment study, reflecting the decision maker's preferences in the process of assessing efficiency (R. Allen et al., 1997).

3.5 Network DEA:

So far, the models reviewed in DEA to measure efficiency have considered the DMU as a complete system but ignoring the structure within the system. In other words, the DMU is treated as a black box (Fare & Grosskopf, 2000) where resources used as inputs produce outputs and there is a positive correlation between the two. In their study on efficiency Wang, Gopal and Zionts (1997) showed that bank operations had two processes collection of capital and investment and that the efficiency of the bank depends on the performance of both these divisions. A system may be evaluated as efficient using classical DEA but may be termed inefficient using network DEA. Similarly, there are DMUs where there are many processes that have a bearing on the performance of DMU. Such processes may have a parallel structure, series structure or a mixed structure. These structures are generally called network structures and the DEA technique to measure the efficiency of systems with a network structure is called network DEA (Fare & Grosskopf, 2000).

To understand the transformation process in the black box Fare and Grosskopf (2000) used a new formulation called Network- DEA (NDEA) model and it has proved fruitful in practical applications. They present three general net-work models; 1) A static network model where a finite set of sub-processes or activities are connected to form a network. This model helps to study the processes that are not visible in the black box approach of DEA.2) A dynamic network model structure enables one to analyze a sequence of production technologies. In this situation, a decision at one stage (e.g. time) impacts process at later stages. Here the intermediate products are accounted wherein outputs of one stage becomes inputs in the subsequent stage.3) A technology adoption model is the simplest model that allows analysis of production on different processors (e.g. machines) to allow one to determine the choice of technology.

Castelli et al., (2001) described a network model that evaluates the efficiencies of each of several interdependent sub-units within a larger DMU, called sub DMU. It is important to note that you may get misleading results by ignoring the operations of the component process (Kao &Hwang, 2008). There are also instances in which all the internal processes of a DMU have performances that are worse than those of the DMU and yet the former has better system performance (Kao& Hwang, 2008). Therefore, these findings indicate that while measuring efficiencies network DEA is needed for measuring the performance more accurately. Several models related to the basic two-stage system were reviewed by Cook, Liang and Zhu (2010), where there are only two processes connected in series and the second process consumes all the outputs from the first process for production. Other network models such as *shared flow* and *multilevel models* were reviewed by Castelli, Presenti, and Ukovich (2010). However, there are several types of models that have still not been studied. Some of these models are related to the type of data such as probabilistic data, qualitative data, fuzzy data and incomplete data and some models that study the dynamic analysis of network systems (Kao, 2014).

There are many types of structures in Network systems and every study on network DEA is associated with a structure that is very specific to the business organization. A network model for measuring efficiency is developed based on the needs of the business organization for practical applications. It is found from Studies on NDEA as mentioned in the literature review in Chapter 2 that efficiency of a DMU is very much dependent on the internal structure of the DMUs. Therefore, it is proposed to study in the current research, the internal structure of the DMU so that the cause of inefficiencies in the internal structure if any can be identified.

As indicated in the schematic diagram of the internal structure of DMU, Fig 3.18 the DMU has three components. The three components, the sales operations, service operations and parts operations form the three sub DMUs of the DMU.

All the above operations in a DMU happen independently and are not dependent on each other. In other words, all of the above operations happen independently and therefore follow a parallel structure. The following is the analysis of the structure of the heavy equipment distributor under study.

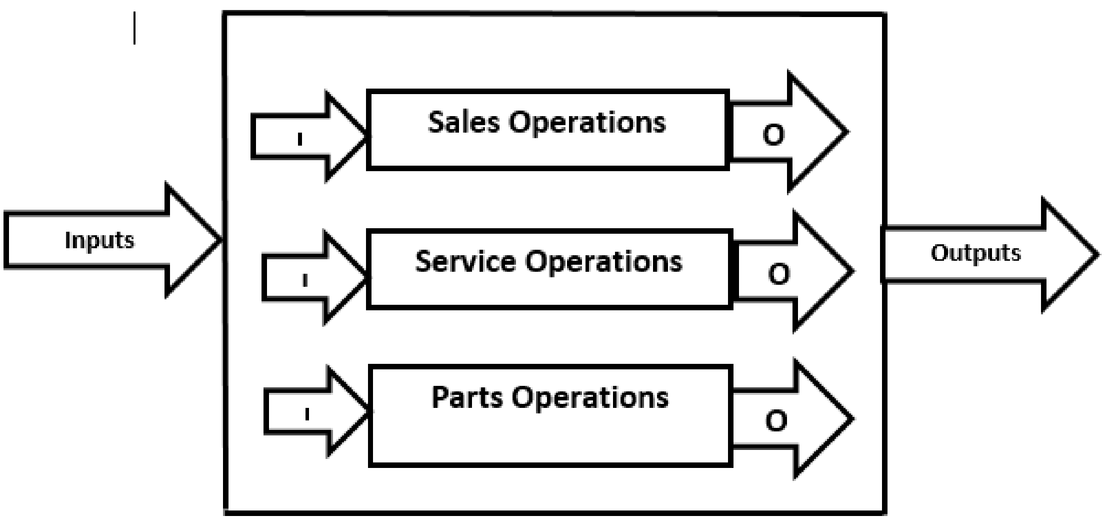

Figure 3.18. Internal Structure of the branch (DMU) under study

a) The Main distributor of the heavy equipment has thirty-three business units across Canada. In other words, each branch is a DMU and is an independent decision-making unit.

b) In each of these units, there are three different subunits called Sales, Service and Parts operations. Each of these sub DMUs, operate independently and are parallel in nature.

Therefore, from the above it is clear that the DMU is a combination of three sub DMU, having a parallel structure. Therefore, the efficiency of a subunit can also be computed as a parallel model. Then Summation of the efficiencies of the three sub DMUs will be the efficiency measure of a single DMU(Kao,2009). The efficiency of the distributor is the summation of the efficiency of thirty-three DMUs under study.

In the current research efficiency of DMU will be studied using the following two Network DEA models.

1) Network DEA: Using Parallel Structure.
2) Network DEA: Two-stage structure

The application of the above two Network DEA models is given in Chapter 5.

3.5.1. Efficiency Measurement of Parallel Structure Model:

The CCR model (Charnes et al., 1978) measures relative efficiency of DMUs that use the same inputs to produce the same outputs. Let us consider there are **n** DMUs. The k_{th} DMU utilizes **m** inputs $\mathbf{X}_{ik}$, i =1, 2, 3… m to produce **s** outputs $\mathbf{Y}_{rk}$, r =1,2,3…s. Its efficiency E_k is calculated by the following CCR model. Model 1 (Kao, 2009).

$$E_k = \max \sum_{r=1}^{s} u_r Y_{rk}$$

$$s.t \ \sum_{i=1}^{m} v_i X_{ik} = 1$$

$$\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \le 0, \ j = 1,2,3...n \qquad \text{.........} \ \mathbf{(3.23)}$$

$$u_r, v_i \ge \varepsilon, \ r = 1,2,3...s, \ i = 1,2,3....m$$

Here $\mathbf{u_r}$ and $\mathbf{v_i}$ are the most favorable multipliers to be applied to the **r** $_{\mathbf{th}}$ output and $\mathbf{i_{th}}$ input for DMU **k** in calculating its efficiency E_k.ε is a small non- Archimedean constant that ensures that all inputs and outputs are included in the calculation however small they may be.

In the real world of heavy equipment distributor efficiency study as above, there are thirty-three DMUs each using the same inputs to produce the same outputs. The firm 's outputs and inputs are the sums of all DMUs inputs and outputs. Each DMU has a parallel structure and has sub DMUs. Let us consider a general case of a DMU k with q production units as in the figure 3.19 below.

Each production unit p, p=1,2…q converts inputs X_{ik}^{p} , i = 1…m into outputs Y_{rk}^{p} r =1,2…s independently. The sums of all X_{ik}^{p} over p, $\sum_{p=1}^{q} X_{ik}^{p}$ and all Y_{rk}^{p} over p, $\sum_{p=1}^{q} Y_{rk}^{p}$ are the input X_{is} and output Y_{rk} of the system respectively (Kao, 2009). A parallel DEA model calculates the efficiency of the entire system as well as the efficiencies of individual production units that are sub-DMUs. The conventional CCR model 3.28 as above, measures the performance of a DMU in terms of efficiency. On the other hand, efficiency can also be measured from the inefficiency point of view.

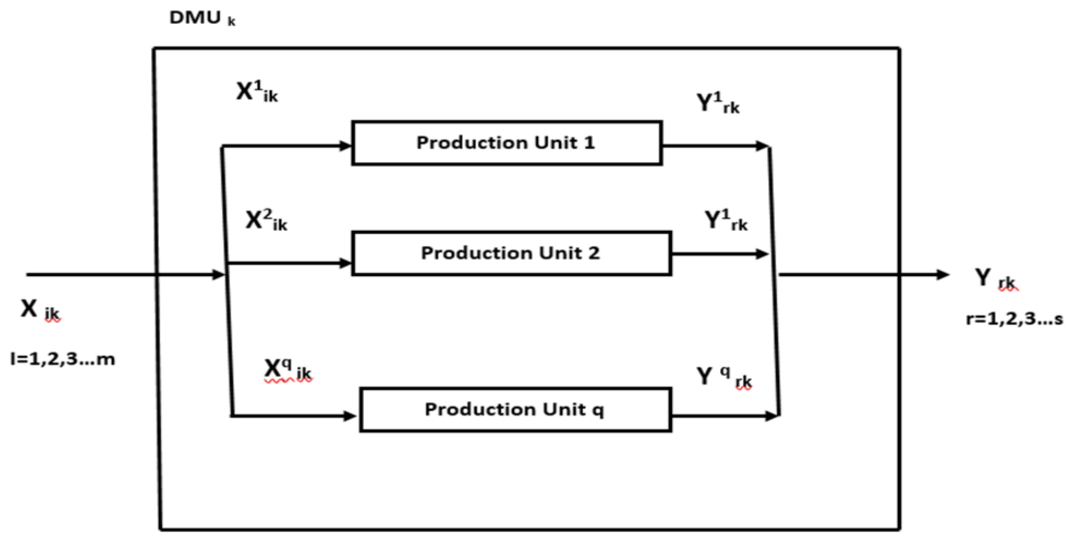

Figure 3.19. Parallel Network Structure (constructed by author. Source: C. Kao, 2009)

as the complement of efficiency E_k, is inefficiency $1\text{-}E_k$. The objective of maximizing efficiency is equivalent to minimizing inefficiency. From, the above model 3.23, the inefficiency of DMU k is

$$= 1-\sum_{r=1}^{s} u_r Y_{rk}$$

This is equal to the slack s_k in the equation,

$$\sum_{r=1}^{s} u_r Y_{rk} - \sum_{i=1}^{m} v_i X_{ik} + s_k = 0$$

Please note that $\sum_{i=1}^{m} v_i X_{ik}$ is equal to 1 as per the condition of model 3.23.

Now model 3.23, is equivalent to the following program (Kao, 2009).

Min s_k ,

$$\sum_{i=1}^{m} v_i X_{ik} = 1$$
$$\sum_{r=1}^{s} u_r Y_{rk} - \sum_{i=1}^{m} v_i X_{ik} + s_k = 0$$
$$\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \le 0 \quad j = 1,2,3....n,\ j \ne k,$$
$$u_r, v_i \ge \varepsilon, r = 1,2...s, i = 1,2....m$$

………… **(3.24)**

The slack variable S_k represents the inefficiency score.

In the parallel production system, each input/output of the system is the sum of all its production units. Therefore, we have

$$\sum_{r=1}^{s} u_r Y_{rk} - \sum_{i=1}^{m} v_i X_{ik} + s_{k,}$$
$$= \sum_{r=1}^{s} u_r (Y_{rk}^1 + Y_{rk}^2 + ...+ Y_{rk}^q) - \sum_{i=1}^{m} v_i (X_{ik}^1 + X_{ik}^2 + ...+ X_{ik}^q) + s_k$$
$$= \sum_{p=1}^{q} 1\left(\sum_{r=1}^{s} u_r Y_{rk}^p - \sum_{i=1}^{m} v_i X_{ik}^p - \right) + s_k = 0,$$
$$u_r^*, v_i^* \ \ and\ w_p^*$$

.... **(3.25)**

Where X_{ik}^p and Y_{rk}^p are the i^th input and r^th output of the p^th production unit within this DMU k. Please note $(\sum_{r=1}^{s} u_r Y_{rk}^p - \sum_{i=1}^{m} v_i X_{ik}^P)$ in the above equation 3.25 represents the production mechanism of the p^th production unit(Kao,2009). This must be non-positive in determining the most favorable multipliers u_r and v_i for DMU k to fulfill the definition of efficiency.

Let s_k^p denote the slack associated with the p^th^ production unit.

Total slack of the system S_k, can be allocated to its q production units

$s_k = \sum_{p=1}^{q} S_k^p$ and the last equation 3.25 can be written as

$$\sum_{p=1}^{q}\left(\sum_{r=1}^{s} u_r Y_{rk}^p - \sum_{i=1}^{m} v_i X_{ik}^p + s_k^p\right) = 0$$ **3.26** (Kao,2009)

Each quantity in the parentheses is equal to zero, the following are the q constraints.

$$\sum_{r=1}^{s} u_r Y_{rk}^p - \sum_{i=1}^{m} v_i X_{ik}^p + s_k^p = 0,\ p = 1, 2, 3..q$$ .... **(3.27)** (Kao,2009)

The constraint associated with each DMU other than **k** in the model (3.24) is replaced by the same constraints corresponding to its q sub-DMUs. Each DMU can have a different number of sub-DMUs. For simplification of notation, a common number q is used. The DEA model for calculating the relative efficiency of a set of **n** DMUs, that has q parallel sub-DMUs is,

$$\min \sum_{p=1}^{q} S_k^p,$$
$$s.t \sum_{i=1}^{m} v_i X_{ik} = 1,$$
$$\sum_{r=1}^{s} u_r Y_{rk}^p - \sum_{i=1}^{m} v_i X_{ik}^p + s_k^p = 0, p = 1,2,3....q$$
$$\sum_{r=1}^{s} u_r Y_{rj}^p - \sum_{i=1}^{m} v_i X_{ij}^p \leq 0, p = 1,2,3...q, j = 1,2,3..n, j \neq k \quad \text{…….. (3.28)} \quad \text{(Kao,2009)}$$
$$u_{r,} v_i \geq \varepsilon, r = 1,2,3..s, i = 1,2,3..m$$

The above model will be solved **n** times once for each DMU to calculate the inefficiency slacks of the systems as well as their sub-DMUs. The decision maker can identify the sub-DMUs with large inefficiency slacks by decomposing inefficiency, accordingly make improvements (C. Kao,2009).

The efficiency score of the wth, production unit of the kth DMU is not 1- s_k^w. This is because

$\sum_{i=1}^{m} v_i X_{ik}^w$ is is not equal to 1.

As per the second constraint of model 3.28, s_k^w must be divided by $\sum_{i=1}^{m} v_i X_{ik}^w$ to obtain efficiency score of:

$$1 - \frac{s_k^w}{\sum_{i=1}^{m} v_i X_{ik}^w}$$

The main difference between the parallel DEA and the conventional DEA model is that the constraint for each DMU has been replaced by those associated with the sub-DMUs.

As per equation 3.25, the sum of constraints associated with the production units is equal to the constraints of the system. Alternately the constraints of model 3.28 are stronger than the constraints of model 3.24. This makes the efficiency score calculated from the DEA model smaller than that calculated from the conventional model. This will be verified using real-life data from the heavy equipment dealerships of sales, parts and service operations input /output data (C. Kao, 2009).

The above model is under the assumption of constant returns to scale (CRS) which means that a relative increase in the input will result in the proportional increase in its output. The above model can be studied under variable returns to scale, the BCC model by adding a convexity constraint U_{0e}. BCC model measures the pure technical efficiency.

$$
\begin{aligned}
&\min \sum_{p=1}^{q} S_k^p, \\
&s.t \quad \sum_{i=1}^{n} v_i X_{ik} = 1 \\
&\quad \sum_{r=1}^{s} u_r Y_{rk}^p - \sum_{i=1}^{m} v_i X_{ik}^p + S_k^p = 0,\ p = 1,2,3\ldots q \qquad \ldots\ldots\ldots \ \textbf{(3.29)} \\
&\quad \sum_{r=1}^{s} u_r Y_{rj}^p - \sum_{i=1}^{m} v_i X_{ij}^p - U_{0e} \le 0,\ p = 1\ldots q,\ j = 1..n,\ j \ne k \\
&\quad u_r \, and \ v_i \ge \varepsilon, r = 1\ldots..s, i = 1\ldots m
\end{aligned}
$$

The variable U_{0e} determines the type of returns to scale and if less than 0 then it is decreasing returns to scale, greater than zero then it is increasing returns to scale and if equal to 0 then it is constant returns to scale (Amirtemoori et al., 2013). The model is applied to the heavy equipment dealership under study as below. The, figure3.18 shows a sample of network DEA of each sub DMU of sales, service and parts operation each with one input and one output.

There are many types of structures in Network systems and there is a structure associated with every study on network DEA and that is very specific to the business organization. A network model for measuring efficiency is developed based on the needs of the business organization for practical applications. Various network structures have been reviewed in-depth in literature review and methodology.

3.5.2 Network DEA Two Stage Process:

In the current study of the efficiency of the heavy equipment retailing organization, a branch's operation can be viewed as a multistage process in series with intermediate products. In a branch operation, the inputs are number of employees, the area of the facility, total department expenses and total COGS of the branch. With these inputs equipment sales, rental sales, service sales, and parts sales are realized. In other words, realizing sales can be viewed as a first intermediate stage in generating profit. Generation of profit margin by selling equipment, rentals, service, and parts can be viewed as a second stage. In short, the process can be considered as a process in tandem where branch sales are the first intermediate product and generating profit margin is the second stage.

For a system such as the above the relational two-stage model of Kao & Hwang, 2008 can be applied and efficiency can be calculated. The process is depicted in figure 3.20 as below. The inputs are X_{mk} and outputs are Y_{sk} and Z_{qk} are an intermediate process.

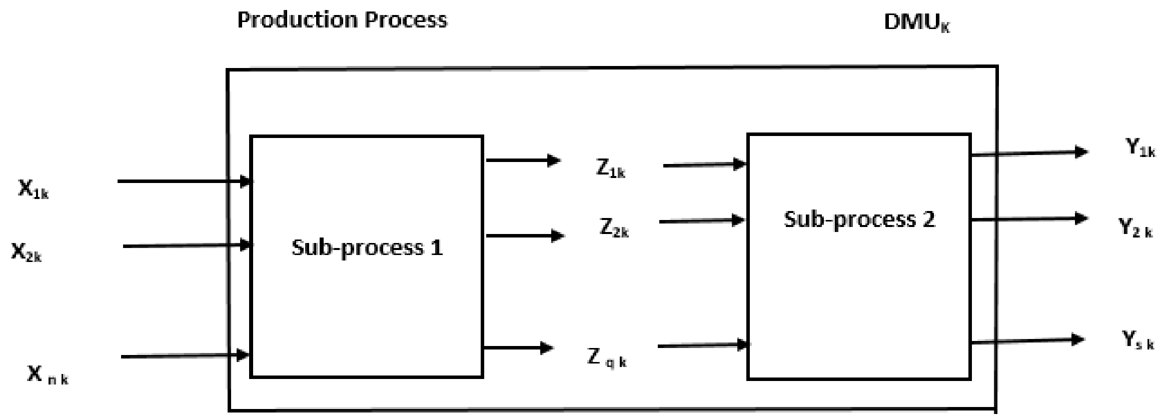

Figure 3.20. A system with inputs X and Y and intermediate products Z (Kao & Hwang, 2008).

$$
\begin{aligned}
&E_k = \max \sum_{r=1}^{s} u_r Y_{rk} \\
&\textit{subject to,} \\
&\quad \sum_{i=1}^{m} v_i X_{ik} = 1, \\
&\quad \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \le 0,\ j = 1,2,....n \\
&\quad \sum_{p=1}^{q} w_p Z_{pj} - \sum_{i=1}^{m} v_i X_{ij} \le 0,\ j = 1,2,....n \\
&\quad \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} w_p Z_{pj} \le 0,\ j = 1,2,....n \\
&\quad u_r, v_i w_p \ge \varepsilon, r = 1.....s, i = 1....m, p = 1...q \\
&\quad X_{ij} = \textit{Inputs}, Y_{rj} = \textit{Outputs}, Z_{pk} = \textit{Intermediate product} \\
&\quad E_K \ \textit{is efficiency.}
\end{aligned}
\qquad \dots\textbf{(3.30)}
$$

After the optimal multipliers $u_r^*, v_i^* \ and\ w_p^*$ are solved the efficiencies are obtained as below.

$$E_k = \sum_{r=1}^{s} u_r^* Y_{rk}\ , \quad E_k^1 = \frac{\sum_{p=1}^{q} w_p^* Z_p^k}{\sum_{i=1}^{m} v_i^* X_i^k} \quad \text{and } E_k^2 = \frac{\sum_{r=1}^{s} u_r^* Y_{rk}}{\sum_{p=1}^{q} w_p^* Z_p^k}$$

Therefore, $E_k = E_k^1 \times E_k^2$ .... (3.31) (Kao&Hwang,2008)

3.6: Bootstrap DEA.

The accuracy of statistical estimates can be determined by a computer based statistical method called bootstrapping. Bootstrapping was first introduced by Efron in 1979 who obtained the sampling properties of random variables using computer-based simulations. Bootstrap is a method where sampling is done repeatedly from a given data set and a new random data set of the same size as the original is created. This new data set is called replicates and the necessary statistics of the new data set can be calculated and this process is repeated to create a sample of replicates. One can draw inferences and conclusions about the distribution of the statistics based on this sample (Bogetoft and Otto,2013).

Let us consider a sample of **n** observations x_1, x_2, ...x_n.. Let us assume that we have observed 7 numbers 90,202,25,48,108,156 and 38. The mean of this data is $\bar{x}$ is 92.43 and the (unbiased) standard error s is 65.54. The standard error is **s/√n** and is equal to 24.78. The standard error is easy to calculate when there is a formula. However, we do not always have such a formula for standard error or variance (Bogetoft and Otto,2013). Similarly, for determining the median and variance of the median, formula is also not available easily. In such situations, the bootstrap method is a handy tool (Bogetoft and Otto,2013).

In this case, a bootstrap sample is a random sample obtained by sampling seven data points as above with replacements from the original sample. The bootstrap sample could be

$x^b = (x_6, x_1, x_4, x_1, x_3, x_3, x_5)$ i.e. 156,90,48,90,25,25 and 108. Based on this bootstrap sample we can estimate the statistic **$t(x^b)$** median that we are interested in. We make B bootstrap replications instead of calculating the standard deviation of the median. We calculate $t(x^b)$, the median for each bootstrap replication (Bogetoft and Otto,2013).

The bootstrap estimate of the standard error of t(x) with B replications is

$$\bar{s}_B = \sqrt{\frac{1}{B-1}\sum_{b=1}^{B}(t(x^b)-\bar{t})^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \textbf{(3.32)}$$

$$where\ \bar{t} = \frac{1}{B\sum_{b=1}^{B} t(x^b)}$$

is the mean over the replications of the statistic that we are looking for. The principle of the bootstrap method is that if the empirical distribution of x^b corresponds to the true distribution x, then the empirical distribution $t(x^b)$ will correspond to the true distribution t(x). If we are interested in the variance of the median t(x), that is difficult to determine, then we can simply use the empirical variance of the median of the bootstrap $t(x^b)$ that is much easier to obtain.

The following is the bootstrap algorithm for estimating standard errors (Bogetoft and Otto,2013).

1) Select B independent bootstrap samples $x^1, x^2, x^3 \ldots\ldots\ldots x^n$, a sample is drawn with replacement from our data set.
2) Calculate the estimate for each bootstrap sample;

   $t(x^b)$ $(b=1\ldots\ldots\ldots B)$
3) Estimate the standard error using the sample standard error of B replications,

$$\sqrt{\frac{1}{B-1}\sum_{b=1}^{B}(t(x^b)-\bar{t})^2}$$

…………………….. **(3.33 & 3.34)**

$$where\ \bar{t}=\frac{1}{B\sum_{b=1}^{B}t(x^b)}$$

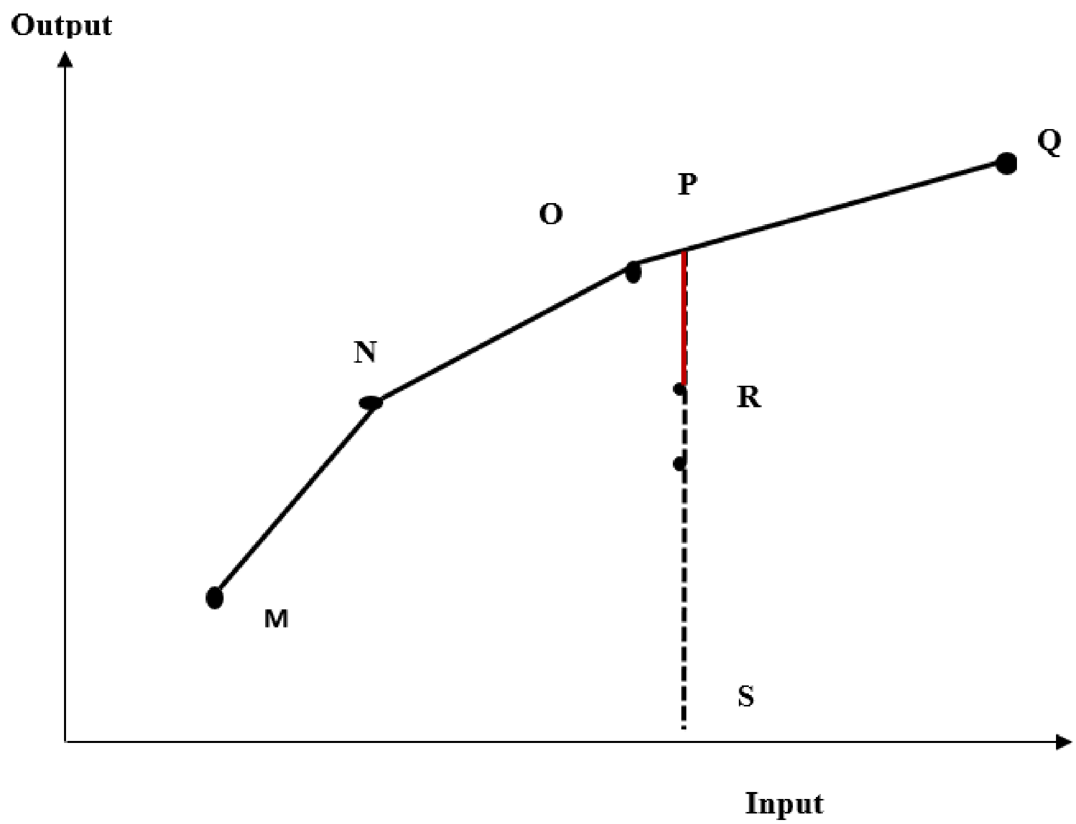

Figure 3.21. Basic DEA Frontier

We can use the above figure to understand how bootstrap DEA works. In the Figure 3.21 the output efficiency of R is RS/PS and the output benchmarks for R is units, O and Q. The opportunity for output increase is RP. One of the criticisms of DEA is that it automatically assumes that all distance between an observation and the efficient boundary (RS) reflects inefficiency.

However, inefficiency and noise are the components in the distance of an observation from the efficient boundary. This is since the data used in DEA may have errors due to measurement, omission or outliers.

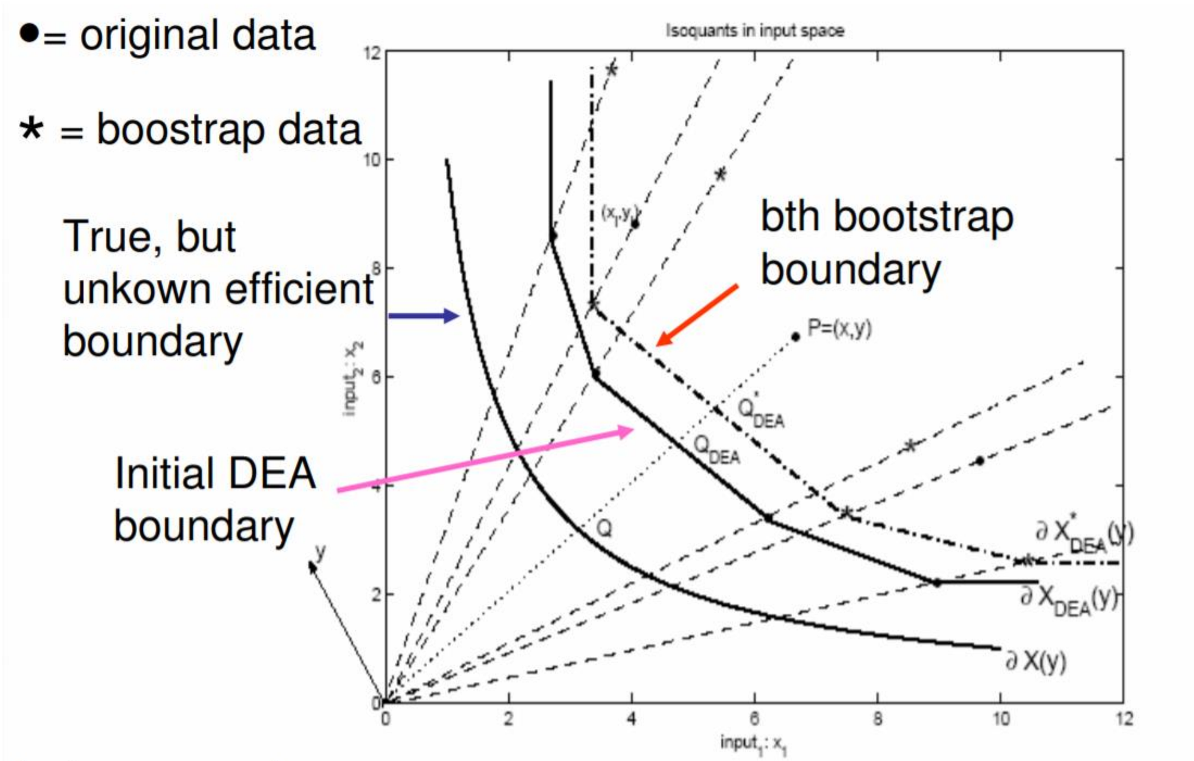

Figure 3.22. Bootstrapping in DEA (Source: Simar & Wilson, 2011)

Bootstrapping can be used in DEA to correct efficiencies for bias and to estimate confidence intervals on the assumption that the data is subject to random noise.

To account for noise in DEA and for more consistent results, several studies by Efron 1987, Xue & Harker,1999; Lothgren &Tambour 1999, have suggested the use of bootstrapping. Simar and Wilson (1998, 2000), constructed confidence intervals for DEA scores using a bootstrapping method. The method used by Simar and Wilson involves smoothing the empirical distribution by employing the simulation of a true sampling distribution by using outputs from

DEA. Drawing a replacement sample from the observed DEA efficiencies is equal to drawing the sample from the population itself. By sampling repeatedly from the observed DEA efficiency score an empirical sampling distribution can be constructed. By this process a new data set is generated and DEA scores are re-estimated using this data set. By repeating this process many times (say 2000 or 3000 times) an approximation of the true distribution population of the DMUs can be obtained(Ozcan,2014).

The data used in DEA has random noise and therefore, bootstrapping is used to correct DEA efficiencies for both bias and to estimate confidence intervals. It is assumed that the probability distribution of observed DEA efficiencies imitates the true but unknown parent population of DEA efficiencies(Thannassoulis,2010). Therefore, if from observed DEA efficiencies a replacement sample is drawn it is like drawing a sample from the population itself. An empirical sampling distribution can be constructed for the DEA efficiencies of units by sampling repeatedly from the observed DEA efficiencies. The confidence intervals of the DEA efficiencies are estimated from the empirical sampling distribution.

In Figure 3.22, it is assumed that the distances Q_{DEA} -Q^*_{DEA} are distributed similar to the distances Q-Q_{DEA} in bootstrapping process.

3.7 Effect of Contextual variables on Efficiency scores.

3.7.1 Introduction:

The CCR and BCC models discussed so far gives efficiency scores and extended models such as *weight restrictions, super efficiency,* and *cross efficiency* were used to get more discriminating power out of the analysis and to rank the efficient units. However, the reasons for differences in efficiency across the DMUs were not explored as the basic models of CCR and BCC only give the efficiency scores under constant returns to scale and variable returns to scale respectively. There have been several studies made to find the differences in efficiency scores and how it is dependent on the external operating environment. The factors of the external operating environment are called contextual variables (Banker and Natarajan,2008). External variables include the form of ownership, location characteristics, labor relations and government regulations to mention a few. The characteristics of the external environment may influence the capacity of DMUs to transform inputs to outputs. Therefore, there has been researches in DEA to analyze factors that contribute to productivity differences.

Studies on the effect of contextual variables on the efficiency can be broadly classified into three categories: the frontier separation approach, the all-in-one approach and the two-stage approach (Fried et al.,1999). In the frontier separation approach, the data set is stratified according to a single categorical variable that characterizes the different external environments e.g., ownership structure. The reference frontiers are calculated for each sub sample and for each pooled data set. The units are evaluated relative to their subsample and the pooled frontier. The sub sample and pooled efficiency scores are compared and then the effect of the external environment on operating inefficiency is determined by projecting all inefficient units on to their respective frontiers. This procedure can handle only one categorical variable and needs a priori selection of the most crucial factor of the operating environment (Fried et al.,1999).

The second approach is known as an all-in-one approach where the variables of the external environment are included directly into the linear programming formulation along with inputs and outputs. This can take as many variables as possible unlike only one categorical variable in the frontier separation approach. However, the external factors must be treated as inputs or outputs in this approach prior to the analysis. If the factors are classified as inputs, then more outputs must be produced and vice versa. If the purpose of the study is to test whether an operating environment is favorable or unfavorable then this method of choosing factors as inputs or outputs in not suitable. The scores assume that all inputs can be reduced in input orientation and all outputs can be expanded in output orientation (Fried et al.,1999).

The third approach is known as the two-stage approach. In this approach, the outputs and inputs are used in the LP formulation and the efficiency is calculated in the first stage. The efficiency scores are then used as the dependent variable and the environmental factors are used as independent variables in a second stage regression. Some studies use ordinary least squares (OLS) and some studies use a Tobit regression model. The two-stage approach has an advantage of testing the effect of contextual variables on the production process in terms of both significance and sign (Fried et al.,1999).

Some of the researches carried out using two-stage DEA are; Eliyasu and Mohammed (2016) used ordinary least squares(OLS) to evaluate contextual factors affecting the technical efficiency of freshwater pond culture systems in peninsular Malaysia. Ko et al.,2017 used Tobit regression to analyze the efficiency of retail chain stores in Korea. Mujasi et al.,2016 used Tobit regression to study the efficiency of hospitals in Uganda. Marschall and Flessa (2011) used Tobit regression to study primary care in rural Burkina Faso. Gillen and Lall (1997) studied airport efficiency using Tobit regression, Tripathy, Yadav, and Sharma (2010) used Tobit regression to study the efficiency of pharmaceutical firms in India.

In this study, the two-stage approach using both OLS (Ordinary Least Squares) and Tobit regression will be used to study the effect of contextual variables on efficiency scores.

3.7.2 Ordinary Least Squares Regression (OLS) method:

Banker and Natarajan (2008), reported that the use of OLS (ordinary least squares) regression modeling as the most appropriate technique to study the effect of contextual variables on the DEA score. Ray (1991) has regressed DEA scores on several socioeconomic factors to identify key performance drivers in school districts. There are many studies where the two-stage approach is used to first calculate productivity scores and then finding if these scores can be correlated to environment variables (Forsund,1999). In a two-stage study based on DEA, the efficiency scores are calculated for each DMU in the first stage based on the data of inputs and outputs. These efficiency scores are then regressed on the environmental factors to find out if the effect of environmental factors on productivity is statistically significant. The regression methods used are OLS (Banker and Natarajan,2008) or Tobit regression. According to Banker and Natarajan (2008), the two-stage method either with OLS or Tobit regression performs statistically better than other methods. The OLS regression model as per Banker and Natarajan (2008) is

$$\text{Technical Efficiency (T.E)} = \beta_0 = \sum_{i=1}^{n} \beta_i z_i + \delta \quad \ldots\ldots\ldots \mathbf{(3.35)}$$

Where TE is the efficiency score from CCR model, β_i denotes unknown parameters to be estimated are contextual variables and δ is the error term.

In the current research dependent variables are DEA bootstrapped CRS and VRS scores and the five contextual variables are the total population of the city where the branch is located, capital expenditure on machinery and equipment by the federal government, competition index, number of competition stores and squared number of competition stores.

The population data of each city where the branch is located and the capital expenditure on machinery and equipment were collected from the Statistics Canada database for the year 2014. The number of competition stores in each city where the branch is located was compiled from the data base of the various equipment manufacturers' organization. The competition index HHI is Herfindahl-Hirschman Index is a statistical measure of concentration used in economics to measure competitive effects (Naunberg et al.,1997). HHI is a measure of the number of firms in a market as well as concentration and is calculated by squaring the market share of all firms in a market and then summing the squares as below.

$$\mathbf{HHI} = \sum_{i=1}^{n}(\boldsymbol{MS}_{\mathrm{i}})^{2} \quad \ldots\ldots\ldots\ldots\ldots \quad \mathbf{(3.36)}$$

MSi represents the market share of firm ***i*** and ***n*** represents the number of firms in the market. The data of market share was obtained from trade and industry journals(statista.com).

3.7.3 Tobit Regression:

According to Chillingerian,1995, DEA scores resemble truncated or censored distributions shown in econometrics and statistical literature. Since DEA scores are not truncated above value the of 1, it is more of a censoring of the efficiency scores at 1 for efficient units. In such cases, Tobit regression may be helpful as an alternative to OLS regression. Tobit regression has been used in the second stage to study efficiency of Finnish secondary schools (Kirjavainen &Loikkanen,1998), Efficiency of retail chain stores in South- Korea (Ko et al.,2017), Efficiency of Pharma firms in India (Tripathy et.al.2013) and Airport productivity and performance (Gillen&Lall,1998).

The Tobit model is used to analyze the factors that affect technical efficiency. Tobit model is also known as truncated or censored regression model. The technical efficiency function of DMUs can be written as

$$\textbf{\textit{Technical Efficiency}} = \beta_o = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \varepsilon_i \quad \text{.....(3.37)}$$

Where β_0 indicates technical efficiency, α is a constant, Tobit coefficients that indicate how a one unit, change in an independent variable, changes the latent dependent variable. $x_1 \ldots\ldots\ldots x_5$, indicate the five contextual variables as mentioned above, $\beta_1 \ldots\ldots\ldots \beta_5$ indicate the coefficient of independent variables (contextual variables) and ε_i is the error term assumed to be normally distributed with mean μ and standard deviation σ

3.8: Outliers in Data Envelopment Analysis.

3.8.1 Introduction:

Data Envelopment Analysis uses extreme observations to identify performance and therefore the efficiency estimates are quite sensitive to the presence of outliers (Sexton et al.,1986). Outliers may be difficult to identify as each record describing an observation is typically a high dimensional vector with multiple inputs and outputs. Outliers may occur due to recording or measurement errors or due to unusual characteristics that relate to factors and external environment or uncontrollable factors (Chen and Johnson,2010). The occurrence of Outliers can be associated with low probabilities. Outliers can give a lot of information when the associated observation differ greatly from the remaining data set (Sexton et al.,1986).

An outlier does not have a generally accepted precise definition in the literature (Novaes et al.,2010). Outlier is often referred to as an observation that appears to be inconsistent with the remainder of the data (Simar,2003). The rigorous definition of an outlier is still elusive (Wen and Johnson,2010). In the context of efficiency estimation many authors term observations with significant influence on other's efficiency estimates as influential observations (Pastor et al.,1999). An influential observation owes its influence on the fact that it is an outlier and forms part of the frontier (Chen and Johnson,1999). According to Pastor et al.,1999 and Simar (2003), an outlier need not be an influential observation as the influential observation is not far from the data cloud.

In the literature of non-parametric efficiency analysis, studies of Wilson (1995), Pastor et al., 1999 and De Sousa and Stosic (2005) focused on efficiency estimates and attempted to detect influential observations. Wilson (1995) and Fox et al., (2004) focused on outliers removed from the data cloud. Wilson (1995), Pastor et al., (1999) and De Sousa and Stosic (2005) used estimates of DEA efficiency and frontier concept to detected outliers. However, Simar (2003) finds that these types of approaches do not take the frontier aspect of the problem into consideration. There seems to be a significant limitation in the study of outliers as researchers focus on overly efficient outliers that have maximum influence on efficiency scores (Chen and Johnson,2010).

The following table 3.4(Naidoo et al.,2016), provides a summary of the most prominent techniques used to detect outliers in DEA literature. Of the various methods listed above the following two methods will be used in this research to detect and remove outliers. Banker and Chang's method and Tran's methods are used widely in the literature.

1) Super efficiency approach of Banker and Chang, 2006.
2) The scalar method of Tran, Shively, and Preckel,2008.

Table 3.4. A Literature review of Outlier studies in Data Envelopment Analysis.

Item Srl#	Methodology	Approach	Authors
1	Probablistic frontier production using a Cobb-Doglas production frontier	Method of removing a pre specified number of efficient firms till the frontier stabilized.	Timmer,1971
2	Using super-efficiency model to screen outobservations with gross data errors	Banker and Gifford super-efficiency model	Banker and Gifford ,1988
3	DEA-Super-efficincy	Proposed a modified approach to rank eficient units based on super efficiency model	Andeson and Petersen,1993
4	DEA	Extended methodology of Anderson and Peterson(1993) to cater for multiple outputs.The author finds that although an observation had a low probability of occurrence , it cannot be concluded that it is an outlier.	Wilson,1993
5	DEA	Method in 4 was modified using super-efficiency models and a model that was computationally less extensive was proposed.	Wison, 1995
6	DEA with Bootstrapping	Proposed a bootstrap method and approximated the sampling variation of the estimated frontier.	Simar and Wilson,1998
7	DEA with Bootstrapping	The restrictive method of 1998 was alleviated by allowing for heterogenity in the structure of efficiency.	Simar and Wilson,2000
8	Non-parametric estimator based on the expected minimum function or maximum output function.	The approach is related to DEA/FDH estimators of efficiency but is robust to outliers,noise and extreme values.	Cazals,Florence and Simar,2002

9	Non parametric frontier model	This is based on the above work and demonstrates how the method can bes used to detect outliers.	Simar,2003
10	Super-efficiency	Devised a method to find outliers using screen level.	Banker and Chang,2006
11	Two stage semiparametric DEA	Mehod used both efficient and inefficient frontiers to detect outliers.	Johnson and McGinnis,2008
12	Super -efficiency DEA	Proposed a new mthod based on two scalar measures to detect outliers.	Tran,Shively and Preckel,2010
13	DEA	Identified a set of axioms and developed an approach consistent with the axioms.	Chen and Johnson,2010
14	DEA and Bootstrapping	This approach introduced two parameters,probability level and tolerance.Bot these parameters must be specified externally and bootstrap was used to approximate the true distribution.	Yang,Wang and Sun,2010
15	Super -efficiency and DEA	The superefficiency model is merged with a forward search and introduced a distance to be monitored along the search.The distance was obtained by integrating a regression model with DEA superefficiency model.	Bellini,2012
16	DEA and Bootstrapping	Modified the above method and applied to measurement of efficiency in hospitals.	Bahari and Emrouznejad,2014
17	BiSuper DEA,Predictive DEA	Super DEA method is used to detect outliers and this method constructed both frontiers.A predictive DEA method is also used to address the performance of the method.	Yang,Wang and Zheng,2014
18	Preidentification of outliers	In this method authors identify otliers in the data before using it in the DEA model.	Naidoo,Yadavalli and Naidoo,2016
19	Stochastic Threshold value	The methodology used stochastic threshold value to identify outliers.	Ahmed,Naidu and Reddy,2016

3.8.2: Super -Efficiency approach to detect outliers (Banker and Chang, 2006):

Of the many methods listed in table 4.5.1 to detect and remove outliers in DEA, the super-efficiency model of Andersen & Peter 1993, Banker & Chang,2006 is well accepted in the literature.

DEA assigns an efficiency score of one to efficient units and a score of less than one to inefficient units. A score that is less than one means that by using less inputs a linear combination of efficient units from the sample could produce the same level of outputs(Novaes,2011). This score reflects the radial distance of the DMU under evaluation from the production frontier. From the basic DEA model all efficient units have a score of one and therefore no ranking is possible as all units have the same efficiency score 1. Banker and Chang,2006 argued that the super-efficiency model should not be used for ranking but they recommend it for screening out possible outliers thereby obtaining more reliable efficiency estimates.

The super-efficiency model compares the efficient DMU under evaluation with a linear combination of all other units and this is done by excluding the DMU itself from the sample (Banker & Chang,2006). Considering the BCC radial model, output-oriented VRS, the equivalent super efficiency model is obtained by not including the observation **k** under evaluation in the reference set (Cook et al .,2009).

Max ϕ,

Subject to,

$$\sum_{\substack{j=1\\ j\neq k}}^{n} \lambda_j x_{ij} \leq x_{ik},\; i = 1,2...,m$$

$$\sum_{\substack{j=1\\ j\neq k}}^{n} \lambda_j y_{rj} \leq \phi y_{rk},\; r = 1,2...,s$$

$$\sum_{\substack{j=1\\ j\neq k}}^{n} \lambda_j = 1 \qquad \ldots\ldots\ldots(3.40)$$

$$\lambda_j \geq 0,\, j = 1,2,3...,n$$

$$\phi \geq 0, \qquad j \neq k$$

Under the condition of excluding the DMU under observation, an efficient DMU may decrease its output vector while retaining efficiency. In such a case a DMU assumes an efficiency score greater than one. This score that is greater than one reflects the radial distance from the DMU under evaluation to the production frontier estimated with that DMU excluded from the sample. In other words, the DMU is subject of the maximum proportional decrease in outputs while retaining efficiency (Andersen&Petersen,1993). Banker and Chang (2006) suggested using a screen based on the super efficiency score to identify those observations that are more likely to be contaminated with noise. This is done by eliminating from the sample those observations with super-efficiency scores higher than a preselected screen (Novaes et al.,2011).

3.8.3: Tran et al., 's method to detect outliers in DEA:

Tran et al.2010 suggested an easy and effective method to detect super-efficient outliers. The lambda λ_j in CRS model and VRS model represents the weight assigned to the j th DMU to construct the virtually efficient DMU for evaluating DMU_0. To find the efficiency scores of all j

DMUs, the corresponding model has to be solved j times generating jxj matrix. The resulting λ values containing all λ's can be organized as follows.

$$M_{\lambda} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} & \lambda_{1J} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} & \lambda_{2J} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} & \lambda_{3J} \\ \lambda_{J1} & \lambda_{J2} & \lambda_{J3} & \lambda_{JJ} \end{bmatrix} \qquad \textbf{(3.41)}$$

The ith row and jth column of M_{λ} represent the weight assigned to the j th DMU to construct the virtually efficient DMU for evaluating the i th DMU. The ones that would always be selected to construct the virtually efficient DMUs are the outlier DMUs that perform significantly better than other DMUs. Therefore, they can be identified by the number of occurrences during the construction of virtually efficient DMUs as

$$C_j = \sum_{i=1}^{n} \mathbf{1}\left\{\lambda_j \geq \mathbf{0}\right\} \qquad \ldots\ldots \textbf{(3.42)}$$

Where $\mathbf{1}\left\{\lambda_j \geq \mathbf{0}\right\}$ are an indicator function and it returns 1 if $\lambda_j \geq \mathbf{0}$ is true; otherwise 0.

The outliers can also be identified through the cumulative weight during the construction of virtually efficient DMUs as

$$\mathbf{S_j} = \sum_{i=1}^{n} \lambda_{ij} \qquad \ldots. \textbf{(3.43)}$$

The DMUs that perform significantly better than the peer DMUs are considered outliers as they have a high number of occurrences and have a high value of cumulative weight. The value of C_j

and $S_{j,}$ (j=1,2,3…n) should be calculated after running a model and value of C_j and S_j , with a certain higher threshold can be identified as an outlier and then removed from the data set. The selection of threshold is subjective and is not discussed in the literature. The process stops once the desired degree of convergence in the weights has been reached (Tran et al.,2010). Jun Wang (2017) in his Ph.D. thesis suggests the use of median plus 2x standard deviation as the threshold. Any DMU that has both, number of occurrences and cumulative weight higher than median plus 2x standard deviation can be considered significantly larger than vast majority and therefore can be identified as an outlier (Jun Wang, 2017).

3.9 Time Series Analysis Using DEA:

3.9.1. Introduction:

The study of variations of the efficiency of DMUs over time is referred to as time series analysis can help in understanding and making important conclusions. There are two ways of measuring the performance over time (Ramanathan, 2003).

1) Window Analysis, 2) Malmquist productivity index.

The various DEA models discussed so far has dealt with data for a single period to calculate the efficiency score. In other words, the above DEA models calculate efficiency under static conditions. The business environment in heavy equipment industry and dealerships is dynamic and therefore may show varying performances over time and this may depend on many external factors such as government regulations, the effect of related industries, business cycle to name a few. These dealerships may make profits or losses depending on how they respond to the various external business environment and internal influences within the organizations. Therefore, measurement of efficiency under dynamic conditions may give more information.

This study will use two non-parametric DEA models Window analysis and Malmquist index under dynamic (time-dependent) situations.

3.9.2 Window Analysis:

The basic concept of Window Analysis and name are due to G. Klopp (1985), who developed this technique for US Army Recruiting Command (USAREC). Klopp used DEA with 3 outputs and 10 inputs to analyze performance in various organizational units.

Window analysis consists of a series of analysis of DMUs over multiple time periods to study the change in efficiency over a time period. In such an analysis it is possible to perform DEA over time by using a moving average method where a DMU in each different period is treated as if it were a different DMU. In other words, a DMU's performance is compared with its performance in another period in addition to the performance of other DMUs. For example, analysis #1 could be result of analysis of data for first, second, and third year of operations, analysis #2 could be result of analysis of data of second, third and fourth year of operations, and analysis #3 could include third and fourth year data from one year and first quarter data for the following year(Bowlin,2001). Therefore, each analysis has a new and a different set of DMUs resulting in different efficiency ratings. The technique that operationalizes the above procedure is called window analysis and helps in analyzing the stability and trend that are time dependent behaviors(Bowlin,2001).

Window analysis is a measure of efficiency changes over time (Charnes, 1994). This technique was used by Charnes et al., (1985) in studying the operations of aircraft maintenance. In this study, the data were collected for fourteen tactical fighter wings in the U.S. Air force over a period of seven months. The analysis was performed using a three-month window. Window analysis works on the principle of moving averages(Bowlin,2001). A DMU in period p_1 is treated as a completely new unit in another period p_2 in the analysis. When there are n units in a given time period and each window has a width of k periods, then there will be (n*k) units in each window. In the above case, there were 42 (3x14=42) DMUs (Cooper et al., 2007). When there are a small number of units with a large number of inputs and outputs, this feature is important in DEA analysis as it increases the discriminating power of DEA (Sowlati, 2004). The width of each window and choosing the number of time periods to be included in the window for analysis is now subjective and a judicious decision (Charnes et al., 1994). Discriminating power of DEA decreases if the width of the window is small and a larger width gives misleading results since changes occur over a longer period (Sowlati, 2004). Pjevcevic (2012) used a window analysis to study port efficiency in Serbia. Asmild et al., (2004) used a window analysis to study the Canadian banking industry over a twenty-year time period. Culliane et al., (2004) used window analysis to study the efficiency of container port production. Yang and Chang (2009) used window analysis to study the efficiency of Taiwan’s telecommunication firms. There has been no literature found that measures efficiency changes over time in heavy equipment retailing organizations using window analysis. In this research window analysis is carried out on a five-year data of an equipment retailing organization in Canada that has thirty-three retailing branches (DMUs) for the period 2010 to 2014. The formula for calculating the number of data points (Cooper et al.,2007) is as below.

In other words, **Δ (delta)** represents an additional 264 DMUs that are now available to calculate the change in efficiency scores as compared to the original 33 DMUs. With three inputs and two outputs, there are now 264x5= 1320 data entries available to which DEA model can be applied to study the variation in technical efficiency, pure technical efficiency and the scale efficiency. The column views in the result enable us to analyze the stability of results across different data sets and row views help in determining the trends in efficiency scores within the same data set (Cooper, Seiford, and Tone,2007).

$$
\begin{aligned}
&p = \text{length of window} (p \leq k) \\
&w = \text{number of windows} \\
&k = \text{number of periods} \\
&\text{is calculated by the formula} \\
&w = k - p + 1 \quad .... \\
&\text{In the current research} \\
&n = 33, k = 5, p = 3, \text{and } w = 3. \\
&\text{Number of different DMUs} = npw \\
&\quad = 33x3x3 = 297 \\
&\quad 297 \text{ different DMUs} \\
&\Delta = \text{number of DMUs} = n(p-1)(k-p) \\
&\quad = 33(5-1)(5-3) = 264
\end{aligned}
\tag{3.44}
$$

3.9.3: Malmquist Productivity Index:

Malmquist Productivity Index is a concept first introduced by Prof. Sten Malmquist (1953). Malmquist Index also called Malmquist Productivity Index (MPI) is an index that was originally used to compare the production technology of two economies and based on the concept of the production function. This approach has been further studied and developed in a non-parametric environment called DEA based productivity index. Fare, Grosskopf, Lindgren, and Roos (1992) combined the ideas of measurement of efficiency from Farrell (1957) and the measurement of productivity from Caves, Christensen and Diewart (1982) to construct a Malmquist productivity index directly from input and output data using DEA. This is an index representing Total Factor Productivity (TFP) growth of DMUs and it reflects either progress or regress in efficiency along with progress or regress in frontier technology between two periods of time when there are multiple inputs and outputs (Cooper, Seiford, and Tone, 2007).

Malmquist index and window analysis have been used in conjunction with DEA to study efficiency in hospitals and banks. The constant change in the banking environment with deregulation has been the main reason for the continued interest in this area of research. In the healthcare industry in countries like Canada where the government is giving free health care, the interest in continued research is to make the health care services as efficient as possible (Paradi et al.,2012). Similar is the heavy equipment industry in Canada that is greatly affected by the business cycles due to fluctuating world oil prices and commodity prices (statcanada.com).

The DEA- based Malmquist productivity index has been found to be a useful tool for measuring productivity change in organizations. For example, Fare, Grosskopf, Lindgren and Roos (1994) studied the productivity change in Swedish hospitals, Lothgren and Tambour (1999) studied productivity change in the Swedish eyecare service provisions, Grifell, Tatje, and Lovell (1996) studied the effect of deregulation on Spanish saving banks.

Paradi et al., (2004) used the Malmquist index and window analysis to study Canadian bank branch efficiencies. Darijani and Taboli (2014) analyzed the total factor productivity index in the automotive industry in Iran using Malmquist Productivity index. Yao Chen (2011) studied the productivity of automobile industries in the USA using the Malmquist productivity index. Refaie et al., (2015) used both window analysis and Malmquist index to study efficiency in pharmaceutical industry in Jordan.

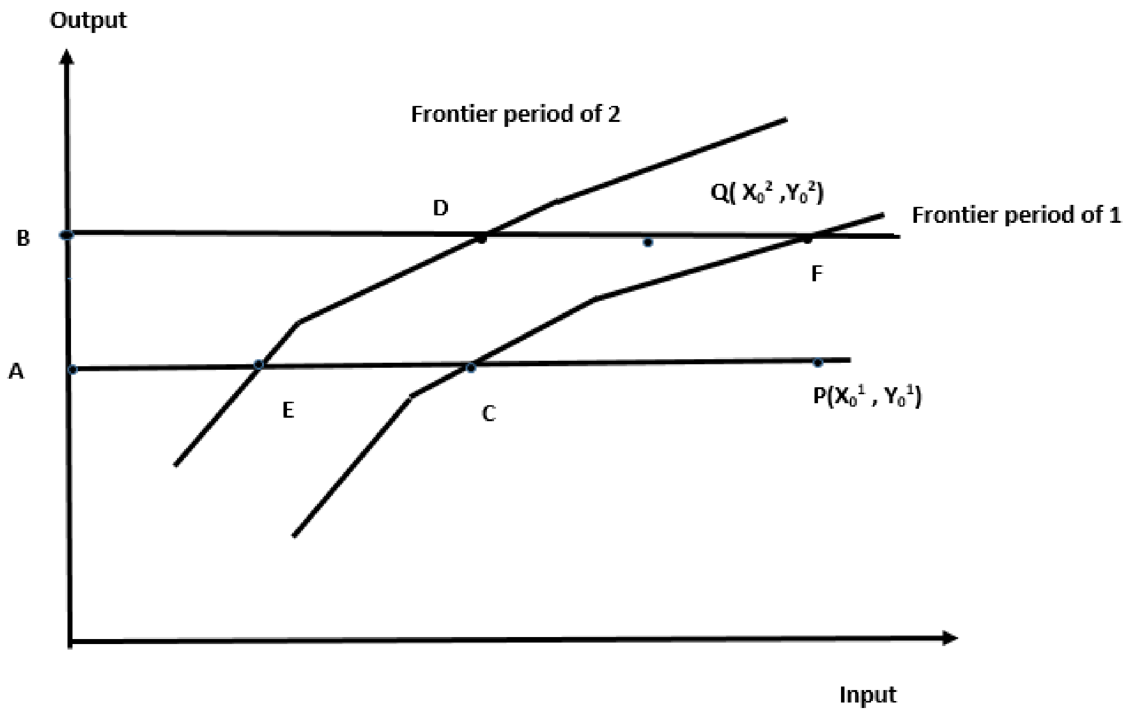

Figure 3.23. Illustration of frontier shift - Catch-up (Constructed by the author, Source: Cooper et al., 2007)

The performance change of an equipment retail organization between two-time periods is shown in the above figure3.23. In this figure a branch P in the time period “**1** “with X_0 level input and Y_0 level output reduces the level of input to point Q and increases the level of output to point B at time 2. This particular retail organization seems to be inefficient at time 1 as it is not on the frontier. The frontier as defined by other efficient retail organizations shifts higher in time period 2. However, the retail organization 2 falls behind the frontier again despite the fact that it has reduced its input and increased its output. Q is made once more inefficient as other branches have moved on to the frontier and have become efficient. While Q is efficient as compared to P, it is inefficient as compared to branches on frontier 2. The question is now whether Q will ever catch up with frontier and how it can catch up? The other questions are the reasons that are causing the frontier to shift. The Malmquist index can assess these productivity changes by indicating how much a branch can improve its efficiency in two-time periods as well as the impact of changes for the frontier shift. In DEA literature, the change in efficiency for a given DMU(branch) is termed as Catch-Up or recovery. The frontier shift occurs due to technological change. Therefore, the Malmquist index is calculated as a product of technical change and efficiency change.

Catch-up= $\dfrac{\boldsymbol{BD}}{\boldsymbol{BQ}} \Big/ \dfrac{\boldsymbol{AC}}{\boldsymbol{AP}}$ ……… **(3.45)**

Or = $\dfrac{\boldsymbol{Period\,1}}{\boldsymbol{Period\,2}}$ ………. **(3.46)**

The numerator of the formula calculates the efficiency at period 2 with respect to frontier 2 and the denominator calculates the efficiency at period 1 with respect to period 1.

If catch up is greater than 1, efficiency is increased from period 1 to period 2, if catch –up=1, there is no change in efficiency from period 1 to period 2 and if catch-up is less than1 then efficiency is decreased from period 1 to period 2.

The frontier-shift effect (innovation or technical change) portion of the Malmquist index can be constructed by measuring the distances between the respective frontiers. In the above example point, C from frontier1 shifted to point E in frontier 2.

Frontier shift for period 1= $\frac{AC}{AP}/\frac{AE}{AP} = \frac{AC}{AE}$ ………. **(3.47)**

$= \dfrac{Efficiency\ of\ (X_0Y_0\ )1\ wrt\ period\ 1\ frontier}{Efficiency\ of\ (X_0Y_0\ )1\ wrt\ period\ 2\ frontier}$ ……. **(3.48)**

Frontier Shift $=\frac{AP}{BQ}\sqrt{\frac{BF}{AC}x\frac{BD}{AE}}$ ……… **(3.49)**

MI = Catch-up x Frontier Shift …… **(3.50)**

If Frontier shift > 1, efficiency increased from period 1 to period 2

If Frontier shift =1, then there is no change in efficiency from period 1 to period 2.

If Frontier shift<1, then efficiency decreased from period 1 to period 2.

In the current research Malmquist index (catch-up), CRS efficiency change, and Technology change are calculated for the period 2010 to 2014.

3.10 Strengths and Limitations of DEA:

Data Envelopment Analysis(DEA) DEA presents a comprehensive picture of organizational performance and is a powerful technique for performance measurement. This is supported by approximately 15,000 research papers on DEA that includes applications in various fields (Paradi et al.,2017). These research papers are an evidence of the strength of DEA. Some of these strengths are discussed below.

3.10.1 Strengths of DEA:

1) Objectivity is the main strength of DEA. DEA provides efficiency scores based on numerical data and not on the subjective opinion of people. Since DEA is data-oriented it is a valuable analytic tool. If one accepts the principle of frontier analysis, DEA results are very useful(Ramanathan,2003)
2) DEA can handle multiple inputs and multiple outputs and they can be measured in different units. For example, in the various inputs and outputs presented in the literature review in

Chapter 2, some of the inputs are a number of employees measured in number units, the area of facility measured in square feet, and the outputs of sales revenue is measured in dollars.

3) DEA being a non-parametric technique does not require a functional form between inputs and outputs like other methods where you need to assume a production function.
4) DEA presents three extremely useful features depending upon the orientation of the problem; input oriented or output-oriented (Charnes et al., 1994). Firstly, it identifies each DMU by a single efficiency score and this facilitates the ranking and comparison of DMUs in multiple output frameworks as opposed to having a separate ratio measure for each output.
5) By projecting inefficient units on the efficient frontier, it indicates areas of improvements for every single DMU. and thirdly it enables making inferences on the DMUs' general profile.
6) It enables making inferences on the DMUs' general profile (Ramanathan,2003).
7) DEA focusses on a best practice frontier instead of the average behavior of all DMUs in a data set as in regression. As every unit is compared to an efficient unit or a group of efficient units, such a comparison leads to sources of the inefficiency of units that are not on the frontier (Zhu,2004).
8) DMUs are benchmarked against actual performance rather than a theoretical benchmark that may not be achievable in real life (Sherman and Zhu,2006).

There are also limitations to this powerful tool as the same characteristics that make it powerful also creates some issues.

3.10 2 Limitations of DEA:

1) Measurement errors (noise) may pose some issues in efficiency measurement as it is assumed inefficiency is due to deviation from the frontier and there is no allowance made for errors in measurement or other factors(Ramanathan,2003). DEA efficiencies are very sensitive to small errors and therefore post DEA analysis, sensitivity analysis should be carried out.

2) DEA is unable to model small sample sizes as the model will tend to produce higher than normal average efficiency scores with many units appearing on the frontier.

3) While DEA can help set targets for performance improvement, it does not tell the analyst how to reach the targets. DEA results are a good starting point but not the final analysis (N. AvKiran,1999).

4) DEA will not be able to discriminate the scores well if the ratio of DMUs to the product of the sum of the inputs and outputs is low (N. Av Kiran,1999).

5) The process of DEA cannot be explained intuitively to a non-technical audience in case of more than two inputs and outputs as the efficiency scores are obtained after running a number of LP problems(Ramanathan,2003).

6) DEA provides a relative efficiency score that is based on specific DMUs studied. If a very

efficient DMU is excluded from the analysis, the scores provided will not be accurate.

7) Outliers operating with unfair advantages can greatly skew resulting efficiency scores. Other DMUs may experience lower overall efficiency score.

8) DEA measures the relative efficiency of a DMU but not absolute efficiency. In other words, DEA tells us how well the DMU is doing compared to the peers (set of efficient units) but not compared to a theoretical maximum.

9) One of the inconvenience is to rank efficient units as all the efficient units have a score of 100%. From a managerial point of view, it may be useful to compare a DMU to absolute best performance to know true inefficiencies.

10) Another issue with DEA is the way in which efficiencies are calculated. The values of the weights chosen are from the solution of the LP but not under the control of the management. Although weight restrictions can be used it does not solve the problem completely as if the choice of weights lies with the management, it will increase the flexibility of DEA methodology(Ramanathan,2003).

We have reviewed the concepts of efficiency, the theoretical perspectives of DEA, bootstrap DEA, effect of environmental factors on efficiency scores, detection of outliers and the efficiency change over time and the strengths and limitations of DEA. In the following chapter 4, research methodology for the research based on theoretical aspects of DEA will be discussed. Research questions will also be formulated in the next chapter that will address the research objectives and aim that was stated in Chapter 1, introduction.

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Chapter IV: Research Methodology

4.1 Introduction:

This chapter describes the research methodology that will be used in the thesis to measure the performance of a heavy equipment retailing organization(**HERO**). As described in the previous chapter the performance of a heavy equipment retailing organization will be measured using the linear programming technique, Data Envelopment Analysis. Various authors have proposed different application procedures of DEA and these will be discussed briefly. Based on the application procedures of these authors, the author of the thesis has drawn out an application reference that would be used as a framework for measuring the performance of heavy equipment retailing organizations.

4.2 The Heavy equipment retailing organization under study:

The retailing organization under study is a major multiline mobile equipment dealer with operations across Canada. The dealer sells, rents and services equipment used in diverse sectors such as construction, infrastructure, mining, oil and gas, utilities, municipalities, waste management and forestry. The company has thirty-three branches in Canada and represents globally recognized brands such as Volvo, Case, Manitowoc, National and Grove Cranes, Terex Cedarapids, Terex Trucks, Fassi, Sennebogen to mention a few. The company is listed on the Toronto Stock Exchange. The name of the company is withheld for confidentiality.

4.3. Application Procedures used in DEA:

There are four application procedures available in the literature for carrying out DEA studies and they are listed below.

4.3.1: Golany and Roll's Application Procedure (Golany and Roll, 1989):

This is one of the earliest frameworks suggested that can serve as a general reference. According to the author, the three main stages in carrying out an efficiency study using DEA are selection of DMUs that are to be analyzed and its characteristics, selection of input and output factors that are appropriate in assessing the relative efficiency of the selected DMUs and use of the DEA models and analysis of results.

4.3.2: N. AvKirans's Guidelines for DEA Study (1999):

Av Kiran (1999) provided a checklist of twelve questions that can serve as a guideline for DEA researchers, that is similar to the above application procedure. Of these twelve questions, the first question relates to selection of DMUs and the second and third relate to selection of inputs and outputs. The balance of the questions relates to the analysis of results, but the author has not provided a structured step by step approach for application of DEA.

4.3.3: William Cooper Framework:

Emrouznejad and Witte (2010) proposed a comprehensive model called the "COOPER-Framework" for carrying out DEA studies and they said that DEA studies cannot be considered as push button technology. The framework consists of the following six interrelated phases: **C**oncepts and objectives, **O**n structuring data, **O**perational Models, **P**erformance comparison model, **E**valuation and **R**esults and deployment. The unified standard process is shown in the following diagram.

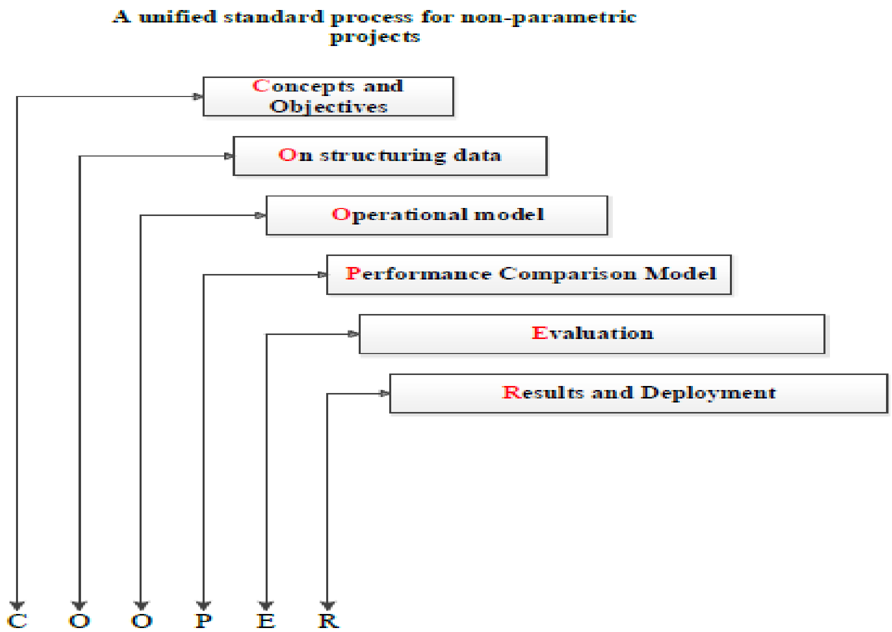

Figure 4.1. Cooper Frame work (Constructed by author, Source: Emrouznezad & Witte, 2010)

4.3.4: Components in DEA Applications (Paradi et al., 2017):

The following ten components are suggested by Paradi et al., 2017 in developing a DEA model to be used in applications.

1) Analyze the objectives
2) To identify the operations of the DMUs.
3) Selection of inputs and outputs and verifying for adequacy and completeness of the data.
4) Run preliminary DEA analysis for testing the reasonableness of the results.
5) Analyzing the efficiency scores and its limitations in ranking.
6) Using the information to find if there is excess resources and excess capacity.
7) Increasing the discriminatory power of the analysis.
8) Impact of environmental variables on DMU scores.

9) Benchmarking with best practice DMUs.

10) Implementing DEA results into initiatives to improve performance and management of the process.

4.4 Framework for the research:

Based on the broad guidelines suggested by above authors in the literature, the author has drawn out a framework that is specific to the study of efficiency of heavy equipment retailing organization in Canada. The frame work of the study is as shown in figure 4.2.

The research will study the efficiency, of a heavy equipment dealership in Canada that has thirty-three business units across Canada from east to west coast (Name not disclosed for confidentiality). Each of the branch in the dealer network is a profit center and contributes to the profit of the organization and hence a decision-making unit (DMU).

According to the framework below, in this research four methodologies namely DEA (Black-Box approach, Network DEA, Bootstrapped DEA), Efficiency change over time (Window Analysis and Malmquist productivity index), Second stage analysis (OLS and Tobit regression) and Outlier detection will be used. Of these the first methodology DEA will help in formulating models to study the efficiencies of the heavy equipment retailing organization. The second methodology Window Analysis and MPI will help in determining efficiency change over time. The third methodology of regression will help to know if the environmental factors have any bearing on efficiency. The fourth methodology will help in identifying if there is any outlier DMUs in the study that shows a very high performance. The various models that will be used in the above methodologies are given in the architecture of the research in the figure 4.3.

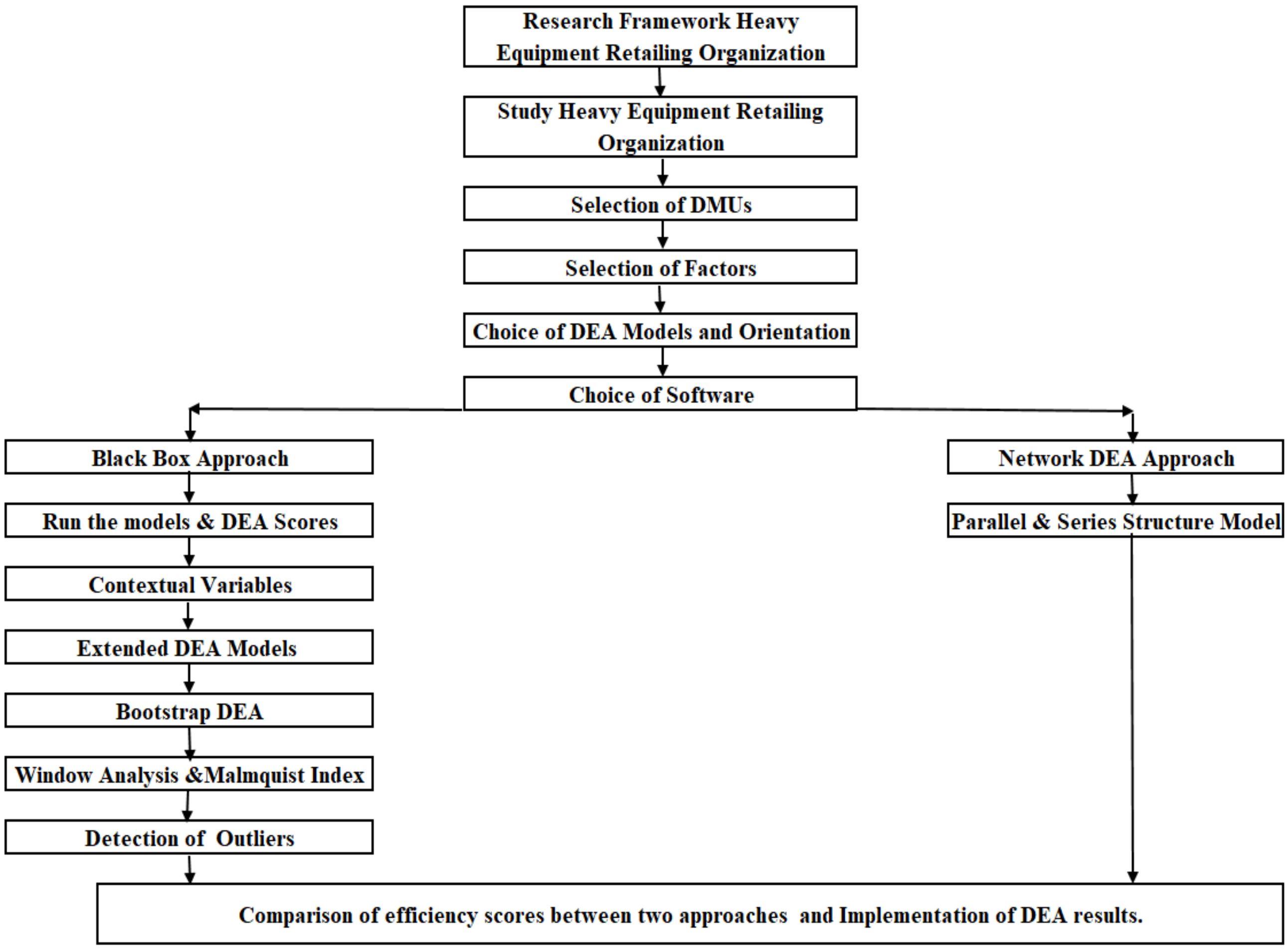

Figure 4.2. Framework of the research (constructed by author)

4.5: Architecture of the models in Research

Data Envelopment Analysis is a linear programming technique to measure efficiency. As per the theory of linear programming, every linear programming problem (usually called the ***primal*** *problem*) has another closely related linear program called its ***dual*** (Anderson, Sweeney and Williams,2015). DEA programs involving weights of inputs and outputs (**u** and **v**) are called Multiplier DEA program. Those involving weights of the firms (**ɵ** and **λ)** are called

Envelopment DEA programs(Ramanathan,2003). For more details please refer to Chapter 3, on theoretical aspects of DEA. Various types of extended DEA models should be used based

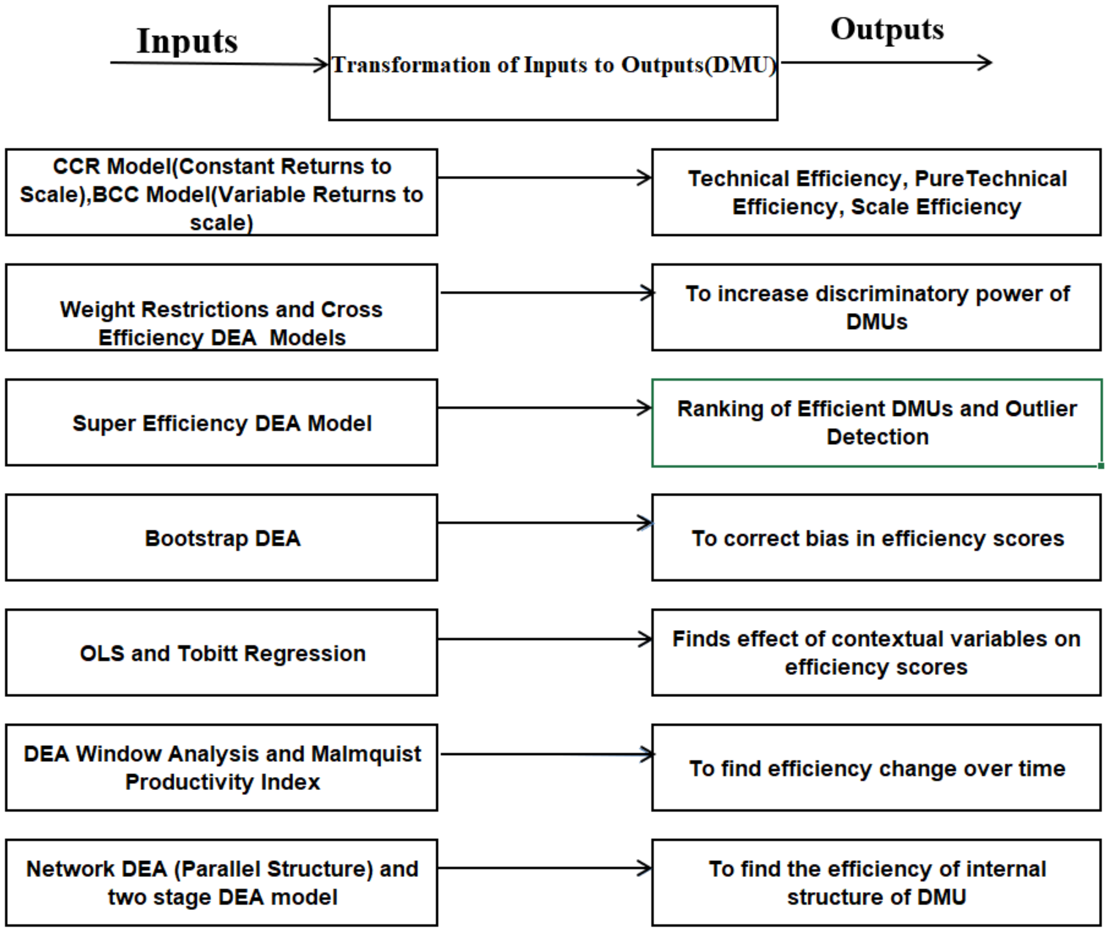

Figure 4.3. Architecture of the models in Research Methodology (Constructed by author)

upon the requirements of the study (Cook and Seiford,2009; Adler and Yazhemsky,2010). The figure above lists various models that would be used in the research to measure the efficiency of the heavy equipment retailing organizations.

The basic CCR model under constant returns to scale will be used to determine technical efficiency whereas the BCC model under variable returns to scale will be used to determine pure technical efficiency and scale efficiency. However, no judgement is made about the importance of one input versus another while using various factors in the above models and it is assumed that all variables had the same importance (Thompson et al.,1986, Allen et al.,1997). Therefore, this research uses weight restriction models that can accommodate the importance of various factors used in the analysis. The conventional DEA model does pure self-evaluation of its DMUs. Therefore, cross efficiency model is used to compare the scores of each DMU with other DMUs to find the performance using the peer optimal multipliers (Sexton 1984, Doyle 1994, Cook and Zhu,2015). The analysis, of above models yields a number of efficient units all of which have a score of unity. To rank these efficient units that have the same score, super-efficiency model is used to rank the efficient units (Anderson &Peterson,1993; Cook and Seiford,2009; Adler and Yazhemsky,2010). In all the above analysis the error which is intrinsic to the data was not factored in. To account for bias bootstrapping method is used to correct the efficiency scores (Simar and Wilson,1998). The efficiency scores thus determined may be influenced by environmental factors. To find the effect of environmental factors on the corrected DEA scores Ordinary Least Square(OLS) Regression and Tobit Regression has been used (Banker and Natarajan,2008). Further envelopment model of DEA will be used to find slacks and set targets for improvement (Charnes, Cooper, Rhodes 1978, Banker, Charnes and Cooper,1984).

Malmquist productivity index and Window analysis (Cooper, Seiford and Tone,2007) are used to find the change in efficiency scores in the data from period 2010 to 2014.Malmquist index is a method that compares the performance of the branch from one period to another. In Window analysis the efficiency of the DMUs is evaluated, to observe overall trend in the performance in a specific window.

Finally, in the analysis of all these models the DMUs are considered as a black box and has not taken into consideration the internal structure of the DMU. The internal structure of the DMU is taken into consideration and Network DEA (C. Kao,2009) is used to find efficiency score of internal structure. In Network DEA, a two stage -network DEA (Kao and Hwang,2008) model is used where in the outputs in the in the first stage is taken as the input in the second stage. Further the internal structure is considered parallel in nature and efficiency is calculated. Since DEA is an extreme point method outlier (Banker and Chang, 2006) will be detected if any in the analysis. Please refer to Chapter 3 for all theoretical aspects of all the above methodologies that will be used in the research.

4.6: Research Questions:

The research objectives and research aim stated in Chapter 1(Introduction) are as below.

1. How do the branches of the heavy equipment retailing organization under study compare to each other in terms of their efficiency?
2. What conditions may account for the differences in the efficiency of the branches.
3. What factors or constraints create varying scores amongst inefficient branches?
4. Is there any change in efficiency over time and if so how it can be measured?
5. How can the efficiency of the departments in a heavy equipment retailing organization be measured?
6. Can the efficiency of the branch have relation to the efficiency of individual department?

The purpose of the study is to develop models and methods that are more appropriate and suitable to measure the performance and relative efficiency of the heavy equipment retailing organizations than the existing conventional methods.

This thesis addresses different research questions based on the objectives delineated in Chapter 1 and referred again above for easy reference. Each of the above objectives has its separate question set. The, research question related to each objective is given below.

Objective 1: How do the branches of the heavy equipment retailing organization under study compare to each other in terms of their efficiency? This can be addressed by the research question of how to construct a model to measure the performance and relative efficiency of a heavy equipment dealer in Canada treating it as a Black box without considering its internal structure.

Objectives 2 and 3: What conditions may account for the differences in the efficiency of the branches and what factors or constraints create varying scores amongst inefficient branches? This can be addressed by the research question of finding if there is any effect of environmental factors on the efficiency scores.

Objective 4: Is there any change in efficiency over time and if so how it can be measured? This can be addressed by the research question of finding the impact of change in efficiency under dynamic conditions.

Objective 5: How can the efficiency of the departments in a heavy equipment retailing organization be measured? This can be addressed by the research question of finding methods to find the efficiency of individual departments in a branch

Objective 6: Can the efficiency of the branch have relation to the efficiency of individual departments? This can be addressed by research question that finds out methods that can relate efficiency of the branch and its individual departments.

Therefore, the research questions for the thesis can be summed as below based on the above stated research aim and objectives.

1. *To construct a model to measure the performance and relative efficiency of a heavy equipment dealer in Canada treating it as a Black box without considering its internal structure.*
2. *Measure the performance and relative efficiency of heavy equipment dealer considering the internal structure of each of its branches using Network Data Envelopment Analysis.*
3. *Find ways of maximizing outputs to improve efficiency.*
4. *Find ways of minimizing inputs to improve efficiency.*
5. *Compare the efficiency between the two approaches of Black Box and Network DEA approach.*
6. *To study the variations in efficiency scores over time using Window Analysis and Malmquist Index.*
7. *To find the effect of environmental (contextual) variables on the efficiency scores.*
8. *To find out if there are any DMUs that are outliers*.
9. *How to improve the efficiency of inefficient units so that they also become efficient.*

4.7: Selection of DMUs:

As described above, the heavy equipment retailing organization under study has thirty-three business units in its network. Each of these branches has three business operations viz. Sales Operations, Service Operations and Parts Operations that are business drivers that generates profit. Each of these operations is independent in nature and considered as profit centers. Therefore, each of these branches will be considered as a Decision-Making Unit (DMU). The functions of DMUs and its structure have been discussed in chapter 1.

4.8: Selection of Factors:

The selection of input and output variables is the most important step in the application of modeling using data envelopment analysis(Bowlin,1998).In other words DEA and its applications depend on the data set of inputs and outputs.All resources used by a unit should be included as input.A unit will convert resources to produce outputs so that outputs should include the amounts of products or services produced by the unit.In DEA inputs are minimized and outputs are maximized or vice-versa. In Chapter two on literature review various factors used in DEA applications in the retail sector and in retail automotive industry has been discussed in details.Based on this list, a list of factors appropriate to the heavy equipment retailing organization will be chosen for the research.

One of the key consideration in using DEA is defining input and output variables.For the management to accept the results of DEA analysis it is very important that correct choice of inputs and outputs is made before beginning the analysis (Bowlin,1998).One of the important considerations in DEA is isotonicity(increase in any input should result in some output increase and not a decrease in any output(Bowlin,1998). According to Sarkis (2002),
when ascertaining the size of the data set there are two considerations that are conflicting.

1) When the number of DMUs are larger, in relation to the inputs and outputs, the discriminatory power of the analysis increases.
2) When the data set is large there is a chance that the homogeneity of the data set may decrease. This means some exogenous factors that are beyond control of manager or analyst may affect

the results (Golany & Roll, 1989). The larger the data set the larger is the computational requirement.

There are some thumb rules on the number of inputs and outputs to be selected. According to Boussofiane et al., (1991) the lower limit on the number of DMUs should be the product of the number of inputs and number of outputs. According to Banker et al., (1989) the number of DMUs should be at least three times the number of inputs and outputs combined.

Golany and Roll (1989), established a rule of thumb that the number of DMUs should be at least be two times the number of inputs and outputs considered. Bowlin (1998) mentions the need to have three times the number of DMUs as there are input and output variables. Dyson et al., (2001) recommend a total of two times the product of the number of input and output variables. As per Cooper et al., (2007) sufficient number of DMUs is required to perform DEA. The number of degrees of freedom increases with the number of DMUs and decreases with the number of inputs and outputs. As proposed by Cooper et al., (2007) a general rule for minimum number of DMUs (**n**) is that it should exceed the greater of the product of the input (**m**) and output(**s**) variables or three times the sum of the number of input (**m**) and output (**s**) variables.

$$n \geq \max\{m * s, 3(m+s)\}$$

As per Cook et al., (2014) "*such a rule is neither imperative nor does it have a statistical basis, but rather is often imposed for convenience*".

Table 4.1. Degrees of Freedom in DEA

Author's Suggestion	Formula for recommended sample size m=number of inputs ,s=number of outputs	Number of DMUs	Max Inputs	Max Outputs	Appropriateness of Sample Size
Banker 1984	3x(m+s)	33	4	4	Sufficient
Golany and Roll (1989)	2x(m+s)	33	7	7	Sufficient
Bowlin 1998	3x(m+s)	33	4	4	Sufficient
Bouusafianne (1991)	mxs	33	2(15)	15(2)	Sufficient
Dyson et al(2001)	2xmxs	33	3(4)	4(3)	Sufficient
Cooper et al.2007	n≥max{mxs,3(m+s)}	33	7	7	Sufficient

The inputs and outputs for the research would be determined by using the Judgmental process used by Golany and Roll (1999) and with the inputs and outputs used for similar studies on DEA application in retail sector that is found in the literature and discussed in Chapter 2, literature review. This is done to broaden the set of factors and to select the most appropriate factors from among this broad set that suits the model. As per judgmental process team of experts were contacted in the field of sales, parts and service operations to find factors that will influence the performance of respective departments. As mentioned there are thirty-three business units (DMUs) under study and therefore in each DMU there are experts such as a Sales manager who looks after sales operations, a Parts Manager responsible for parts operations and a Service Manager responsible for service operations.

All these managers are professionals and very experienced and experts in their field. The minimum qualification for a sales manager is ten years of sales experience selling industrial equipment preferably with a university degree or college diploma. Similarly, the minimum qualification for a Parts manager is college diploma with ten or more years of experience in parts management in heavy equipment industry. There are Parts managers who were service technicians and therefore are well versed in both the technical and commercial aspects of the business. The minimum qualification for a service manager is a four-year technical training that leads to college diploma as a technician. After years of working as a mechanic they grow vertically to become service manager. In short, all these managers are experts in the field as some of them would have worked in more than a couple of manufacturers of heavy equipment.

Therefore, there is a vast pool of thirty-three sales managers, parts managers and service managers and each of them is an expert in their field. From among this expert pool, managers with a minimum of twenty years of experience in their field were selected for judgmental process and it was found that there are six sales managers, five parts managers and seven service managers. These managers were met individually to find out all the possible inputs and outputs in their field of expertise and some of the inputs and outputs matched the one found in the literature review discussed in chapter 2. From the set of these factors (10 inputs and 11outputs) listed in chapter 5, appropriate factors will be chosen based on the model requirements and will be discussed in Chapter 5.

4.9 Data Collection:

Empirical data is collected for business research and various types of data collection methods are used in any research. Specialized knowledge and skills are required to collect data from each of these data collection methods. The empirical data collected by the researchers through interviews, observation, asking participants to write etc. are called *primary data*. Primary data is collected by various qualitative research methods like questionnaire survey, observation, ethnographic study etc. There are empirical data that is already available and existing and these are called *secondary data*.

Many of the DEA research in applications such as banking, universities, hospitals, educational institutions etc. have used secondary data from the archives of the respective organizations. The current research concerning efficiency measurement in heavy equipment dealerships will be carried out using secondary data. The data will be collected from internal sources and external sources.

The internal sources will consist of management documents, financial documents and database, operations document and database and marketing documents and database. All these documents are audited financial reports of the heavy equipment company under study. The data from the financial statements of the last five financial calendar years viz., 2010,2011,2012,2013 and 2014 will be used for the study.

While the financial statements give details only at the company level special approval has been obtained from the management of the company to access the performance details of thirty-three DMUs at the branch level. Therefore, this data is highly confidential, and the name of the company is kept in strict confidence for security reasons. A check was also made with the research ethics officer of Athabasca University to find out if secondary data needs approval from research ethics board. Research ethics officer clarified that since there is no research involving humans such a clearance from research ethics board is not needed for this research. The external sources will consist of relevant data from business association of equipment distributors, trade journals and other related sources.

4.10: Formulation of the model:

Having understood the industry, set in the context research questions, defined the DMUs and have selected the inputs and outputs. The next step is formulation of the model using the most appropriate inputs and outputs with respect to the model.

The efficiency measures can be made by viewing each DMU in two ways.

1) As a black box without knowing the internal structure of the DMU as the overall efficiency is studied at a higher level i.e., the organizational level. In this study DEA does not reveal sources of inefficiency that is within the divisions of the organization. DEA analysis is carried out using CCR, BCC and other extended models of DEA as discussed in Chapter 3 on theoretical aspects of DEA.
2) The specific sources of inefficiency among the components of DMU as specified above viz., Sales, Service and Parts operations, can be found using Network DEA. Network DEA provides fuller access to find efficiency of the internal structure of the DMU under study.

There will be four models formulated under Black-Box approach. The models are Branch Production process model, Branch Profit maximization model, Branch Expense minimization model and Branch Asset maximization model. Network DEA will be used to study the efficiency of the individual operations of the DMU using the production process model. In the production, process model extended DEA models such as Super efficiency and Cross efficiency models will be used. Similarly, in the profit maximization model weight restrictions will be used to increase the discriminatory power of the model. In the other two models only, basic models will be used for the analysis.

As detailed in the architecture of the study in Fig 3, models will be formulated to study variations in efficiency over time using Window analysis and Malmquist productivity index. Further OLS (Ordinary least squares) and Tobit regression will be used to find if environmental factors have any effect on the efficiency scores as a second stage analysis. Bootstrapped DEA will be used to find the bias in the scores and outliers if any in DMUs will also be detected.

As per Cooper, Seiford &Tone,2007, the researcher is advised to try different models if the researcher cannot identify in the preliminary analysis the characteristics of the production frontier. It may be risky to rely only on one model. If the application has important consequences it is wise to try different models and methods and compare the results to arrive at a definitive conclusion (Cooper, Seiford &Tone,2007). This is the first study in measuring the performance of heavy equipment retailing organizations using DEA and as such DEA has to be accepted as an effective tool to measure performance by the industry that is currently using many different methods to measure performance as outlined in chapter 1. Therefore, a comprehensive analysis is made by using the basic models and other extended models as described above in the architecture of the study to make it as a more effective tool. The formulations of models will be dealt in detail in Chapter 5,

where DEA models will be formulated to study the efficiency of heavy equipment retail organizations.

4.11: Choice of Software:

As described in Chapter 3, in the theoretical framework of DEA, DEA is a linear programming technique and therefore any software package that solves linear programming formulations can be used to solve DEA problems. DEA applications require a separate linear programming problem to be solved for each DMU in the data set. Therefore, if there are **n** DMUs, then the software must be used **n** times to solve the linear programming problem for each DMU. If there are a considerable number of DMUs in the dataset then this computation can become tedious(Ramanathan,2003).

Therefore, this study requires a specialized DEA software package. There are a number of specialized DEA software packages that are developed by leading researchers in the field of DEA. A comparative study of such software is made by Iliyasu et.al., (2015) that can quickly solve the DEA problems with many DMUs. There are eight such software tools that have been classified as commercial and non-commercial. Some of these software developers have kept pace with the developments in DEA. A review of the capabilities of all this software resulted in the selection of Max DEA as the most suitable software for the study as it has many current models for use and can export the results to MS-Excel. This selection was also approved by the supervisor as the appropriate software to study the performance of heavy equipment retailing organization in this research.

4.12: Summary:

As can be seen from the research framework the heavy equipment retailing organization, performance will be studied using both as a black box approach and considering the internal components using a parallel structure using Network DEA approach.

Inputs and outputs that are most appropriate for each model will be used and the optimization problem will be solved using Max DEA software and the following analysis will be made.

4.13. Recommendation and Reporting:

Based on the above analysis and findings recommendations will be made to the management for improving the efficiency of inefficient units. The immediate goal of the research is to integrate the efficiency model with the Dealer Management systems of the organization. The goal is to convince the Association of Equipment Distributors and the manufactures of heavy equipment and the retailing organizations to use DEA as a performance measurement tool for benchmarking their dealers.

Table 4.2. Purpose of models used in the research

Item#	Name of the Model	Purpose of Using the Model
1	CCR-Model	This model gives the technical efficiency scores of each branch.
2	BCC-Model	This model decomposes the technical efficiency score into pure technical efficiency and
		Scale Efficiency.
3	Super efficiency Model	This model ranks the efficient units that all have a score of 1.
4	Cross-Efficiency Model	This model evaluates the efficiency scores by the peer DMUs. This increases
		discriminatory power of the model.
5	Weight Restriction Model	This model imposes restrictions on the weights that each input and output can use.This
		increases discriminatory power of the model.
6	Bootstrap-DEA Model	This model finds out the bias and confidence interval in efficiency scores.
7	OLS and Tobit Regression Model	This model finds the effect of contextual variables on efficiency scores
8	Network DEA-Model	This model find the efficiency of internal structure of a branch that has sales ,service and parts
		operations
9	Window Analysis Model	This model finds the stability and trend in efficiency score over time.
10	Malmquist Index Model	This model finds efficiency change over time and decomposes it into efficiency change and
		technological change.
11	Outlier detection -Model	This model finds if there are outliers present in the DMUs being analyzed.

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Chapter V: Developing DEA Models

5.1: Introduction:

In this chapter models will be developed to find the efficiency of heavy equipment retailing organizations based on the theoretical framework of DEA as described in chapter 3, the research methodology as described in detail in chapter 4, using various inputs and outputs as used by researchers found in Chapter 2 on Literature review and from the judgmental screening process from experts in the field. There will be four different models that will be developed. The first model will consider all possible factors of production in a branch and will be modeled to maximize the production process. The second model will consider factors that maximize the profit, the third model will use factors that will minimize the expenses in the branch and the fourth model will use factors that will help in maximizing the assets used in the branch. All these four models will analyze the efficiency from different perspectives. From the business point of view, each of these models will appeal to different divisions within the organizations. The asset and expense model will appeal to the Finance division as they want to minimize expenses and maximize asset utilization, production process model will appeal to the Operations division and the profit maximization model will appeal to the CEO of the organization as they are interested in maximizing the ROI (return on investment).

DEA and its applications are heavily dependent on the data set that is used for the research (Paradi et al.,2017) and therefore, the first step in the present research is the selection of factors.

5.2 Factors of Production:

The research is conducted to measure the performance of a heavy equipment dealership operating in Canada with a network of thirty-three branches operating in different locations from East to the West coast . Each of these branches is a decision-making unit (DMU). In the selection of factors, the primary question is what the inputs are and what are the outputs in a service setting like a retail branch operation of a heavy equipment organization where the activities are to sell equipment, rent equipment, sell parts and services for this equipment and make a profit margin to sustain the business. As detailed in the chapter on research methodology, the data that will be used for the research will be the accounting information such as COGS (cost of goods sold), Department expenses etc. and non-accounting information, such as the area of facility and labor. Therefore, the question is how inputs and outputs are defined in such an organization.

Achabel, Heienke, and McIntyre (1984) have defined output and input in the retail industry. According to them, retail production is a process that transforms manufactured goods along with several types of labor and capital into rather complex offerings customers will demand. They define output (extended product) as a function of the level of resource utilization that measures the capability of the firm to meet demand i.e. sales, gross margin, units sold, customer satisfaction, number of customers served, customer conversion ratio etc. They define inputs as all the factors of production used by the retailing firm such as personnel, information systems, number of stock keeping units and other components of the firm's offer to the customers.

In production theory input is defined as the resources expended and outputs as the outcome of the process that has an extended value (Bogetoft,2013). The author adds that contextual variables (non-controllable) inputs and outputs can also be handled in a similar way.

According to Cook and Zhu,2007, in some situations, however, certain performance measures can play either input or output roles. These performance measures are referred to as flexible measures. “For example, a variable like deposits used in Bank branch efficiency can be argued both as an input and output. It can also be argued that in the evaluation of research productivity by universities, Beasley (1990, 1995) ‘‘research income’’ can be viewed as both an output and input, in a situation where research – granting agencies (e.g., NSERC in Canada and NSF in the USA) wish to allocate funds to those researchers and universities to have the greatest impact. In this environment, graduate students can play the role of either an input (a resource available to the faculty member, (effecting his/her productivity) or as an output (trained personnel, hence a benefit resulting from research funding).

In a very different environment, W. Cook et al., (1990), use the measure ‘‘average pavement rating’’ as an input that (negatively) influences the outputs, in evaluating the efficiency of highway maintenance crew. At the same time, it can also be argued that this measure can as well, be an output that clearly is influenced by the level of annual maintenance expenditure. There are instances where there is ambiguity in defining inputs and outputs. There appears to be at least two possible approaches for deciding the status of the flexible variables in a DEA setting. The first and most obvious approach is to examine the issue from the point of view of the individual DMU and the second approach is to view the situation from the manager’s perspective (Cook and Zhu,2007).

The choice of variables should also consider the structure of the model and the variables of the model should be relevant, complete, operational, independent and non - redundant (Bogetoft,2013). The variables are termed as relevant when they reflect the industry’s comprehension of the system. In other words, the relevant variables defined should be used by the decision makers in daily practice. Completeness of the variable indicates that the variables should capture the resources (inputs) that go into production and the outputs that come out of the production process. The variables that are defined unambiguously and measurable are termed operational. Independence indicates that values of one set of inputs do not affect the values of outputs. Non-redundancy indicates that the variables chosen should be free of overlap (Bogetoft,2013).

A list of various efficiency studies made in the automotive industry that has similarities to heavy equipment retailing organization and efficiency studies made in retail chains that covers grocery stores, supermarkets, wine distribution etc., are listed in Table in2.2 and 2.3 in Chapter 2 on literature review. There are in all forty-four studies listing inputs and outputs used in all these retail organizations. It is found from the table that most of the studies have used sales revenues and profits as outputs and number of employees, the area of the facility, cost of goods sold, number of labor hours, capital employed, and assets etc. as inputs.

In the retailing of heavy equipment, the data set consists of the following possible inputs and outputs as found from the literature review and from the judgmental screening process.

Parameters of Inputs:

I_1, Department Expenses: The expenses are a component of the production process in retailing of heavy equipment and have an impact on the productivity of a branch (Thomas et al., (1998); Keh et al. (2006,).

I_2, Depreciation &Amortization expenses: This component of expenses is a part of total expenses in branch operations (Seong-Jong-Joo et al., (2011).

I$_3$, Number of employees: Labor makes a crucial contribution in the production process in a branch and does affect the productivity of the branch (Thomas et al.,1998; Kamakura et al., (1996); Barros&Alves (2003); Dubelaar et al., (2002).

I$_4$, Area of the Branch: Productivity of the branch is related to the size of the store available to the branch (Kamakura et al., (1996); Donthu &Yoo, (1998); De Jorge, (2008);

I$_5$, Cost of Goods Sold(COGS) of Equipment is an indication of depletion of inventory of equipment (Seong-Jong-Joo et al., (2011), Barth, (2007).

I$_6$, Cost of Goods sold of rental equipment is an indication of depletion of inventory of rental equipment (Seong-Jong-Joo et al., (2011), Barth, (2007).

I$_7$, Cost of Goods Sold of Service done in the workshop for repairing equipment (Seong-Jong-Joo et al., (2011), Barth, (2007).

I$_8$, Cost of goods sold by Parts is an indication of depletion of inventory of parts sold to realize parts revenues (Seong-Jong- Joo et al., (2011), Barth, (2007).

I$_9$, Cost of Goods Sold of Total Sales, (Equipment+ Rentals) (Seong-Jong- Joo et al., (2011), Barth, (2007).

I$_{10}$, Cost of goods sold for the branch (COGS of equipment+ rental+ service+ parts) (Seong-Jong- Joo et al., (2011), Barth, (2007).

Parameters of Outputs:

O$_1$: Revenue from Sale of equipment (Ingene &Lusch (1999); Donthu and Yoo (1998), Seong-Jong-Joo et al., (2011), Barth, (2007).

O$_2$: Revenue from Rental Equipment (Ingene & Lusch (1999); Donthu and Yoo (1998), Seong-Jong-Joo et al., (2011), Barth, (2007).

O_3: Revenue from Servicing of equipment in the branch Ingene & Lusch (1999); Donthu and Yoo (1998), Seong-Jong-Joo et al., (2011), Barth, (2007).

O_4: Revenue from the sale of Parts Ingene &Lusch (1999); Donthu and Yoo (1998), Seong-Jong-Joo et al., (2011), Barth, (2007).

O_5: Revenue from sales of equipment, rental, service, and Parts Ingene &Lusch (1999); Donthu and Yoo (1998), Seong-Jong-Joo et al., (2011), Barth, (2007).

O_6: Total Gross Margin for the branch (Equipment + Rental+ Service+ Parts) Ingene &Lusch (1999); Donthu and Yoo (1998), Seong-Jong-Joo et al., (2011), Barth, (2007).

O_7: Gross Margin for Equipment Sales

O_8: Gross Margin for Rental Sales

O_9: Gross margin from Total sales (Equipment+ Rental)

O_{10}: Gross Margin from Service Sales

O_{11}: Gross Margin from Parts Sales

While many authors have used monetary values from accounting data as inputs and outputs in DEA studies, it is important to establish how the accounting metrics can be used as factors of production. In the next section, we will explore how accounting information can be decomposed into factors of production in a retail branch operation.

5.3: DEA and Accounting Performance Measurement:

There is a complementary relationship between DEA and Accounting Performance Measurement (APM) (Harrison and Rouse,2016). DEA can be applied to profit-making organizations by converting financial performance indicators as production factors in calculating their technical efficiency equivalents (Feroz et al.,2003). One such approach is to decompose the Return on equity using the DuPont model. For example, ROE can be decomposed as follows:

$$\text{ROE}= \frac{NI}{S} \text{X} \frac{S}{A} \text{X} \frac{A}{E}$$

In the above equation, the profit margin is net income (NI) divided by sales(S), asset utilization is sales(S) divided by Total Assets(A) and equity multiplier is total assets(A) divided by common equity (E). This decomposition enables analysis of ROE in terms of a measure of profitability (profit margin), assets required to generate sales (asset utilization) and the financing of those assets(equity). In other words, measure of ROE is a measure of sales, net income, total assets and equity and these components define important dimensions of technical efficiency in a profit-making organization (Feroz et al.,2003). Alternately cost of sales, total assets and equity can be minimized as inputs and net income can be maximized as outputs. This approach identifies a technically efficient firm as using a minimum of resources and producing a maximum of net income (Harrison and Rouse,2016). However, sometimes organizations may have losses and therefore in such cases, net income may be negative. DEA models work with positive data and therefore net income may be replaced by gross margin. In other words, financial information can be incorporated into the operational definition of efficiency by maximizing revenues subject to the constraints from employing long terms assets and equity and short-term costs(resources). Similarly, ROA (return on assets) can be decomposed using the DuPont model.

$$\text{ROA} = \frac{Profit\ Margin}{Asset\ Turnover}$$

Profit Margin = $\frac{Net\ Income}{Sales}$, Net Income = Sales- Total Costs

And Total Costs = Cost of Sales+ Labor Expenses+ SG&A Expenses

Asset Turnover = $\frac{Sales}{Total\ Assets}$

Total Assets = Current Assets + Non-Current Assets

Current Assets = Cash+ Account Receivables +Inventory

Non-Current Assets = Plant &Machinery +Land and Buildings

The above decomposition into component parts helps in identifying the source of increase and or decrease in a branch's performance related to ROA. The accounting performance measures often use partial productivity ratios with the numerator in the form of an output and denominator in the form of an input. e.g. sales per employee, net income per branch. In the field of revenue management, there are ratios that combine profitability with capacity. e.g. RevPASH represents the amount of revenue available per available seat hour (Kimes and Singh,2009).

There are several studies that use accounting information in productivity measurement using DEA models and lists of such studies in Automotive industry and retail industry have been dealt in detail in literature review Chapter 2.

Based on the above discussion, the efficiency of heavy equipment branches will be analyzed using the four models mentioned below by using as factors of production from accounting information and other factors related to land and labor.

5.4: Selection of mix of Variables:

The selection of a right mix of input and output variables is the most important step in the application of modeling using data envelopment analysis. For the management to accept the results of DEA analysis it is very important that the correct choice of inputs and outputs is made before beginning the analysis (Bowlin,1998). One of the important considerations in DEA is isotonicity (increase in any input should result in some output increase and not a decrease in any output.Bowlin,1998).

There are now ten input variables and eleven output variables identified for the research. In the black box approach the ten inputs and eleven outputs do not satisfy the rule proposed by Cooper et al., (2007): three times sum of inputs and outputs, is sixty-three and is more than the number of business units under study thirty-three (Refer section 4.6, Chapter 4). However, there are only thirty DMUs in 2010 and 2011 and therefore if all the available inputs and outputs are considered for constructing the model, it does not satisfy the rule of degree of freedom proposed by Cooper and other authors. One of the reasons for the inputs and outputs to follow the above rule is that it increases the discriminatory power of the model as otherwise, the model finds many DMUs as efficient. Now there is a paradox of choosing the right variables from among the twenty-one variables available for analysis. As in any statistical model, in the design of a relative efficiency model, the choice of variables must be justified. The following are some of the methods to select the right variables for the analysis from among the twenty-one variables identified (ten inputs and eleven outputs) so that the discriminatory power of the model is retained.

5.5: Variable selection techniques in DEA:

There is no guidance provided by DEA for specifying the production function and the input and output variables, but these are left to the user's discretion, judgment and expertise(Nataraja&Johnson,2011). Unavailability of data, high dimension of the production process and the inclusion of irrelevant inputs and outputs in the analysis are some of the issues in selecting the variables. Nataraja and Johnson (2011) have listed eight different variable selection methods to, identify the relevant variables and have also offered guidelines in choosing the most appropriate method.

The impact on efficiency scores due to model misspecification was demonstrated by Sexton et al.,1986 and Smith ,1997. They showed that efficiency scores may not differ by adding more inputs or outputs, but the shape of the frontier can be changed by the inclusion of a variable and this will affect the ranking of the efficiency estimate. Since DEA is a non-parametric approach, it loses discriminatory power as the dimensionality of the production space increases and therefore variable selection methods are important(Nataraja&Johnson,2011). There are several methods in the literature that addresses the issue of determining the relevant variables and all of these approaches are statistical in nature. The eight most cited methods as per Nataraja and Johnson (2011) are as below.

1) **Efficiency Contribution measure (ECM):** This method was proposed by Pastor et al., 2002, where the relevance of a variable is determined based on its contribution to efficiency. The variable being tested is called the candidate. The efficiency scores are evaluated one with candidate variable and one without it and a binomial statistical test is done to determine if the candidate variable is important to the contribution process.
2) **Principal Component Analysis (PCA)-DEA:** This is a general statistical method used to reduce the dimensionality of the data where the weighted linear combination of variables determines the variance structure of a matrix. This was independently developed by Ueda and Hoshiai,1997 and Adler and Golany,2001. Each principal component obtained from the weighted linear combination of original variables and setting in decreasing order of percentage of variance accounts for a maximal variance.
3) **Regression-based Test:** This was suggested by Ruggiero,2005 where an initial measure of efficiency is obtained through known production variables. The efficiency score is then regressed against a set of candidate variables. The variables are relevant to the production

process if the coefficients in the regression are statistically significant and have a proper sign. The values of the coefficient should be positive for inputs and negative for outputs. One variable is done at a time and the procedure is repeated until there are no further variables.

4) **Bootstrapping for variable selection:** Simar and Wilson 2001, used a bootstrap estimation procedure to identify relevant variables.
5) **Banker (1996)** used three statistical tests to indicate the significance of an input and output variable to the production process.
6) **Fanchon (2003)** suggested a recursive method to determine the variables to be included by iteratively using DEA to analyze the increase in the number of efficient observations.
7) **Jenkins and Anderson (2003)** used partial correlation to omit variables that contained minimum information that had no effect on efficiency scores.
8) **Dario and Simar (2007)** aggregated highly correlated inputs and outputs to reduce the dimensionality of the production possibility set into a single input and single output using eigenvalues.
9) **Norman and Stoker (1991)** used correlation coefficients to reduce the variables.
10) **Wagner and Shimshak** (2007) used average change in efficiency scores to reduce variables in a stepwise process.

Of the above ten methods in the literature, Stepwise approach of Wagner and Shimshak (2007) is used in selecting the variables for the research as it uses the average change in efficiency scores.

5.5.1: Variable selection with Stepwise Approach:

The results of DEA rely heavily on the data set of inputs and output variables that are used in the analysis. However, little attention has been paid in literature, to how these variables should be chosen in the real-world application (Wagner&Shimshak,2007). The input and output variables used in DEA studies are simply treated as givens in many of the existing papers. Golany and Roll (1989), noted that few papers focus on the choice of data variables when carrying out DEA studies. According to Jenkins and Anderson (2003), it is crucial to give attention to variable selection as model weights assigned to inputs and outputs are less constrained and the discriminatory power of DEA results are less if the number of input and output variables are greater in relation to the number of DMUs as per Cooper et al., (2007) formula. Although it is advantageous to limit the number of variables, till now there is no consensus on the best method for the selection of variables for a study (Wagner&Shamshak,2007). In other words, DEA itself does not provide a method for selection of variables nor provides guidance for the specification of production function (Nataraja&Johnson,2011).

A stepwise procedure using a backwards approach for modeling DEA was proposed by Wagner and Shimshak (2007). In the backward approach, the process starts by considering all the inputs and outputs that are available in the DEA model. The efficiency score of the model is calculated by dropping one variable each at a time. Theoretically, the method can continue until only one input and one output variable are available in the model. This method can be used to create a parsimonious DEA model (Wagner &Shimshak).

In the research there is a set of ten input variables i= (1,2,3……10) and 11 output variables s= (1,2…11). The objective is to reduce as many input and output variables as possible. Groups of inputs and outputs with similar attributes can be combined into a single measure of input or output thus creating a composite data to reduce the number of variables (Paradi, Sherman and Tam,2017). Of, the 10 input variables COGS (Cost of Goods Sold) of equipment and COGS of rentals can be combined into one variable of COGS of Equipment sales. Similarly, depending on the model, the COGS of Equipment, Rentals, Parts, and Service can be combined into one variable called COGS of the branch. This will bring down the number of input variables to 9 and 6. Similarly among the output variables revenue of Equipment and revenue of rentals can be combined to revenue of total sales bringing down the output variables to 10. Similarly, revenue of equipment, rentals, parts, and service can be combined into one variable called Revenue of the branch thus making a composite variable from four variables. Similarly, gross margins of equipment, rentals, parts, and service can be combined to form one composite variable called the gross margin of the branch. This brings down the number of variables to eight. Such composite variables of inputs and outputs can be formed from input and output variables based on intuition and judgment (Paradi et al., 2017).

The process was started by running a DEA CCR model with output orientation, with eight (I_1 to I_8) input variables and five output variables (O_1 to O_5) for thirty-three DMUs using data for the year 2014 as the data was available for all thirty-three DMUS. With thirteen variables eight inputs and five outputs, it was found that all the DMUs were shown as efficient. This is because the sum of inputs and outputs times three is thirty-nine which is more than the number of DMUs under analysis (thirty-three). Therefore, a composite variable was formed by combining inputs COGS of equipment sales and rental sales and the new variable is called COGS Total sales. Similarly, for outputs, equipment sales and rental sales were combined to form a composite output variable revenue total sale. This reduced the number of inputs to seven and keeping the outputs to four. The model was again run with eleven variables and it was found that twenty-eight DMUs had a score of 1 and all the other DMUs had an average score of 0.99425. In other words, with eleven variables all the DMUs are shown as efficient with a score of 1indicating that the number of variables is too many in relation to the number of DMUs. The details of the score are shown in table 5.1, Stepwise approach to reduction of variables.

Three more models were run by dropping each time an output and efficiency score was calculated retaining the same number of input variables seven and reducing output variables one by one up to model 8. Please refer to the summary of stepwise approach in Table 5.1. It is found that average efficiency score between model 3 to model 8 ranged from 0.949106273 to 0.999134091 and the change in efficiency score is 0.0500278(5%).

From Model 9 to Model 15, each time an input was dropped and the same number of outputs two was retained. The average efficiency score ranged from 0.972327879 to 0.326532394 and the change in efficiency score is 0.6457954 (64.57%). In other words, as the number of inputs and outputs used in the analysis decreases in relation to the number of DMUs the discriminatory power of the analysis increases.

It is also found from the table 5.1 that as you move from left column to right column and as each output and input is dropped step by step thus reducing the number of variables in the analysis, discriminating power of the analysis increases. The number of efficient units found from the analysis varies from 33 when thirteen variables were used as compared to 2 efficient units when the number of variables were reduced to two. Therefore, the number of variables used in the analysis should conform to the formula that the sum of outputs plus inputs times three should be less than the number of DMUs under analysis (Cooper et al.,2007).

It can, therefore, be concluded from the stepwise variable reduction approach that a minimum number of variables should be chosen based on the requirement of the model.

Table 5.1. Stepwise Approach for reducing one variable each time - CCR Model

DMU	CRSScore I1,I2,I3,I4,I5,I6,I7,I8& O1,O2,O3, O4,O5,O6 (8I,5O)	CRS ScoreCombined Equip sales and Rental Sales and COGS of Equp and Rental (7I,4O)	CRS Score Dropped Output Branch Gross Margin O6output.(7I3O)	CRSScore Dropped output Parts Sales O3,(7I,2O)	CRS Score Dropped Output Service Sales O2.(7I,1O)	CRS Score Two outputs O5,O6.(7I,2O)	CRS Score Dropped Output Total Branch GM O6(7I,1O)	CRS Score Dropped Output Total Branch Sales O5.(7I,1O)	CRS Score Dropped Input COGS Service I7.(6I,2O)	CRS Score.Dropped Input COGS Parts I8.(5I2O)	CRS Score Dropped Input COGS(Total Sales)(I5+I6).(4I20)	CRS Score Dropped Input Area of Facility I4.(3I,2O)	CRS Score.Dropped Input Dep&Amortization I2.(2I,2O)	CRS Score.Dropped Input Dept ExpensesI1.(1I,20)	CRS Score.Dropped Input number of Staff I3.(2I2O)
	Model 1	**Model2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**	**Model 8**	**Model 9**	**Model 10**	**Model 11**	**Model 12**	**Model 13**	**Model 14**	**Model 15**
DMU1	1	0.989593	0.989593	0.967235	0.967235	0.977342	0.977342	0.889585	0.928058	0.693864	0.42182	0.42182	0.42182	0.42182	0.208662
DMU2	1	1	1	1	0.931569	0.971906	0.971906	0.870701	0.971906	0.785988	0.48919	0.318888	0.318888	0.318888	0.168403
DMU3	1	0.998365	0.998365	0.984499	0.970653	0.988632	0.988632	0.942526	0.98498	0.762284	0.58176	0.546022	0.546022	0.542423	0.33709
DMU4	1	1	1	0.953069	0.942635	0.961119	0.961119	0.761873	0.901602	0.732432	0.69211	0.688834	0.688834	0.678309	0.252646
DMU5	1	1	1	0.965248	0.957689	0.976565	0.976565	0.894933	0.976565	0.776375	0.50281	0.502812	0.502812	0.502812	0.122217
DMU6	1	1	1	1	1	1	1	1	0.941989	0.748317	0.6729	0.642753	0.642753	0.636646	0.261159
DMU7	1	1	1	1	1	1	1	1	1	1	1	0.945157	0.945157	0.766065	0.605358
DMU8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.616789
DMU9	1	1	1	1	0.948003	0.991238	0.991238	0.954671	0.907007	0.653734	0.30494	0.304781	0.304781	0.304781	0.147471
DMU10	1	1	1	0.98827	0.98827	1	1	1	1	1	0.93799	0.937082	0.937082	0.937082	0.445497
DMU11	1	0.998356	0.998356	0.91126	0.893869	0.957636	0.957636	0.832463	0.920183	0.884298	0.39217	0.392167	0.392167	0.392167	0.155423
DMU12	1	1	1	1	0.959242	1	1	1	1	1	0.75145	0.460149	0.460149	0.460149	0.219101
DMU13	1	1	1	1	1	1	1	1	1	1	0.50983	0.509832	0.509832	0.509832	0.147368
DMU14	1	1	1	1	1	1	1	1	0.922221	0.770769	0.64765	0.624646	0.624646	0.623513	0.200038
DMU15	1	1	1	1	0.965812	1	1	1	1	1	0.80135	0.801347	0.448844	0.448844	0.801347
DMU16	1	0.9915	0.9915	0.981481	0.981481	0.98093	0.98093	0.9017	0.931733	0.736034	0.60045	0.591086	0.591086	0.591086	0.164936
DMU17	1	1	1	0.970485	0.970485	0.983763	0.983763	0.91465	0.973481	0.792111	0.61986	0.426007	0.426007	0.406701	0.264097
DMU18	1	0.993611	0.993611	0.986225	0.986225	0.983603	0.983603	0.917549	0.943158	0.730722	0.58141	0.581413	0.581413	0.581413	0.133591
DMU19	1	1	1	0.991991	0.989624	0.98893	0.98893	0.931857	0.974962	0.875669	0.80662	0.726809	0.726809	0.690322	0.441428
DMU20	1	1	1	1	1	1	1	1	1	1	1	0.893622	0.893622	0.860661	0.432218
DMU21	1	1	1	1	0.994468	1	1	1	1	0.799856	0.78335	0.766515	0.766515	0.764688	0.360825
DMU22	1	1	1	1	0.970969	1	1	1	0.965412	0.636026	0.51627	0.510207	0.510207	0.510207	0.171629
DMU23	1	1	1	1	1	1	1	1	1	1	0.7735	0.773502	0.773502	0.773502	0.1068
DMU24	1	1	1	0.976204	0.976204	0.973628	0.973628	0.872343	0.952329	0.709545	0.4723	0.471206	0.471206	0.471206	0.248825
DMU25	1	1	1	1	0.943806	0.9973	0.9973	0.976813	0.9973	0.772235	0.41322	0.413215	0.413215	0.413215	0.192186
DMU26	1	1	1	0.991861	0.991861	0.994919	0.994919	0.967683	0.978184	0.850353	0.84594	0.845941	0.845941	0.821446	0.365767
DMU27	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
DMU28	1	1	1	1	0.96057	0.999133	0.999133	0.99652	0.978893	0.745567	0.41078	0.308094	0.308094	0.308094	0.14829
DMU29	1	1	1	1	1	1	1	1	1	1	1	1	1	0.448159	1
DMU30	1	1	1	1	1	0.981137	0.981137	0.860889	0.981137	0.72878	0.71615	0.709188	0.709188	0.683975	0.33626
DMU31	1	1	1	1	1	0.961897	0.961897	0.833751	0.95572	0.730766	0.50287	0.43418	0.43418	0.43418	0.20335
DMU32	1	1	1	1	1	1	1	1	1	0.850797	0.72375	0.653394	0.653394	0.653394	0.222665
DMU33	1	1	1	1	1	1	1	1	1	0.998608	0.99861	0.971882	0.971882	0.971882	0.294133
	33	32.971425	32.971425	32.667828	32.29067	32.669678	32.669678	31.320507	32.08682	27.76513	22.4711	21.17255	20.82005	19.927462	10.775569
Average	1	0.9991341	0.9991341	0.9899342	0.9785052	0.989990242	0.989990242	0.9491063	0.972328	0.841368	0.68094	0.641592	0.630911	0.6038625	0.3265324
Change in		0.0008659	0	0.0091999	0.011429	-0.011485091	0	0.040884	-0.02322	0.13096	0.16043	0.039349	0.010682	0.0270481	0.2773301
Efficiency															
Number of Efficient Branches	33	28	28	22	13	12	16	16	13	10	5	3	3	2	2

Table 5.2. Summary of Stepwise Approach reducing one variable each time - CCR Model

Models	Inputs	Outputs	Dropped	Efficiency Score	Change In Efficiency Score	Number of efficient Units
Model 1(8I,5O)	Department Expenses(I1),Dep&Amortization(I2),	Sales of Equipment(O1),Sales of Rentals(O2)	None	1	None	**33**
	Number of Staff(I3),Area of facility(I4),COGS Eq	Sales of Service(O3),Sale of Parts(O4),				
	COGSRental Sales(I6),COGS Service(I7),COGS	Total Branch Sales(O5)Total Gross Margin (O6)				
Model 2(7I,4O)	I1,I2,I3,I4,(I5+I6),I7,I8	(O1+O2),O3,O4,O5,O6	Composite Variables	0.9991341	0.000865909	**28**
			(I5+I6)&(O1+O2)			
Model 3(7I,3O)	I1,I2,I3,I4,(I5+I6),I7,I8	(O1+O2),O3,O4,O5	Dropped O6	0.99913409	0	**28**
Model 4(7I,2O)	I1,I2,I3,I4,(I5+I6),I7,I8	(O1+O2),O4,O5	Dropped O3	0.98993418	0.009199909	**22**
Model 5(7I,1O)	I1,I2,I3,I4,(I5+I6),I7,I8	(O1+O2),O6	Dropped O4	0.97850515	0.01142903	**13**
Model6(7I,2O)	I1,I2,I3,I4,(I5+I6),I7,I8	O5,O6	Dropped O1,O2	0.98999024	-0.011485091	**12**
Model 7(7I,1O)	I1,I2,I3,I4,(I5+I6),I7,I8	O6	Dropped O5	0.98999024	0	**16**
Model 8(7I,1O)	I1,I2,I3,I4,(I5+I6),I7,I8	O5	DroppedO6	0.94910627	0.04088397	**16**
Model 9(6I,2O)	I1,I2,I3,I4,(I5+I6),I8	O5,O6	Dropped I7	0.97232788	-0.023221606	**13**
Model 10(5I,2O	I1,I2,I3,I4,(I5+I6),	O5,O6	Dropped I8	0.84136758	0.130960303	**10**
Model 11(4I,2O	I1,I2,I3,I4,	O5,O6	Dropped(I5+I6)	0.68094112	0.160426455	**5**
Model 12(3I,2O	I1,I2,I3,	O5,O6	Dropped I4	0.64159245	0.039348667	**3**
Model 13(2I,2O	I1,I3,	O5,O6	Dropped I2	0.63091055	0.010681909	**3**
Model 14(1I,2O	I3	O5,O6	Dropped I1	0.60386248	0.027048061	**2**
Model 15(2I,2O	I1,I2,	O5,O6	Dropped I3	0.32653239	0.277330091	**2**

In conclusion from the stepwise approach to variable reduction, it is found that a lesser number of variables used yield efficiency scores with better discrimination. However, the choice of the variables used will depend on the requirement of the model used to analyze efficiency.

5.6: Returns to Scale:

We have now collected the data, defined the DMUs and the number of variables (inputs and outputs) to be used in the research study. We must now define the DEA model with respect to returns to scale whether constant or variable returns to scale. According to Banker and Natarajan, 2004 it is possible to choose the most suitable scale for the sample by testing the hypothesis of constant returns to scale. Banker and Natarajan,2004 suggest using the two-sample Kolmogorov-Smirnov test to test the null hypothesis H_0: The scale is constant returns to scale. Kolmogorov-Smirnov test (K-S, Test) is non-parametric and is based on the maximum distance of the cumulative distribution of efficiency scores of the DEA–CRS and DEA-VRS models(Banker&Natarajan,2004).

The above test evaluates the null hypothesis of constant returns to scale against the alternative hypothesis of variable return to scale (Perico et al.,2016). By construction, the above statistic takes the value between 0 and 1 and a high value closer to 1 rejects the null hypothesis and accepts the alternative hypothesis and vice versa (Banker& Natarajan,2004).

The two sample K-S test was done using 2013 data and 2014 data. These two years were chosen as data were available for maximum number of DMUs. For both, these data test results are shown in the table below.

Table 5.3. K-S Test statistic at α=0.05

Year	**Confidence Level**	**K-S Test Statistic**
2013	**95%**	**0.2404(0.145-p value)**
	99%	**0.2878(0.065-pvalue)**
2014	**95%**	**0.2367(0.157-pvalue)**
	99%	**0.2833(0.071-p value)**

It is evident from K-S Test statistic from the above table 5.3, that the test statistic for both years 2013 and 2014 at confidence interval 95% and 99%, are closer to zero and as per Banker and Natarajan, 2004, it accepts the null hypothesis H_0, of constant returns to scale. Therefore, all the analysis in the research will be based on constant returns to scale. However, for sake of comparison BCC model will also be used to study the behavior of the dealerships under variable returns to scale. However, as per Necmi Av Kiran (1999), returns to scale can also be established by finding CRS and VRS scores and if the scores do not match then there is scale efficiency and therefore BCC model can be used.

5.7: Orientation of the Model:

Identification of orientation of the model will help in defining what is to be achieved from the analysis. There are two orientations in DEA analysis input orientation and output orientation. There are instances such as hospital efficiency study where inputs are number of bed days available and hospital budget while outputs are number of patients recovered and the number of support staff trained. If the objective is to identify units that are over-utilizing resources, then it is obvious that input reduction would be the goal of the exercise. In such cases, input-oriented DEA model would be a choice. There are instances on the other hand like the study of bank efficiency, where the inputs could be interest expense and operating expense and output could be net-interest margin, investments, and deposits (Das,1997), the objective will be to maximize the outputs such as deposits and investments. In such instances output-oriented DEA model seems more suitable.

There are cases in an application the goal is input reduction and output enhancement simultaneously, then a slack-based model would be appropriate to study efficiency using DEA (Cook et al.,2014). The slack based model deals with the slacks directly and it is a non-radial model. It puts aside the assumption of proportionate changes in inputs and outputs.

In the current research, there are ten inputs and eleven outputs. From, among the twenty-one variables, many models can be formulated. The orientation of the model will be based on what needs to be accomplished with the model. If the objective is to maximize the gross margins for a given level of cost of goods sold, then it will be output orientation and if the objective is to minimize the expenses then it will be input orientation. Therefore, the orientation of the model will be based on the objective of the analysis.

In DEA analysis under a specific, returns to scale assumption (RTS), constant returns to scale or variable returns to scale, both inputs oriented, and output-oriented models yield the same efficient or best practice frontier. The orientations may not matter if the interest is in finding out the best practice frontier. However, depending upon the orientations, the reference set of the inefficient units may vary (Cook et.al, 2014).

5.8: DEA Model Development:

One of the basic assumptions in DEA analysis is that if a given branch A is capable of Y_s units of outputs (e.g., sales) with X_r units of inputs, then other branches should also be able to do in the same way if they were to function efficiently. Similarly, if branch-B is capable of Y_o units of outputs with X_i units of inputs, then other branches should be able to perform at the same level (Grewal et al.,1999).

It is also assumed in DEA branches A and B can be combined to form a composite branch with composite inputs and outputs. This composite branch may not necessarily exist in reality, and therefore is called a virtual branch and the inputs and outputs associated with the virtual branch are called virtual inputs and outputs. The basis of the DEA analysis lies in finding the virtual branch for each real branch. If the virtual branch performs better than the real branch being assessed by either producing more outputs with existing inputs or producing more outputs with fewer inputs, then the real branch is inefficient (Grewal et al.,1999).

The process for finding the best virtual branch is to formulate a linear program to analyze the efficiency of thirty-three branches and then solving thirty-three linear programming problems. The DEA models described in the literature section will be used with appropriate inputs and outputs to model the efficiency of the branches.

Based on the above discussion, the efficiency of heavy equipment branches will be analyzed using the following four models by using factors of production from accounting information and other factors related to land and labor. All these models will use a Black-Box approach where the internal structure of the DMU will not be analyzed. However, the factors of production in the production process model will be used to analyze efficiency scores using Network DEA. All other three models will use only Black-Box approach. The models are listed as below.

1) **Model 1: Branch Production Process Maximization(PPM) Model**

Inputs: Number of employees, Area of Facility, Total COGS of the branch, Total Department Expenses.

Outputs: Total Branch Sales revenue, Total Gross Margin of the branch

2) **Model 2: Branch Profit Maximization Model.**

Inputs: Number of employees, Area of Facility, COGS of Sales, COGS of Service, COGS of Parts. **Output**: Total Equipment Sales Revenue, Parts Sales Revenue, Total Service Revenue

3) **Model 3: Branch Expenses Minimization Model.**

Input: COGS of Sales, COGS of Service, COGS of Parts, departmental expenses, Depreciation and Amortization **Output**: Total Sales Revenues.

4) **Model 4: Branch Assets Maximization Model.**

Inputs: Total COGS (Sales+ Service+ Parts), Current Assets, Fixed Assets, Other Assets

Output: Total Sales (Equipment +Parts +Service) Revenues

5.9: Branch Production Process Maximization Model(PPM) (Model 1)

This model is called the production process maximization model as all the inputs and outputs represent the production process in the branch operation. The four inputs used in the model are number of employees, the area of the facility, the total cost of goods sold for the branch (COGS) and total department expenses. The total cost of goods sold is the sum of the cost of goods sold for equipment and rentals, cost of goods sold for service and cost of goods sold for parts. The two outputs used are total branch sales revenue and total gross margin for the branch. The total branch sales revenue is the sum of sales revenue for equipment and rentals, sales revenue of service and sales revenue of parts. Similarly, the total gross margin of the branch is the sum of gross margin from equipment and rentals sales, gross margin from service sales and gross margin from parts sales. Please note all the gross margin data are real numbers and not ratios and are all positive.

Inputs: Number of Staff, Area of Facility, Total COGS of the branch, Total Dep Expenses.

Outputs: Total Branch Sales revenue, Total Gross Margin of the branch.

There are a total of six factors and a minimum of thirty-three DMUs. The sum of inputs and outputs times three is eighteen that is less than thirty-three the number of DMUs and therefore complies with the formula of degree of freedom, that relates the number of DMUs and number of factors.

In this production process model of the branch, efficiency scores will be found using the following DEA multiplier models that have been discussed in detail in Chapter 3, Theoretical Framework. The orientation of the model will be output oriented as the objective is to maximize the production process. The data for the period 2010-2014 will be used for analysis.

a) **CCR and BCC Models:** These models are the basic DEA models CCR and BCC models. These two models will give the efficiency score under the assumption of constant returns to scale and variable returns to scale. The CCR and VRS scores will then be used to find the scale efficiency of the branch.

b) **Super Efficiency Model:** The above two models will find more than one DMU as efficient. In other words, there will be a number of DMUs that will be efficient and a number of DMUs will be inefficient. The super efficiency model will help in assigning a score greater than one for efficient DMUs and retain the same score for inefficient DMUs. The scores greater than one are called super efficiency score and it helps in ranking the DMUs and then in benchmarking.

c) **Cross Efficiency Model:** The cross-efficiency model helps in evaluating the efficiency of a DMU by its peers and will also identify the DMUs that are far from the normal DMUs on the frontier. Such DMUs are called maverick and such DMUs can be identified with a score called maverick index.

d) **Envelopment Model:** The envelopment model is the dual of the multiplier model in linear programming parlance. The envelopment model is chosen to find out the efficiency score, reference benchmarking, slack variable and target values.

e) **Bootstrapped DEA:** There is noise or error in the CCR and VRS scores obtained by CCR and BCC models. Bootstrapped DEA helps in finding the bias in the score and thereby helps in correcting the score.

f) **Contextual Variables:** There are environmental factors that are called contextual variables that have a bearing on the efficiency scores. Ordinary least square regression and Tobit regression will be used to find the effect of contextual variables on efficiency scores.

g) **Efficiency Change over time:** The change of efficiency scores over period 2010-2014 will be analyzed using the methods of Window Analysis and Malmquist Productivity Index. Window analysis helps in understanding the trend in the efficiency scores and the Malmquist index helps in decomposing the efficiency change into technological change and efficiency change.

h) **Detection of Outliers:** The estimates of efficiency are sensitive to the presence of outliers as in DEA superior performance is identified using extreme observations. The two methods used will be the super efficiency method proposed by Banker and Cheung (2006) and Tran's method that uses lambda values.

i) **Network DEA:** All the above analysis has been done considering the heavy equipment retailing organization's branch as a complete system ignoring the structure of the system. Each branch of a heavy equipment retailing organization has equipment sales, equipment service, and equipment service organization. All these three operations operate in parallel independent of each other. Therefore, the inputs and outputs of the branch will be decomposed into inputs and outputs of the individual sales, service and parts operations and efficiency found using the network parallel structure. The system can also be viewed as a series operation in two stages wherein the first stage the equipment is sold and in the second stage the services are performed that includes repairing the machine using labor and the parts are sold to service the equipment. The series structure analysis is found to understand the efficiency of two-stage operation.

5.9.1: CCR and BCC Models:

The CCR and BCC models were run using the following four inputs and two outputs.

Inputs: Number of Staff, Area of Facility, Total COGS of the branch, Total Dep Expenses.

Outputs: Total Branch Sales revenue, Total Gross Margin of the branch.

The orientation of the model is output oriented as it is to maximize the outputs, revenue and gross margin from the sale of equipment, service, and parts utilizing the number of employees, the area of the facility, department expenses and total cost of goods sold as inputs.

LP formulation of the model

$$\text{Efficiency} = \frac{\text{Output(U)}}{\text{Input(V)}} =$$

$$\frac{\text{Total sales revenue of branch(Y1)} + \text{Total G.M of the branch(Y2)}}{\text{Nbr of Staff(X1)} + \text{Area of Facility(X2)} + \text{Total Exp(X3)} + \text{Total COGS of the branch(X4)}}$$

Let, $u_1 =$ Weight attached to output Y1,

$u_2 =$ Weight attached to output Y2

v1= Weight attached to input X1

v2= Weight attached to input X2

v3= Weight attached to input X3

v4 = Weight attached to input X4

$$Max\,TE_i = 0 \le \frac{\sum_{r=1}^{2} u_r y_{ro}}{\sum_{i=1}^{4} v_i x_{io}} \le 1$$

$$\textit{Fractional program problem is}$$

$$\max \theta = \frac{u_1 y_{1j} + u_2 y_{2j}}{v_1 x_{1j} + v_2 x_{2j} + v_3 x_{3j} + v_4 x_{4j}}$$

$$\textit{Subject to,} \frac{u_1 y_{1j} + u_2 y_{2j}}{v_1 x_{1j} + v_2 x_{2j} + v_3 x_{3j} + v_4 x_{4j}} \le 1 (j = 1, 2, 3 ... 33)$$

$$v_1, v_2 ....v_{33} \ge 0$$

$$u_1, u_2 ....u_{33} \ge 0$$

Transferring fractional programming problem to Linear programming problem using Charnes-Cooper transformation, the LP problem is

$$\max \theta = u_1 y_{1j} + u_2 y_{2j}$$

$$\textit{Subject to,}$$

$$v_1 x_{1j} + v_2 x_{2j} + v_3 x_{3j} + v_4 x_{4j} = 1$$

$$u_1 y_{1j} + u_2 y_{2j} \le v_1 x_{1j} + v_2 x_{2j} + v_3 x_{3j} + v_4 x_{4j}$$

$$v_1, v_2, v_3, v_4 \ge 0$$

$$u_1, u_2 \ge 0$$

An example of DEA LP formulation for DMU1 is as below,

$$y_{11} = 2282, Sales\,revenue\ DMU1(output1)$$
$$y_{12} = 45, Gross\ M\arg in\ DMU1(output2)$$
$$x_{11} = 47, Number\ of\ staff\ DMU1\left(Input1\right)$$
$$x_{21} = 93635, Area\ of\ facility\ DMU1(Input\ 2)$$
$$x_{31} = 34, Total\ department\ \exp enses\ DMU1(Input3)$$
$$x_{41} = 171, Total\ COGS\ for\ the\ branch\ DMU1(Input4)$$

Outputs and Inputs 3 and 4 are times 10,000

$$Max\theta = 228u_1 + 45u_2$$
$$Subject\ to,$$
$$47v_1 + 93635v_2 + 34v_3 + 171v_4 = 1$$
$$DMU2: 40u_3 + 8u_4 \le 12v_5 + 5550v_6 + 7v_7 + 3v_8$$
$$contd.......till\ DMU33$$

$$DMU33: 87u_{34} + 14u_{35} \le 7v_{36} + 8780v_{37} + 8v_{38} + 65v_{39}$$
$$u_{1,}u_2.....u_{35} \ge 0$$
$$v_1, v_2.......v_{39} \ge 0$$

The primal problem above is rewritten as below.

$$Max\theta = 228u_1 + 45u_2$$

$$Subject\ to,$$

$$47v_1 + 93635v_2 + 34v_3 + 171v_4 = 1$$

$$DMU\,2: 40u_3 + 8u_4 - 12v_5 - 5550v_6 - 7v_7 - 3v_8 \le 0$$

$$contd.......till\ DMU\,33$$

$$DMU\,33: 87u_{34} + 14u_{35} - 7v_{36} - 8780v_{37} - 8v_{38} - 65v_{39} \le 0$$

$$u_{1,}u_2.....u_{35} \ge 0$$

$$v_1, v_2.......v_{39} \ge 0$$

The dual of the above primal problem is written as below,

$$Min\,\theta,$$

$$Subject\ to,$$

$$40\lambda_1 + x_{12}\lambda_2 + ... + 87\lambda_{33} \ge 228$$

$$89\lambda_1 + x_{22}\lambda_2 + ... + 14\lambda_{33} \ge 45$$

$$47\theta - x_{11}\lambda_1 - .... - x_{133}\lambda_{33} \ge 0$$

$$9\theta - 5550\lambda_1 - .... - x_{233}\lambda_{33} \ge 0$$

$$34\theta - 7\lambda_1 - ....x_{333}\lambda_{33} \ge 0$$

$$171\theta - 3\lambda_1 - ....x_{433}\lambda_{33} \ge 0$$

$$\lambda_1, \lambda_2, ...\lambda_{33} \ge 0$$

$$\theta, sign\ free$$

The above is the LP primal and dual problem for four inputs and two outputs for DMU1. Similar LP problems can be written for all other 32 DMUs. However, this is a voluminous work and hence Max DEA software is used to formulate and solve the complete model to find the efficiency scores.

The solution to the above problem will give the values of **θ**, the efficiency score under CCR and BCC models, value of **λ** the coefficients to define the hypothetical efficient DMU, weights for each outputs $\mathbf{u_r}$ and inputs $\mathbf{v_i}$**,** efficient targets for inputs and outputs and the ratio of CCR score to BCC efficiency scores provides a measure of scale efficiency.

5.9.1A. Efficiency Scores:

The CRS efficiency scores for the period 2010-2014 for all the DMUs are given in table 5.4, and the VRS efficiency scores for the same period is given in table able 5.5 and Scale efficiency scores for the above period is shown in table5.6. The descriptive statistics of CRS (Constant returns to scale), VRS (Variable returns to scale) and Scale efficiency scores are shown in table 5.7, table 5.8 and table5.9 respectively.

The CRS efficiency scores indicate the technical efficiency of the DMUs in each period. The technical efficiency is a composition of pure technical efficiency and scale efficiency. The VRS scores decompose the technical efficiency into pure technical efficiency and scale efficiency. The scale efficiency is computed by dividing the CRS scores by VRS scores.

The results indicate that the thirty-three branches have been characterized by significant difference between branches as regards their overall technical efficiency that ranges between 89.77%-100%(2014),84.79%-100%(2013),90.86%-100%(2012),86.93%-100%(2011) and 86.34%-100%(2010) under constant returns to scale assumption. This indicates that a branch has opportunity to increase output by 10.23%(2014),14.21%(2013),9.14%(2012),13.07%(2011) and 13.64%(2010) to reach the efficient frontier. In other words, the magnitude of overall technical inefficiency (1-technical inefficiency) that exists in branches ranges from, 9.14%-14.21%. This indicates that by adopting best practice technology the branches can increase their gross margin and revenue from sales from 9.41%to 14.21% with the same level of inputs in terms of a number of staff, the area of the facility, total department expenses and total COGS for the branch. However, the potential increase in outputs from the existing inputs varies from branches to branches. Alternatively, each branch has the scope of producing 1.11 times (1/0.8977) in 2014,1.18(1/0.8479) times in 2013,1.10(1/0.9086) times in 2012,1.15(1/0.8693) times in 2011 and 1.16(1/0.8634) times in 2010, as much outputs from the same level of inputs.

Similarly, under variable returns to scale the efficiency varies from 90.67%-100% in 2014,85.75% -100% in 2013,91.51%-100% in 2012,89.80%-100% in 2011 and 87.58%-100% in 2010. Similarly, this indicates that by adopting best practice technology under VRS, the branches can increase their gross margin and sales revenue by 9.33% in 2014,14.25% in 2013,8.49% in 2012,10.2% in 2011 and 12.42% in 2010. Alternately under variable returns to scale each branch has the scope of producing 1.10 times (1/0.90672) in 2014,1.17 times (1/0.8575) in 2013,1.09 times (1/0.9151) in 2012,1.11 times (1/0.8980) in 2011 and 1.4 times (1/0.8758) in 2010, as much outputs from the same level of inputs.

5.9.1B. Technical Efficiency Scores (CRS Scores)- from CCR Model:

The technical efficiency scores for the period 2010-2014 are given in table 5.4. On analyzing the technical efficiency scores for the year 2014 it is found that there are eight branches under CRS assumption have acquired the status of globally efficient and lie on the frontier with an efficiency score of 1. These branches (DMU 7,8,12,13,20,23,27,29) define the best practice or efficient frontier and thus form the reference set for the inefficient branches. The utilization of resources in these eight branches is functioning well meaning that the production process is not characterized by wastage of inputs. In DEA terminology these eight branches are called *peers* and they set an example of best-operating practices for other inefficient branches to follow. The remaining twenty-five branches have an efficiency score of less than 1, meaning they are technically inefficient. Therefore, from the analysis, it is found that there is a presence of a marked deviation in the efficiency score of all the branches.

These branches that are inefficient can improve their efficiency by utilizing the inputs more efficiently. The technical inefficiency score of the twenty-five branches ranges from 0.8977 for DMU 9 to 0.966245 for DMU 28. This implies that the above two DMUs can reduce their input by 10.23% and 3.38%. This interpretation of technical inefficiency score can be extended to other branches in the year 2014 and other years too.

5.9.1C.: Decomposition of Technical Efficiency – (BCC Model):

The technical efficiency score obtained from the CCR model is composed of pure technical efficiency and scale efficiency. In other words, technical efficiency helps to measure the combined efficiency that is due to pure technical efficiency and efficiency due to inappropriate branch size (scale efficiency). The pure technical efficiency score or VRS score is derived from the BCC model under the assumption of variable returns to scale that is free of scale effects. Therefore, the pure technical efficiency score indicates that all the inefficiencies in a branch directly result from managerial underperformance or managerial inefficiency (Kumar &Gulati,2008). The efficiency scores of the branches under variable returns to scale increases because BCC model envelops the data points more tightly than the CRS model and hence provides efficiency scores that are greater than or equal to those obtained using CCR model (Cooper, Seiford, Tone,2007), a model under CRS assumption. In DEA literature, the branches obtaining technical efficiency and pure technical efficiency scores equal to one are known as globally efficient and locally efficient branches (Kumar and Gulati,2008).

The table 5.4 lists the DMUs that are CCR efficient, VRS efficient and scale efficient. In the VRS scores for the year 2014there are 13DMUs that have acquired the status of locally efficient branches as they attained the pure technical efficiency of 1. There are 8 branches in the same year 2014 under CRS assumption have acquired the status of globally efficient and lie on the frontier under CRS assumption. The remaining twenty-five branches have an efficiency score less than 1. Five DMUs 10,19,28,32 and 33 attained the pure technical efficiency score of 1 and lie on the efficient frontier under VRS assumption. These five branches that have become efficient under VRS assumption but inefficient under CRS assumption, we can infer that the overall technical inefficiency in these branches is not caused by poor utilization of inputs (managerial inefficiency) but caused by the operation of the branches with inappropriate scale size. In the remaining twenty branches where the pure technical efficiency (VRS Score) is less than 1, managerial inefficiency exits but of a different nature. In these twenty branches overall, technical inefficiency stems from both pure technical inefficiency and scale inefficiency as indicated by the fact that of these twenty branches, sixteen branches have VRS score less than Scale Efficiency (Refer tables 5.5 and 5.6). This indicates that the in these sixteen branches the inefficiency is attributed to managerial inefficiency rather than scale inefficiency.

On analyzing the pure technical efficiency score and scale efficiency score for all the branches of this heavy equipment retailing organization, it is found that overall technical inefficiency is both due to pure technical inefficiency (poor input utilization) and scale efficiency (unable to operate at the most productive scale size).

5.9.1D.: Returns to Scale (Scale Efficiency):

As per microeconomic theory, one of the basic objectives of a firm is to operate at its most productive scale size i.e., with constant returns to scale(CRS) in order to maximize revenue and minimize cost(Kumar&Gulati,2008). The firms may operate in increasing returns to scale(IRS) or decreasing returns to scale(DRS) in the short run but will move towards CRS in the long run by becoming larger or smaller to survive in a competitor's market (Kumar & Gulati,2008). The firm may change its operating strategy to scale down or up the operations and the scale efficiency indicated above to find out if the size of the firm is appropriate to the equipment industry. In this research, it is found that there are only 6,11,9,7 and 8 branches operating at the most productive scale size in years 2010-2014 respectively (table5.4).

In other words, the results indicate that 18.75%, 33.33% 27.27%,21.21%, and 24.24% of the branches are operating at most productive scale size under constant returns to scale in the period 2010-2014 respectively. Further 25%,27.27%,30.30%,30.39% and 15.15% of branches are operating under increasing returns to scale in the year 2010,2011,2012,2013 and 2014 respectively indicating that these branches are operating below their optimal scale size can enhance the technical efficiency by increasing their size. The branches that are operating under decreasing returns to scale are 50%, 33.3%, 39.39%, 36.36% and 60.60 % in the years 2010-2014 respectively. under decreasing returns to scale indicating that the branch is operating under supra-optimal size. The strategic option is to downsize these branches so as to reduce costs. From the above analysis it is evident that decreasing returns to scale is the most predominant form of scale efficiency in the dealership under study.

Table 5.4. Summary of CRS Scores: 2010-2014 (Production Process Maximization Model)

DMU	2014Technical Efficiency Score(CRS)	2013Technical Efficiency Score(CRS)	2012Technical Efficiency Score(CRS)	2011Technical Efficiency Score(CRS)	2010Technical Efficiency Score(CRS)
DMU1	0.930246	0.909605	0.931374	0.892344	0.878887
DMU2	0.945015	0.93342	0.997941	0.982162	0.974435
DMU3	0.965043	0.965099	0.950019	0.958615	0.959912
DMU4	0.9029	0.916736	0.938968	0.988051	0.962324
DMU5	0.950534	0.856296	0.931752	0.760054	0.986504
DMU6	0.951304	1	0.957465	1	0.997952
DMU7	1	0.993479	1	1	1
DMU8	1	0.992927	0.986958	1	0.902284
DMU9	0.897789	0.913078	0.914275	0.900003	1
DMU10	0.99902	1	1	1	0.979374
DMU11	0.938974	0.997149	0.982101	0.973535	1
DMU12	1	1	1	1	1
DMU13	1	1	1	1	0.874374
DMU14	0.916691	0.880852	0.9239	0.908951	0.933018
DMU15	0.944756	0.932221	0.946444	0.970717	0.938022
DMU16	0.941799	0.979694	0.960434	0.959842	0.8942
DMU17	0.950098	0.915935	0.936257	0.927245	0.960531
DMU18	0.921278	0.89202	1	1	0.971762
DMU19	0.963665	0.968675	0.975476	0.98117	0.904202
DMU20	1	0.974648	0.965074	0.955843	0.871228
DMU21	0.971495	0.962607	0.908628	0.921262	1
DMU22	0.913958	0.847937	1	1	0.934469
DMU23	1	1	0.911565	0.891073	0.930432
DMU24	0.944889	0.915095	0.911387	0.894267	0.906407
DMU25	0.965188	0.948104	0.916269	0.899284	1
DMU26	0.949302	0.914337	1	1	0.934732
DMU27	1	0.9474	0.944022	0.910519	0.863465
DMU28	0.966245	0.936525	0.934245	0.869369	0.900804
DMU29	1	0.938573	1	1	0.96646
DMU30	0.92872	1	1	1	0.964236
DMU31	0.94526	0.98728	0.990772	0.993787	
DMU32	0.998663	1	NA	NA	NA
DMU33	0.998608	NA	NA	NA	NA

Please Note: **NA indicates that the DMU was not operational during that year**

Table 5.5. Summary of VRS Scores 2010-2014 (Production Process Maximization Model)

DMU	2014Pure Technical Efficiency Score(VRS)	2013Pure Technical Efficiency Score(VRS)	2012 Pure Technical Efficiency Score(VRS)	2011 Pure Technical Efficiency Score(VRS)	2010Pure Technical Efficiency Score(VRS)
DMU1	0.949093	0.936882	0.974071	0.970136	0.987462
DMU2	0.980016	0.970611	1	1	0.976322
DMU3	0.974518	0.965116	0.957558	0.970153	0.991311
DMU4	0.906721	0.93337	0.949942	1	1
DMU5	0.950544	0.875524	0.948191	1	0.986519
DMU6	0.951843	1	0.966077	1	1
DMU7	1	1	1	1	1
DMU8	1	1	1	1	0.990579
DMU9	0.924842	0.930347	0.927724	0.94214	1
DMU10	1	1	1	1	1
DMU11	0.965348	1	0.989367	0.986068	1
DMU12	1	1	1	1	1
DMU13	1	1	1	1	0.875815
DMU14	0.917547	0.883851	0.930313	0.911928	0.943952
DMU15	0.973827	1	0.947707	0.974064	0.950182
DMU16	0.9424	0.982196	0.96062	0.960442	0.894508
DMU17	0.953294	0.920335	0.949247	0.928613	1
DMU18	0.926717	0.905725	1	1	0.981346
DMU19	1	1	0.977769	0.983972	0.91851
DMU20	1	0.975476	0.966566	0.956838	0.879032
DMU21	0.973795	0.964022	0.917062	0.930282	1
DMU22	0.916848	0.857559	1	1	1
DMU23	1	1	0.920805	0.917336	0.940305
DMU24	0.958991	0.921516	0.9151	0.902041	0.929111
DMU25	0.968978	0.954077	0.953981	0.932615	1
DMU26	0.953865	0.945376	1	1	1
DMU27	1	1	1	1	0.92533
DMU28	1	1	0.944973	0.898021	0.910909
DMU29	1	1	1	1	1
DMU30	0.979714	1	1	1	0.966613
DMU31	0.948497	1	0.996238	1	
DMU32	1	1	NA	NA	NA
DMU33	1	NA	NA	NA	NA

Table 5.6. Summary of Scale Efficiency scores 2010-2014 (PPM-Model)

DMU	2014Scale Efficiency Score	RTS	2013Scale Efficiency Score	RTS	2012 Scale Efficiency Score	RTS	2011 Scale Efficiency Score	RTS	2010 Scale Efficiency Score	RTS
DMU1	0.980142	Decreasing	0.970885	Decreasing	0.956166	Decreasing	0.919814	Decreasing	0.890047	Decreasing
DMU2	0.964285	Increasing	0.961683	Increasing	0.997941	Increasing	0.982162	Increasing	0.998068	Increasing
DMU3	0.990278	Decreasing	0.999983	Decreasing	0.992127	Decreasing	0.988107	Decreasing	0.968325	Decreasing
DMU4	0.995785	Decreasing	0.982179	Decreasing	0.988447	Decreasing	0.988051	Decreasing	0.962324	Decreasing
DMU5	0.99999	Decreasing	0.978038	Increasing	0.982663	Increasing	0.760054	Increasing	0.999985	Increasing
DMU6	0.999435	Decreasing	1	Constant	0.991085	Increasing	1	Constant	0.997952	Increasing
DMU7	1	Constant	0.993479	Increasing	1	Constant	1	Constant	1	Constant
DMU8	1	Constant	0.992927	Increasing	0.986958	Increasing	1	Constant	0.910865	Decreasing
DMU9	0.970748	Decreasing	0.981439	Decreasing	0.985503	Decreasing	0.955275	Decreasing	1	Constant
DMU10	0.99902	Decreasing	1	Constant	1	Constant	1	Constant	0.979374	Decreasing
DMU11	0.972679	Decreasing	0.997149	Decreasing	0.992656	Decreasing	0.98729	Decreasing	1	Constant
DMU12	1	Constant	1	Constant	1	Constant	1	Constant	1	Constant
DMU13	1	Constant	1	Constant	1	Constant	1	Constant	0.998355	Decreasing
DMU14	0.999067	Decreasing	0.996607	Decreasing	0.993106	Decreasing	0.996736	Increasing	0.988417	Decreasing
DMU15	0.970147	Increasing	0.932221	Increasing	0.998667	Increasing	0.996564	Decreasing	0.987203	Decreasing
DMU16	0.999362	Decreasing	0.997453	Increasing	0.999807	Increasing	0.999376	Decreasing	0.999656	Increasing
DMU17	0.996648	Decreasing	0.995219	Increasing	0.986315	Increasing	0.998527	Increasing	0.960531	Decreasing
DMU18	0.994131	Increasing	0.984868	Increasing	1	Constant	1	Constant	0.990234	Decreasing
DMU19	0.963665	Decreasing	0.968675	Decreasing	0.997654	Decreasing	0.997153	Increasing	0.984422	Decreasing
DMU20	1	Constant	0.999151	Increasing	0.998457	Increasing	0.99896	Increasing	0.991121	Decreasing
DMU21	0.997638	Decreasing	0.998533	Decreasing	0.990804	Decreasing	0.990304	Decreasing	1	Constant
DMU22	0.996848	Decreasing	0.98878	Decreasing	1	Constant	1	Constant	0.934469	Decreasing
DMU23	1	Constant	1	Constant	0.989965	Decreasing	0.97137	Decreasing	0.9895	Decreasing
DMU24	0.985295	Decreasing	0.993033	Decreasing	0.995942	Decreasing	0.991382	Decreasing	0.975564	Increasing
DMU25	0.996088	Decreasing	0.99374	Increasing	0.960469	Increasing	0.96426	Increasing	1	Constant
DMU26	0.995217	Increasing	0.967167	Increasing	1	Constant	1	Constant	0.934732	Increasing
DMU27	1	Constant	0.9474	Increasing	0.944022	Increasing	0.910519	Increasing	0.933142	Decreasing
DMU28	0.966245	Increasing	0.936525	Increasing	0.988648	Decreasing	0.968094	Decreasing	0.988907	Decreasing
DMU29	1	Constant	0.938573	Decreasing	1	Constant	1	Constant	0.96646	Decreasing
DMU30	0.94795	Decreasing	1	Constant	1	Constant	1	Constant	0.997541	Increasing
DMU31	0.996587	Decreasing	0.98728	Decreasing	0.994513	Increasing	0.993787	Increasing		
DMU32	0.998663	Decreasing	1	Constant	NA	NA	NA	NA	NA	NA
DMU33	0.998608	Decreasing	NA	NA	NA	NA	NA	NA	NA	NA

Table 5.7. Descriptive Statistics of CCR Scores 2010-2014 (PPM- model)

Descriptive Statistics	2014Technical Efficiency Score(CRS)	2013Technical Efficiency Score(CRS)	2012Technical Efficiency Score(CRS)	2011Technical Efficiency Score(CRS)	2010Technical Efficiency Score(CRS)
Mean	0.960649697	0.921808848	0.903494727	0.895093727	0.887187938
Standard Error	0.005680398	0.029825098	0.040970584	0.041305831	0.041853671
Median	0.951304	0.948104	0.957465	0.970717	0.948967
Mode	1	1	1	1	1
Standard Deviation	0.032631402	0.171332142	0.235358084	0.237283932	0.236760119
Range	0.102211	1	1	1	1
Minimum	0.897789	0.847937	0.908628	0.869369	0.863465
Maximum	1	1	1	1	1
Count	33	33	33	33	32

Table 5.8. Descriptive Statistics of VRS Scores 2010-2014 (PPM- model)

Descriptive Statistics	2014Pure Technical Efficiency Score(VRS)	2013Pure Technical Efficiency Score(VRS)	2012 Pure Technical Efficiency Score(VRS)	2011 Pure Technical Efficiency Score(VRS)	2010Pure Technical Efficiency Score(VRS)
Mean	0.970224182	0.93702979	0.91191852	0.914080273	0.907743938
Standard Error	0.00519402	0.03019	0.04125142	0.041459552	0.04266474
Median	0.973827	0.982196	0.966566	0.986068	0.9869905
Mode	1	1	1	1	1
Standard Deviation	0.02983737	0.17342832	0.23697137	0.238166991	0.241348218
Range	0.093279	1	1	1	1
Minimum	0.906721	0.857559	0.9151	0.898021	0.875815
Maximum	1	1	1	1	1
Count	33	33	33	33	32

Table 5.9. Descriptive Statistics of Scale Eff Scores 2010-2014 (PPM- model)

Descriptive Statistics	2014Scale Efficiency Score	2013Scale Efficiency Score	2012 Scale Efficiency Score	2011 Scale Efficiency Score	2010 Scale Efficiency Score
Mean	0.990137	0.9540299	0.9306641	0.919932879	0.91647481
Standard Error	0.00249387	0.0300204	0.041851	0.042042734	0.0427967
Median	0.996848	0.993033	0.993106	0.996564	0.988662
Mode	1	1	1	1	1
Standard Deviation	0.0143262	0.1724542	0.2404157	0.241517117	0.24209471
Range	0.05205	1	1	1	1
Minimum	0.94795	0.932221	0.944022	0.760054	0.890047
Maximum	1	1	1	1	1
Count	33	33	33	33	32

Table 5.10. Summary of Efficient branches

Subject	2014	2013	2012	2011	2010
Number of Efficient Branches under CRS	7,8,12,13,20,23,27,29**(8)**	6,10,12,13,23,31,33**(7)**	7,10,12,13,19,23,27,31,32**(9)**	6,7,8,10,12,13,19,23,	8,10,12,13,23,27(6)
				27,31,32**(11)**	
Number of Efficient Branches under VRS	7,8,10,12,13,19,20,23,27,	6,7,8,10,11,12,13,15,19,23,27,	2,7,8,10,12,13,19,23,27	2,4,5,6,7,8,10,12,13,19,	4,7,8,10,11,12,13,19,23,24,
	28,29,32,33**(13)**	28,30,31,32,33**(16)**	28,31,32**(12)**	27,28,31,32,33**(16)**	27,28,32**(13)**
Number of Branches under Constant Scale Efficiency	8	7	9	11	6
Number of Branches under Increasing Scale Efficiency	5	13	10	9	8
Number of Branches under Decresing Scale Efficiency	20	12	13	11	16

5.9.2: Discrimination of Efficient Branches:

In the above evaluation of the efficiency of the thirty-three branches, it is found that there are 6,11,9,7 and 8 efficient branches during the period 2010-2014 respectively under CRS assumption and 13,16,12,16 and 13 branches efficient during the period 2010-2014 under VRS assumption (Table 5.10). The question now is how to discriminate these efficient units and rank them. Two models that are widely used in discriminating the efficient units under production process model will be used in this research. They are;

a) Super Efficiency Model b) Cross Efficiency Model

5.9.3: Super-Efficiency Model:

The model description and theoretical aspects of Super Efficiency are covered in detail in Chapter 3(Theoretical aspects of DEA). The super efficiency model is run under both CRS and VRS with output orientations as below.

Super-efficiency model was run using the same inputs and outputs for the Production process model both under CRS and VRS. The super efficiency scores for CRS and

VRS models are given in the table 5.11 and table 5.12. In these tables, the super efficiency scores are shown alongside CRS and VRS scores for comparison.

Analyzing the super efficiency score under CRS for the year 2014, it is found that in the super efficiency model the scores of the inefficient DMUs remain the same and the scores of the efficient DMUs are greater than 1. The CRS scores of efficient DMUs DMU 7,8,12,13,20,23,27 and 29 are in 2014 are 1.072082, 1.0332947, 1.66419, 1.046709, 2.036087, 1.315397,1.037421 and 2.01309 respectively. These scores can be organized in the ascending order and the DMUs 20,29,12,23,13,27 and 7 falls in that order. This means DMU 20 is the most efficient and DMU 7 is the least efficient of the eight efficient DMUs. This is how super efficiency scores can be used to rank the efficient units. Similar interpretations can be done for the scores in the rest of the periods.

Analyzing the VRS super-efficiency model from table 5.12 for the period 2014, it is found that the DMUs that were efficient under VRS have a super efficiency score more than 1 and this can be used to rank the efficient units. The efficiency score of the inefficient units remains the same under the VRS super efficiency model. The super-efficient DMUs under VRS model are DMU 7,8,10,12,13,19,20,23,27,28,29,32 and 33 and the scores are1.097122,1(Inf),1.2191,1.2166,1.0485,1.4361,2.1827,1.3364,1.7932,1(inf),1(inf),1.3616 and 1.0513. It is found that DMU8,28 and 29 are infeasible meaning that the LP program has unbounded solutions.

There are several models that have been proposed to deal with infeasibility in super-efficiency DEA models. Lovell and Rouse (2003) proposed an oriented method for tackling the infeasibility problem. This method uses the scaling procedure applied to either input (input orientation) or outputs (output orientation) of the efficient units for which the calculation of super efficiency score is infeasible. However, the issue with this procedure is that it assigns equal super efficiency scores to infeasible DMUs. Chen (2005) tried solving the problem by substituting the inefficient units with their efficient projections under VRS but it solved the problem partially as it failed to define a feasible solution in both orientations. Cook et al., (2009) introduced an approach which proposes one directional input-output movements so that unit under evaluation that is infeasible in SE models reaches the frontier formed by the rest of the DMUs. Ray (2008) used a directional distance function to solve the infeasibility issue but was not an oriented analysis. Chen et al., (2011) proposed a combinatorial input-and output-oriented method that provides targets for evaluated DMUs with radial movements of both inputs and outputs. Cheng and Zervopoulos (2012) used a proxy approach to solve the infeasibility problem. The concept of the proxy approach is to find a virtual proxy unit for the efficient DMU. The authors called the approach as FPA (Frontier Proxy Approach) approach. This approach is used in this research in solving the infeasibility problem and the scores of FPA approach are given in Table 5.13 alongside VRS and VRS super efficiency scores. It can be seen from this table that the DMUs 8,28 and 29 that were infeasible under VRS super efficiency now have a feasible solution under FPA approach. The DMUs can now be ranked based on the Super Efficiency scores as given in Table 5.13.

Therefore, the DMUs can now be ranked based on the CRS and VRS super efficiency scores. Based on CRS super efficiency scores the ranking in ascending order is DMU20, DMU29, DMU12, DMU8, DMU23, DMU13, DMU27, and DMU7. Similarly, based on VRS, FPA scores the DMUs can be ranked in the ascending order as DMU8, DMU20, DMU19, DMU32, DMU23, DMU28, DMU10, DMU12, DMU27, DMU7, DMU33, DMU13and DMU29. These rankings can help in benchmarking other inefficient branches.

Table 5.11. CRS Super-Efficiency of DMUs 2010-2014 (O-O, PPM- Model)

DMU	Technical Efficiency Score(CRS)	2014Super Efficiency Score(CRS)	Technical Efficiency Score(CRS)	2013Super Efficiency Score(CRS)	Technical Efficiency Score(CRS)	2012Super Efficiency Score(CRS)	Technical Efficiency Score(CRS)	2011Super Efficiency Score(CRS)	Technical Efficiency Score(CRS	2010Super Efficiency Score(CRS
DMU1	0.930246	0.930246	0.909605	0.909605	0.931374	0.931374	0.892344	0.892344	0.878887	0.878887
DMU2	0.945015	0.945015	0.93342	0.93342	0.997941	0.997941	0.982162	0.982162	0.974435	0.974435
DMU3	0.965043	0.965043	0.965099	0.965099	0.950019	0.950019	0.958615	0.958615	0.959912	0.959912
DMU4	0.9029	0.9029	0.916736	0.916736	0.938968	0.938968	0.988051	0.988051	0.962324	0.962324
DMU5	0.950534	0.950534	0.856296	0.856296	0.931752	0.931752	0.760054	0.760054	0.986504	0.986504
DMU6	0.951304	0.951304	1	1.085768	0.957465	0.957465	1	1.073789	0.997952	0.997952
DMU7	1	1.072082	0.993479	0.993479	1	1.185578	1	1.080064	1	1.835835
DMU8	1	1.332947	0.992927	0.992927	0.986958	0.986958	1	1.106874	0.902284	0.902284
DMU9	0.897789	0.897789	0.913078	0.913078	0.914275	0.914275	0.900003	0.900003	1	1.216853
DMU10	0.99902	0.99902	1	1.087425	1	1.018098	1	1.072793	0.979374	0.979374
DMU11	0.938974	0.938974	0.997149	0.997149	0.982101	0.982101	0.973535	0.973535	1	1.366884
DMU12	1	1.166419	1	1.016649	1	1.069261	1	1.034942	1	1.897264
DMU13	1	1.046769	1	1.209996	1	1.141654	1	1.122056	0.874374	0.874374
DMU14	0.916691	0.916691	0.880852	0.880852	0.9239	0.9239	0.908951	0.908951	0.933018	0.933018
DMU15	0.944756	0.944756	0.932221	0.932221	0.946444	0.946444	0.970717	0.970717	0.938022	0.938022
DMU16	0.941799	0.941799	0.979694	0.979694	0.960434	0.960434	0.959842	0.959842	0.8942	0.8942
DMU17	0.950098	0.950098	0.915935	0.915935	0.936257	0.936257	0.927245	0.927245	0.960531	0.960531
DMU18	0.921278	0.921278	0.89202	0.89202	1	1.049106	1	1.253471	0.971762	0.971762
DMU19	0.963665	0.963665	0.968675	0.968675	0.975476	0.975476	0.98117	0.98117	0.904202	0.904202
DMU20	1	2.036087	0.974648	0.974648	0.965074	0.965074	0.955843	0.955843	0.871228	0.871228
DMU21	0.971495	0.971495	0.962607	0.962607	0.908628	0.908628	0.921262	0.921262	1	1.503247
DMU22	0.913958	0.913958	0.847937	0.847937	1	2.322999	1	2.874382	0.934469	0.934469
DMU23	1	1.315397	1	1.843306	0.911565	0.911565	0.891073	0.891073	0.930432	0.930432
DMU24	0.944889	0.944889	0.915095	0.915095	0.911387	0.911387	0.894267	0.894267	0.906407	0.906407
DMU25	0.965188	0.965188	0.948104	0.948104	0.916269	0.916269	0.899284	0.899284	1	2.447973
DMU26	0.949302	0.949302	0.914337	0.914337	1	1.890749	1	2.095936	0.934732	0.934732
DMU27	1	1.037421	0.9474	0.9474	0.944022	0.944022	0.910519	0.910519	0.863465	0.863465
DMU28	0.966245	0.966245	0.936525	0.936525	0.934245	0.934245	0.869369	0.869369	0.900804	0.900804
DMU29	1	2.01309	0.938573	0.938573	1	1.485373	1	1.164014	0.96646	0.96646
DMU30	0.92872	0.92872	1	1.039994	1	1.140921	1	1.030598	0.964236	0.964236
DMU31	0.94526	0.94526	0.98728	0.98728	0.990772	0.990772	0.993787	0.993787	NA	NA
DMU32	0.998663	0.998663	1	1.359632	NA	NA	NA	NA	NA	NA
DMU33	0.998608	0.998608	NA	NA	NA	NA	NA	NA	NA	NA

Table 5.12. VRS Super-Efficiency of DMU 2010-2014 (O-O, PPM- Model)

DMU	2014Pure Technical Efficiency Score(VRS)	2014 Super Efficiency Score(VRS)	2013Pure Technical Efficiency Score(VRS)	2013 Super Efficiency Score(VRS)	2012 Pure Technical Efficiency Score(VRS)	2012 Super Efficiency Score(VRS)	2011 Pure Technical Efficiency Score(VRS)	2011 Super Efficiency Score(VRS)	2010Pure Technical Efficiency Score(VRS)	2010 Super Efficiency Score(VRS)
DMU1	0.949093	0.949093	0.936882	0.936882	0.974071	0.974071	0.970136	0.970136	0.987462	0.987462
DMU2	0.980016	0.980016	0.970611	0.970611	1	1.116987	1	1.021764	0.976322	0.976322
DMU3	0.974518	0.974518	0.965116	0.965116	0.957558	0.957558	0.970153	0.970153	0.991311	0.991311
DMU4	0.906721	0.906721	0.93337	0.93337	0.949942	0.949942	1	1.056103	1	1.116406
DMU5	0.950544	0.950544	0.875524	0.875524	0.948191	0.948191	1	1(Infeasible)	0.986519	0.986519
DMU6	0.951843	0.951843	1	1.099753	0.966077	0.966077	1	1.080975	1	1.355157(Inf)
DMU7	1	1.097122	1	1.032825	1	1.263066	1	1.744991	1	1(Inf)
DMU8	1	1(Inf)	1	1(Infeasible)	1	1(Infeasible)	1	1(Infeasible)	0.990579	0.990579
DMU9	0.924842	0.924842	0.930347	0.930347	0.927724	0.927724	0.94214	0.94214	1	1.245585
DMU10	1	1.249168	1	1.340466	1	1.03288	1	1.07781	1	1.163685
DMU11	0.965348	0.965348	1	1.196918	0.989367	0.989367	0.986068	0.986068	1	1.392262
DMU12	1	1.216633	1	1.076076	1	1.073829	1	1.101808	1	1(Inf)
DMU13	1	1.048585	1	1.210731	1	1.162911	1	1.517734	0.875815	0.875815
DMU14	0.917547	0.917547	0.883851	0.883851	0.930313	0.930313	0.911928	0.911928	0.943952	0.943952
DMU15	0.973827	0.973827	1	1(Infeasible)	0.947707	0.947707	0.974064	0.974064	0.950182	0.950182
DMU16	0.9424	0.9424	0.982196	0.982196	0.96062	0.96062	0.960442	0.960442	0.894508	0.894508
DMU17	0.953294	0.953294	0.920335	0.920335	0.949247	0.949247	0.928613	0.928613	1	1.204361
DMU18	0.926717	0.926717	0.905725	0.905725	1	1.769267	1	1.608837	0.981346	0.981346
DMU19	1	1.436189	1	1.134197	0.977769	0.977769	0.983972	0.983972	0.91851	0.91851
DMU20	1	2.182778	0.975476	0.975476	0.966566	0.966566	0.956838	0.956838	0.879032	0.879032
DMU21	0.973795	0.973795	0.964022	0.964022	0.917062	0.917062	0.930282	0.930282	1	1.505214
DMU22	0.916848	0.916848	0.857559	0.857559	1	1(Inf)	1	1(Inf)	1	1.143522
DMU23	1	1.336403	1	2.74677	0.920805	0.920805(Inf)	0.917336	0.917336(Inf)	0.940305	0.940305
DMU24	0.958991	0.958991	0.921516	0.921516	0.9151	0.9151	0.902041	0.902041	0.929111	0.929111
DMU25	0.968978	0.968978	0.954077	0.954077	0.953981	0.953981	0.932615	0.932615	1	1(Inf)
DMU26	0.953865	0.953865	0.945376	0.945376	1	1(Inf)	1	1(Inf)	1	2.349032
DMU27	1	1.179321	1	1(Infeasible)	1	1(Infeasible)	1	1.749443(Inf)	0.92533	0.92533(Inf)
DMU28	1	1(Inf)	1	1(Infeasible)	0.944973	0.944973(Inf)	0.898021	0.898021	0.910909	0.910909
DMU29	1	1(Inf)	1	1.005002	1	1.520974	1	1.223321	1	1.004517
DMU30	0.979714	0.979714	1	1.041306	1	1.249039	1	1.311913	0.966613	0.966613
DMU31	0.948497	0.948497	1	1.329488	0.996238	0.996238	1	1.030046	NA	NA
DMU32	1	1.361682	1	1.443594	NA	NA	NA	NA	NA	NA
DMU33	1	1.051372	NA	NA	NA	NA	NA	NA	NA	NA

Table 5.13. VRS Super efficiency score by FPA method (2014)

DMU	2014Pure Technical Efficiency Score(VRS)	2014 Super Efficiency Score(VRS)	2014 Super Efficiency Score(VRS) FPA Method
DMU1	0.949093	0.949093	0.949093
DMU2	0.980016	0.980016	0.980016
DMU3	0.974518	0.974518	0.974518
DMU4	0.906721	0.906721	0.906721
DMU5	0.950544	0.950544	0.950544
DMU6	0.951843	0.951843	0.951843
DMU7	1	1.097122	1.097122
DMU8	1	1(Inf)	3.9837
DMU9	0.924842	0.924842	0.924842
DMU10	1	1.249168	1.249168
DMU11	0.965348	0.965348	0.965348
DMU12	1	1.216633	1.216633
DMU13	1	1.048585	1.048585
DMU14	0.917547	0.917547	0.917547
DMU15	0.973827	0.973827	0.973827
DMU16	0.9424	0.9424	0.9424
DMU17	0.953294	0.953294	0.953294
DMU18	0.926717	0.926717	0.926717
DMU19	1	1.436189	1.436189
DMU20	1	2.182778	2.182778
DMU21	0.973795	0.973795	0.973795
DMU22	0.916848	0.916848	0.916848
DMU23	1	1.336403	1.336403
DMU24	0.958991	0.958991	0.958991
DMU25	0.968978	0.968978	0.968978
DMU26	0.953865	0.953865	0.953865
DMU27	1	1.179321	1.179321
DMU28	1	1(Inf)	1.307694
DMU29	1	1(Inf)	1
DMU30	0.979714	0.979714	0.979714
DMU31	0.948497	0.948497	0.948497
DMU32	1	1.361682	1.361682
DMU33	1	1.051372	1.051372

5.9.4: Cross Efficiency Model:

The model description and theoretical aspects of Cross Efficiency are covered in detail in Chapter 3(Theoretical aspects of DEA). The Cross-efficiency model is run under both CRS and VRS with output orientations as below. The Cross Efficiency is a post DEA analysis.

The following are some of the uses of Cross Efficiency scores(Doyle&Green,1994).

1)It overcomes the problem of "maverick" DMUs. It is used to identify maverick DMUs, the DMUs where the difference between a simple efficiency score of a unit and the average cross efficiency is high then it is called maverick. From the table 5.14 for the period 2014, DMU 29 has a high maverick index of 4.91

2) Assess the similarity of the appraisal by peers.

3) It helps in subclassifying 100% efficient peers. The cross-efficiency score in the matrix and the average cross efficiencies gives insight into whether or not these 100% efficient peers are consistently good performers. In other words, whether they have a high average cross efficiency score.

4) Differentiate truly efficient peers. A truly efficient peer is one which is not only 100% efficient based on simple DEA score but also one whose E_{JJ} score is greater than all other DMUs cross efficiency scores E_{IJ}

5) Identify good all-round performers. The units with a high average cross efficiency scores by peers are representative of good all-round performers.

In this research Cross efficiency scores and the maverick indicator is calculated for all the DMUs for the period 2010-2014 and shown in Table 5.14. The maverick index has been scaled up by 10 to enable easy comparison of numbers. The maverick indicator is used to identify those DMUs that are operating very differently and those DMUs that are all round performers.

From Table 5.14, for the year 2014, DMU 29, has a high maverick score of 4.91 and DMU8 has a lowest maverick index of 0.34. There are DMUs like DMU 10 and 21 that have a lesser maverick index but are efficient meaning they are all round performers. The Maverick index can also be used to rank the DMUs (Doyle and Greene,1994).

5.9.5 Bootstrap DEA of Production Process Maximization Model:

Bootstrap DEA is used to find bias in the efficiency scores. The theoretical aspects of Bootstrap DEA are given in detail in Chapter 3 while covering the theoretical aspects of DEA. Therefore, in this section bootstrapping is done on the CRS and VRS scores and the results are discussed.

In the current research Simar and Wilson (1998) approach has been used where the model uses 2000 bootstrap replicates of the CCR and VRS scores obtained from the basic DEA model for the period 2010 to2014 at 95% confidence level. The results are tabulated in table 5.15 and 5.16 below. From the bootstrapping results, it is found that there is bias in the calculation of the DEA scores for all the periods 2010-2014.

In the CRS scores for the period 2014 the bias ranges from a minimum of 0.65% to a maximum of 4.55%, in 2014,0.65%-5.59% in 2013, 0.58%-4.22% in 2012,0.84% -5.94% in 2011 and 0.95%-6.22% in 2010.In the VRS scores the bias ranges from 0.65% -4.55%, in 2014,0.95%-4.35% in 2013,0.54%-3.37% in 2012,0.83%-3.35% in 2011 and 0.65% -4.03% in

2010. The mean bias in CRS scores is from a minimum of 0.72% to a maximum of 5.30% and the mean bias of VRS scores is from a minimum of 0.73% to a maximum of 3.93%.

Table 5.14. Cross efficiency score with Maverick Index (**PPM-Model**)

DMU	2014 VRS Scores	2014 Mean Cross Efficiency	2014 Maverick Index	2013 VRS Scores	2013 Mean Cross Efficiency	2013 Maverick Index	2012 VRS Scores	2012 Mean Cross Efficiency	2012 Maverick Index	2011 VRS Scores	2011 Mean Cross Efficiency	2011 Maverick Index	2010 VRS Scores	2010 Mean Cross Efficien cy	2010 Maverick Index
DMU1	0.94909	0.753557	2.59	0.93688	0.7591	2.34	0.97407	0.758354	2.84	0.97014	0.750953	2.92	0.98746	0.76871	2.85
DMU2	0.98002	0.73875	3.27	0.97061	0.818382	1.86	1	0.818463	2.22	1	0.742901	3.46	0.97632	0.78089	2.50
DMU3	0.97452	0.850888	1.45	0.96512	0.861653	1.20	0.95756	0.809796	1.82	0.97015	0.793017	2.23	0.99131	0.87038	1.39
DMU4	0.90672	0.750885	2.08	0.93337	0.773754	2.06	0.94994	0.807186	1.77	1	0.807898	2.38	1	0.80448	2.43
DMU5	0.95054	0.741314	2.82	0.87552	0.738912	1.85	0.94819	0.79791	1.88	1	0.171345	48.36	0.98652	NA	NA
DMU6	0.95184	0.808239	1.78	1	0.952791	0.50	0.96608	0.886162	0.90	1	0.872939	1.46	1	0.86734	1.53
DMU7	1	0.888584	1.25	1	0.920933	0.86	1	0.934628	0.70	1	0.851138	1.75	1	0.85978	1.63
DMU8	1	0.966987	0.34	1	0.934728	0.70	1	0.912282	0.96	1	0.708627	4.11	0.99058	0.79399	2.48
DMU9	0.92484	0.70205	3.17	0.93035	0.796433	1.68	0.92772	0.807512	1.49	0.94214	0.791494	1.90	1	0.81254	2.31
DMU10	1	0.95942	0.42	1	0.946665	0.56	1	0.93215	0.73	1	0.878271	1.39	1	0.94661	0.56
DMU11	0.96535	0.757484	2.74	1	0.884593	1.30	0.98937	0.900801	0.98	0.98607	0.862821	1.43	1	0.92734	0.78
DMU12	1	0.844503	1.84	1	0.911737	0.97	1	0.916706	0.91	1	0.877927	1.39	1	0.93975	0.64
DMU13	1	0.780584	2.81	1	0.927157	0.79	1	0.939937	0.64	1	0.837666	1.94	0.87582	0.86368	0.14
DMU14	0.91755	0.754095	2.17	0.88385	0.781986	1.30	0.93031	0.794777	1.71	0.91193	0.747073	2.21	0.94395	0.70024	3.48
DMU15	0.97383	0.729611	3.35	1	0.791256	2.64	0.94771	NA	NA	0.97406	NA	NA	0.95018	NA	NA
DMU16	0.9424	0.765107	2.32	0.9822	0.878076	1.19	0.96062	0.851931	1.28	0.96044	0.820475	1.71	0.89451	0.83543	0.71
DMU17	0.95329	0.78746	2.11	0.92034	0.817298	1.26	0.94925	0.802015	1.84	0.92861	0.771174	2.04	1	0.78641	2.72
DMU18	0.92672	0.708807	3.07	0.90573	0.779908	1.61	1	0.811238	2.33	1	0.698903	4.31	0.98135	0.69074	4.21
DMU19	1	0.881369	1.35	1	0.890344	1.23	0.97777	0.922139	0.60	0.98397	0.891545	1.04	0.91851	0.91307	0.06
DMU20	1	0.871064	1.48	0.97548	0.852082	1.45	0.96657	0.843034	1.47	0.95684	0.768701	2.45	0.87903	0.81277	0.82
DMU21	0.9738	0.888148	0.96	0.96402	0.880897	0.94	0.91706	0.886802	0.34	0.93028	0.785869	1.84	1	0.76217	3.12
DMU22	0.91685	0.733766	2.50	0.85756	0.71737	1.95	1	0.755348	3.24	1	0.733965	3.62	1	0.6764	4.78
DMU23	1	0.802939	2.45	1	0.762992	3.11	0.92081	0.841331	0.94	0.91734	0.700403	3.10	0.94031	0.76844	2.24
DMU24	0.95899	0.800069	1.99	0.92152	0.802898	1.48	0.9151	0.792866	1.54	0.90204	0.73556	2.26	0.92911	0.85157	0.91
DMU25	0.96898	0.763774	2.69	0.95408	0.821252	1.62	0.95398	0.746312	2.78	0.93262	0.686656	3.58	1	0.75176	3.30
DMU26	0.95387	0.813377	1.73	0.94538	0.795004	1.89	1	0.811906	2.32	1	0.686644	4.56	1	0.73337	3.64
DMU27	1	0.818204	2.22	1	0.826506	2.10	1	0.847393	1.80	1	0.723639	3.82	0.92533	0.74719	2.38
DMU28	1	0.721023	3.87	1	0.799926	2.50	0.94497	0.772716	2.23	0.89802	0.674156	3.32	0.91091	0.73942	2.32
DMU29	1	0.670715	4.91	1	NA	NA	1	NA	NA	1	NA	NA	1	NA	NA
DMU30	0.97971	0.824208	1.89	1	0.837516	1.94	1	0.830606	2.04	1	0.730412	3.69	0.96661	0.73551	3.14
DMU31	0.9485	0.782919	2.11	1	0.871396	1.48	0.99624	0.89449	1.14	1	0.803582	2.44		0.77413	-10.00
DMU32	1	0.87152	1.47	1	0.932512	0.72		0.962397	-10.00		0.923292	-10.00		0.87897	-10.00
DMU33	1	0.850581	1.76		0.928561	-10.00		0.911412	-10.00		0.830399	-10.00		0.77104	-10.00

The bias is subtracted from the original score to get the corrected score and the corrected scores are given in table 5.15 and 5.16.

Table 5.15. Bootstrap VRS Efficiency Scores PPM- Model (2010-2014)

	2014			2013			2012			2011			2010		
DMU	**2014 VRS Score(Original)**	**Bias**	**2014Corrected VRS efficiency**	**2013 VRS Score(Original)**	**Bias**	**2013Corrected VRS efficiency**	**2012Score VRS(Original)**	**Bias**	**2012CorrectedVRS efficiency**	**2011Score VRS(Original)**	**Bias**	**2011Corrected VRS efficiency**	**2010Score VRS (Original)**	**Bias**	**2010CorrectedVRS efficiency**
DMU1	0.930246	0.00647	0.923781	0.936882	0.01218	0.924707	0.974071	0.00873	0.96534	0.970136	0.00804	0.9621	0.987462	0.01046	0.977005
DMU2	0.945015	0.02015	0.92487	0.970611	0.01542	0.955187	1	0.03373	0.96627	1	0.02145	0.978552	0.976322	0.01413	0.962188
DMU3	0.965043	0.01437	0.95067	0.965116	0.01002	0.955098	0.957558	0.00643	0.951126	0.970153	0.00935	0.960808	0.991311	0.00809	0.983221
DMU4	0.9029	0.00939	0.89351	0.93337	0.01157	0.921798	0.949942	0.00594	0.944005	1	0.02896	0.971044	1	0.03636	0.963637
DMU5	0.950534	0.01327	0.937262	0.875524	0.01234	0.863186	0.948191	0.00746	0.94073	1	0.0309	0.969097	0.986519	0.01327	0.973252
DMU6	0.951304	0.00941	0.941891	1	0.03816	0.961844	0.966077	0.01103	0.955047	1	0.03103	0.96897	1	0.03728	0.962723
DMU7	1	0.04512	0.954882	1	0.03023	0.969769	1	0.03289	0.967115	1	0.03161	0.968394	1	0.03952	0.960479
DMU8	1	0.04545	0.954548	1	0.04027	0.959729	1	0.03366	0.966345	1	0.03323	0.966772	0.990579	0.01007	0.980514
DMU9	0.897789	0.01044	0.887349	0.930347	0.01415	0.916195	0.927724	0.00978	0.917943	0.94214	0.00998	0.932157	1	0.03828	0.96172
DMU10	0.99902	0.02519	0.97383	1	0.04166	0.958341	1	0.02893	0.971071	1	0.03164	0.968361	1	0.03694	0.963063
DMU11	0.938974	0.01088	0.928092	1	0.0407	0.959299	0.989367	0.01589	0.973481	0.986068	0.01555	0.970522	1	0.03692	0.963078
DMU12	1	0.0438	0.956205	1	0.03747	0.962533	1	0.03391	0.96609	1	0.0335	0.966503	1	0.03874	0.961265
DMU13	1	0.04133	0.958669	1	0.04049	0.959511	1	0.03268	0.967317	1	0.03227	0.967728	0.875815	0.00786	0.867954
DMU14	0.916691	0.01037	0.906317	0.883851	0.01301	0.870843	0.930313	0.00682	0.923497	0.911928	0.00982	0.902109	0.943952	0.0171	0.926848
DMU15	0.944756	0.00598	0.938775	1	0.04088	0.959116	0.947707	0.00948	0.938228	0.974064	0.00987	0.964198	0.950182	0.01095	0.939235
DMU16	0.941799	0.0089	0.932904	0.982196	0.01056	0.971632	0.96062	0.01021	0.950411	0.960442	0.01009	0.950349	0.894508	0.00989	0.884619
DMU17	0.950098	0.0134	0.936701	0.920335	0.00952	0.910811	0.949247	0.00857	0.940675	0.928613	0.01035	0.918267	1	0.03801	0.961995
DMU18	0.921278	0.01475	0.906531	0.905725	0.01039	0.895335	1	0.0337	0.966303	1	0.03264	0.967361	0.981346	0.0118	0.969544
DMU19	0.963665	0.01874	0.944924	1	0.04108	0.958925	0.977769	0.01677	0.961001	0.983972	0.01076	0.973208	0.91851	0.00783	0.910685
DMU20	1	0.04363	0.956372	0.975476	0.01167	0.963808	0.966566	0.0112	0.955368	0.956838	0.00852	0.948322	0.879032	0.00807	0.870965
DMU21	0.971495	0.01384	0.95766	0.964022	0.01074	0.953282	0.917062	0.00563	0.911437	0.930282	0.01031	0.919976	1	0.04032	0.959678
DMU22	0.913958	0.00718	0.906776	0.857559	0.00997	0.847594	1	0.03301	0.966991	1	0.03124	0.968759	1	0.03711	0.962892
DMU23	1	0.04415	0.955847	1	0.03965	0.96035	0.920805	0.00784	0.912969	0.917336	0.00908	0.90826	0.940305	0.00768	0.932623
DMU24	0.944889	0.00697	0.937917	0.921516	0.0114	0.910118	0.9151	0.00789	0.907211	0.902041	0.00863	0.89341	0.929111	0.01664	0.912475
DMU25	0.965188	0.01559	0.949597	0.954077	0.00858	0.945502	0.953981	0.01306	0.940924	0.932615	0.01355	0.919061	1	0.03713	0.962871
DMU26	0.949302	0.0158	0.9335	0.945376	0.01352	0.931857	1	0.03321	0.96679	1	0.03205	0.967951	1	0.03702	0.962983
DMU27	1	0.03733	0.962669	1.0000000	0.04345	0.956547	1	0.03342	0.966579	1	0.03224	0.967763	0.92533	0.00652	0.918814
DMU28	0.966245	0.01828	0.94797	1.0000000	0.04181	0.958187	0.944973	0.01396	0.931016	0.898021	0.00828	0.889742	0.910909	0.00862	0.902288
DMU29	1	0.04397	0.956031	1.0000000	0.02255	0.977451	1	0.0332	0.966804	1	0.0313	0.968702	1	0.01725	0.982751
DMU30	0.92872	0.00919	0.91953	1.0000000	0.03209	0.967915	1	0.03238	0.967621	1	0.03129	0.968712	0.966613	0.01101	0.955607
DMU31	0.94526	0.01179	0.933468	1.0000000	0.03997	0.960026	0.996238	0.01834	0.977898	1	0.02482	0.975179			0
DMU32	0.998663	0.02239	0.976274	1.0000000	0.04155	0.958448			0			0			0
DMU33	0.998608	0.02608	0.972531			0			0			0			0

Table 5.16. Bootstrap CRS Efficiency Scores PPM- Model (2010-2014)

	2014			2013			2012			2011			2010		
DMU	**2014 Score(Original)**	**Bias**	**2014Corrected CRS efficiency**	**2013 Score(Original)**	**Bias**	**2013Corrected CRS efficiency**	**2012Score(Original)**	**Bias**	**2012Corrected CRS efficiency**	**2011 Score(Original)**	**Bias**	**2011Corrected CRS efficiency**	**2010 Score(Original)**	**Bias**	**2010Corrected CRS efficiency**
DMU1	0.909605	0.006465	0.90314	0.909605	0.00699	0.90262	0.93137	0.00759	0.923788	0.892344	0.00949	0.882852	0.878887	0.0137	0.865187
DMU2	0.93342	0.020145	0.913275	1	0.05303	0.94697	0.99794	0.01941	0.978528	0.982162	0.01454	0.967621	0.974435	0.01821	0.956227
DMU3	0.965099	0.014373	0.950726	0.997149	0.0268	0.970347	0.95002	0.00783	0.94219	0.958615	0.01464	0.943971	0.959912	0.0183	0.941608
DMU4	0.916736	0.00939	0.907346	1	0.03877	0.96123	0.93897	0.00813	0.930841	0.988051	0.02345	0.964598	0.962324	0.0331	0.929229
DMU5	0.856296	0.013272	0.843024	1	0.0559	0.944097	0.93175	0.00679	0.924959	0.760054	0.01904	0.741016	0.986504	0.02468	0.961823
DMU6	1	0.009413	0.990587	0.880852	0.01724	0.863614	0.95747	0.01379	0.943671	1	0.04785	0.952148	0.997952	0.03363	0.964318
DMU7	0.993479	0.045118	0.948361	0.932221	0.00579	0.926431	1	0.04222	0.957778	1	0.04859	0.951409	1	0.06225	0.937753
DMU8	0.992927	0.045452	0.947475	0.979694	0.01265	0.967046	0.98696	0.01516	0.971795	1	0.05179	0.948211	0.902284	0.01948	0.882804
DMU9	0.913078	0.01044	0.902638	0.915935	0.01293	0.903001	0.91428	0.0091	0.905171	0.900003	0.01442	0.885583	1	0.06141	0.938595
DMU10	1	0.02519	0.97481	0.89202	0.01208	0.879937	1	0.0315	0.968496	1	0.04671	0.953286	0.979374	0.02607	0.953308
DMU11	0.997149	0.010882	0.986267	0.968675	0.01611	0.952561	0.9821	0.01259	0.969507	0.973535	0.01246	0.961077	1	0.06001	0.939987
DMU12	1	0.043795	0.956205	0.93342	0.0141	0.919316	1	0.04228	0.957722	1	0.03972	0.960282	1	0.06136	0.938638
DMU13	1	0.041331	0.958669	0.974648	0.0114	0.963249	1	0.04224	0.957763	1	0.05233	0.947667	0.874374	0.0095	0.864871
DMU14	0.880852	0.010374	0.870478	0.962607	0.01164	0.950971	0.9239	0.00801	0.915891	0.908951	0.0113	0.897652	0.933018	0.02688	0.906143
DMU15	0.932221	0.005981	0.92624	0.847937	0.01447	0.833469	0.94644	0.00967	0.936778	0.970717	0.01598	0.954733	0.938022	0.02246	0.915562
DMU16	0.979694	0.008895	0.970799	1	0.05583	0.944172	0.96043	0.00824	0.952191	0.959842	0.01277	0.947069	0.8942	0.01045	0.883755
DMU17	0.915935	0.013397	0.902538	0.915095	0.00664	0.908458	0.93626	0.00785	0.928408	0.927245	0.01408	0.913165	0.960531	0.01875	0.941785
DMU18	0.89202	0.014747	0.877273	0.948104	0.00821	0.939892	1	0.04167	0.958335	1	0.05942	0.940576	0.971762	0.02116	0.950599
DMU19	0.968675	0.018741	0.949934	0.914337	0.00646	0.90788	0.97548	0.01873	0.956745	0.98117	0.01532	0.965855	0.904202	0.01229	0.891915
DMU20	0.974648	0.043628	0.93102	0.9474	0.0133	0.934099	0.96507	0.01147	0.953608	0.955843	0.01047	0.945378	0.871228	0.01383	0.8574
DMU21	0.962607	0.013835	0.948772	0.936525	0.01832	0.918205	0.90863	0.00696	0.901665	0.921262	0.01449	0.906774	1	0.06222	0.937785
DMU22	0.847937	0.007182	0.840755	0.965099	0.0119	0.9532	1	0.04259	0.957408	1	0.05683	0.943172	0.934469	0.02062	0.913846
DMU23	1	0.044153	0.955847	0.938573	0.01633	0.922239	0.91157	0.00672	0.904847	0.891073	0.00894	0.882131	0.930432	0.01271	0.917719
DMU24	0.915095	0.006972	0.908123	1	0.04588	0.954124	0.91139	0.00707	0.904318	0.894267	0.01203	0.882242	0.906407	0.01297	0.893442
DMU25	0.948104	0.015591	0.932513	0.98728	0.02352	0.963756	0.91627	0.01257	0.903695	0.899284	0.01306	0.886224	1	0.06014	0.939861
DMU26	0.914337	0.015802	0.898535	1	0.05651	0.943487	1	0.04321	0.956792	1	0.05607	0.943928	0.934732	0.01442	0.920313
DMU27	0.9474	0.037331	0.910069	0.916736	0.01763	0.899103	0.94402	0.00583	0.938193	0.910519	0.00839	0.902126	0.863465	0.01118	0.85229
DMU28	0.936525	0.018275	0.91825	0.856296	0.01578	0.840512	0.93425	0.01171	0.92254	0.869369	0.00869	0.86068	0.900804	0.01435	0.886459
DMU29	0.938573	0.043969	0.894604	1	0.05082	0.94918	1	0.0427	0.957301	1	0.053	0.946998	0.96646	0.01516	0.951297
DMU30	1	0.00919	0.99081	0.993479	0.01441	0.979071	1	0.04212	0.957877	1	0.0373	0.9627	0.964236	0.01623	0.948003
DMU31	0.98728	0.011792	0.975488	0.992927	0.02784	0.965088	0.99077	0.02006	0.97071	0.993787	0.02122	0.972565	0	0	0
DMU32	1	0.022389	0.977611	0.913078	0.01035	0.90273			0			0	0	0	0
DMU33	0.998608	0.026077	0.972531	0	0	0			0			0	0	0	0

The corrected average efficiency scores are given for each year and it can be found that in all cases the new scores corrected with bias are considerably different in magnitude than the original scores. The bootstrapping method helps to find the bias in scores due to errors and adjust in the overestimated scores.

5.9.6 Envelopment Form and Proportionate Movement of PPM-Model:

The solution of the envelopment model corresponds to the dual solution of the multiplier model and vice-versa. The input-output weights of branches (ɵ and λ) in envelopment model is equivalent to the input-output weight coefficients (*u* and *v*) of the multiplier model(Ramanathan,2003). The envelopment model is chosen to find out efficiency score, reference benchmarking, and slack variable and target values.

The results of the output-oriented VRS envelopment model using the data for the year 2014 is shown in Table 5.18. The results show the efficiency score, proportional improvement value, slack improvement value and target value (projection), number of efficient DMUs, lambda values and benchmarking. The target value is equal to the original value plus improved value. If the projection to the strong efficient frontier is selected for the target value, the improved value includes two parts: one is proportionate movement and the other is slack movement(Ramanathan,2003).

$$\begin{aligned} \boldsymbol{Strong\, efficient\, T\, \mathbf{arg}\, et Value = Original Value + \mathbf{Pr}\, oportionate\, \mathbf{Im}\, rovement Value} \\ \boldsymbol{+ Slack\, \mathbf{Im}\, provement Value} \end{aligned}$$

If projection to weak frontier is selected for the target value then improved value includes only the proportional improvement value(Ramanathan,2003).

$$\boldsymbol{Weak - Efficient T\, \mathbf{arg}\, et Value = Original Value + \mathbf{Pr}\, oportionate\, \mathbf{Im}\, provement Value}$$

Strongly efficient DMUs are those that have no slacks and weakly efficient DMUs are those that have a slack (Ramanathan,2003). Proportionate movement, projection values, and slack movements are shown in Table 5.18. The output-oriented VRS envelopment model is as below (Sherman and Zhu,2006).

$$\max \phi + \varepsilon (\sum_{i=1}^{m} s_i^- + s_r^+)$$

subject to

$$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{i0} \quad i = 1, 2, 3...m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = \phi y_{r0} \quad r = 1, 2, 3...s$$

$$\lambda_j \geq 0 \quad j = 1, 2, 3........n$$

$$s_i^-, s_r^+ \geq 0 \quad \left(j = 1....m, r = 1......s \right)$$

$$x_{i0} = \textit{amount of input i used by } DMU_0$$

$$y_{r0} = \textit{amount of output rproduced by } DMU_0$$

$$m = \textit{number of outputs}$$

$$s = \textit{number of inputs}$$

$$n = \textit{number of DMUs}$$

$$\varepsilon = \textit{a small positive number}$$

$$s_i^- = \textit{Input slack}$$

$$s_r^+ = \textit{Output Slack}$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

The solution to the above problem is interpreted as the largest expansion of DMU_0's output by **ϕ**, that can be carried out given the DMU_0 will stay in the reference technology fixing the input at the current level of $\mathbf{X_{io}}$. The first two constraints form the convex reference technology. The third constraint restricts the lambda value or intensity variables, that are dot connectors that will be used to construct the best practice frontier to be greater than 0 and non-negative. The fourth constraint restricts the input and output slack to be non-negative. This model with the sum of lambdas equal to one imposes variable returns to scale on the reference technology and without this constraint it will be a CRS technology. If θ=1 or ϕ=1 then the DMU under evaluation is a frontier point. The left-hand side of the constraint 2 and 3 is called the "Reference Set" and the right-hand side represents a specific DMU under evaluation. The non-zero optimal λ_j^* represents the benchmarks for a specific DMU under evaluation. The reference set provides coefficients λ_j^* to define the hypothetical efficient DMU. The reference set or efficient target shows how inputs can be decreased and outputs can be increased to make the DMU under evaluation efficient(Zhu,2004). The efficiency targets for inputs and outputs can be can be set using the following equation respectively.

$$\hat{x}_{i0} = x_{i0} - s_i^- \quad i = 1, 2...m$$
$$\hat{y}_{r0} = \dot{\phi} y_{r0} + s_r^+ \quad r = 1, 2....s$$

These efficiency targets show how inputs can be decreased and outputs can be increased to make DMU under evaluation efficient.

The results of the solution of the above LP using data for the period 2014 is shown in the following tables. The solution provides non- zero input and output slacks based on input and output constraints. It must be noted that slacks exist only for inefficient DMUs. These slacks provide vital information on the areas where inefficient DMUs have to improve to become efficient. As per Coelli et al., (2005), both technical efficiency and slacks should be reported to provide accurate information about a firm in DEA analysis. Therefore, slacks have to be interpreted with efficiency values. Slacks are needed to push the frontier to the target(Ozcan,2008).

Table 5.18 provides the input and output slacks derived from output oriented VRS model for thirty-three heavy equipment retailing organization under study. For interpreting the contents, let us consider DMU1.The VRS efficiency of the branch is 0.94909. This means that the branch can become technically efficient (under Farrell's definition) if all its inputs are proportionately reduced by 5.1%. However, even with the proportional reduction in inputs, the branch would not be Pareto efficient as it would be operating on the vertical section of the efficient frontier. In order to project the branch to a Pareto-efficient point, some further slack adjustments are necessary for this branch as non-zero input and output slacks appear for this branch. It is found that DMU1 has slack on the inputs, an excess staff of 7.4563 and excess area of the facility of 63,082. On the output side, there is no slack on revenue, but it can be increased by 1,318,131 and the slack on gross margin is 1,071,925. Therefore, DMU1 has to make three adjustments to operate on the efficient frontier. First it has to reduce all inputs by 5.1%, second, it has to reduce staff by 15.86% and reduce the area of the facility by 67.37%. This will result in improved margin by 1,071,925 and the sales will increase by1,318,131. The first type of adjustment is known as radial adjustment while the second, third and fourth adjustments are known as slack adjustments. A similar interpretation can be extended to other inefficient branches.

After analysis, the inefficient DMU is presented with a relevant set of efficient DMUS, called its *Reference Set (*Paradi et al.,2017). This is also referred to as an efficient reference set. The reference set represents the set of efficient DMUs against which the inefficient DMU is judged to be inefficient and the changes to improve the inefficient DMUs can be determined by the efficiency difference between inefficient DMU and its reference set (Paradi et al.,2017). The envelopment form of DEA thus gives actionable advice to the organization on improving efficiency that is perceived to be fair and equitable. One of the most powerful and useful features of DEA is the ability to identify the amount of excess resource consumed and a potential increase in outputs possible in inefficient units as compared to the branches in the efficient reference set.

This perspective offered by DEA is unique and not provided by any other method (Paradi et al.,2017).

Table 5.17. Lambda Values of PPM - Model

DMU	Score	Benchmark(Lambda)
DMU1	0.94909	DMU12(0.203846); DMU23(0.040633); DMU32(0.755521)
DMU2	0.98002	DMU12(0.213333); DMU28(0.558171); DMU8(0.228496)
DMU3	0.97452	DMU10(0.576902); DMU12(0.411703); DMU32(0.011395)
DMU4	0.90672	DMU10(0.724950); DMU12(0.045784); DMU23(0.225093); DMU32(0.004173)
DMU5	0.95054	DMU12(0.215121); DMU13(0.031686); DMU23(0.230710); DMU29(0.446146); DMU8(0.076338)
DMU6	0.95184	DMU10(0.349734); DMU12(0.255778); DMU23(0.048147); DMU8(0.346341)
DMU7	1	DMU7(1.000000)
DMU8	1	DMU8(1.000000)
DMU9	0.92484	DMU12(0.237257); DMU23(0.424054); DMU32(0.338689)
DMU10	1	DMU10(1.000000)
DMU11	0.96535	DMU12(0.070785); DMU23(0.522368); DMU32(0.406846)
DMU12	1	DMU12(1.000000)
DMU13	1	DMU13(1.000000)
DMU14	0.91755	DMU10(0.351284); DMU12(0.073933); DMU23(0.288886); DMU8(0.285898)
DMU15	0.97383	DMU12(0.031287); DMU28(0.249119); DMU29(0.530699); DMU8(0.188895)
DMU16	0.9424	DMU10(0.454680); DMU12(0.250784); DMU23(0.271532); DMU8(0.023004)
DMU17	0.95329	DMU10(0.218831); DMU12(0.713839); DMU20(0.059182); DMU32(0.008149)
DMU18	0.92672	DMU23(0.208118); DMU29(0.468265); DMU8(0.323617)
DMU19	1	DMU19(1.000000)
DMU20	1	DMU20(1.000000)
DMU21	0.9738	DMU10(0.854310); DMU12(0.053394); DMU23(0.001826); DMU32(0.090470)
DMU22	0.91685	DMU10(0.333893); DMU12(0.304666); DMU23(0.333897); DMU8(0.027544)
DMU23	1	DMU23(1.000000)
DMU24	0.95899	DMU10(0.366135); DMU12(0.228366); DMU32(0.405499)
DMU25	0.96898	DMU12(0.598474); DMU13(0.172058); DMU29(0.217481); DMU8(0.011987)
DMU26	0.95387	DMU23(0.023825); DMU29(0.303606); DMU8(0.672570)
DMU27	1	DMU27(1.000000)
DMU28	1	DMU28(1.000000)
DMU29	1	DMU29(1.000000)
DMU30	0.97971	DMU10(0.317650); DMU19(0.616479); DMU32(0.065871)
DMU31	0.9485	DMU10(0.089921); DMU12(0.752542); DMU23(0.046857); DMU32(0.110680)
DMU32	1	DMU32(1.000000)
DMU33	1	DMU33(1.000000)

Table 5.18. Slacks and proportionate movement for improving outputs.

DMU	Score	Proportionate_ Movement(Number of Staff)	Slack_ Movement(Number of Staff)	Projection(Number of Staff)	Proportionate_ Movement(Area of Facility)	Slack_ Movement(Area of Facility)	Projection(Area of Facility)	Proportionate_ Movement(Total Expenses for the branch)	Slack_ Movement(Total Expenses for the branch)	Projection(Total Expenses for the branch)	Proportionate_ Movement(Total COGS for Branch)	Slack_ Movement(Total COGS for Branch)	Projection(Total COGS for Branch)	Proportionate_ Movement(Total Branch Sales)	Slack_ Movement(Total Branch Sales)	Projection(Total Branch Sales)	Proportionate_ Movement(Total Gross Margin for branch)	Slack_ Movement(Total Gross Margin for branch)	Projection(Total Gross Margin for branch)
DMU1	0.94909	-0	-7.4563	39.5437	-0	-63082	30553.5	-0	-0	3581263	-0	-0	2E+07	1318131	0	2.6E+07	246205	1071925	5908271
DMU2	0.98002	-0	-2.7655	9.23451	-0	-0	5550	-0	-174362	566258	-0	-0	3241707	84370.1	0	4221967	18268.1	66104.3	980260
DMU3	0.97452	-0	-2.812	16.188	-0	-4089.8	12125.2	-0	-0	1137495	-0	-0	1E+07	332988	0	1.3E+07	62793	270196	2734395
DMU4	0.90672	-0	-0	15	-0	-5979.6	16492.4	-0	-0	1570163	-0	-0	1.2E+07	1359624	0	1.5E+07	175975	1183651	3070199
DMU5	0.95054	-0	-0	8	-0	-0	10599	-0	-0	979798	-0	-0	3043597	207353	0	4192689	48997.7	158356	1149092
DMU6	0.95184	-0	-0	12	-0	-0	10000	-0	-201472	913532	-0	-0	8125257	494676	0	1E+07	83588.6	411088	2146831
DMU7	1	-0	-0	9	-0	-0	5508	-0	-0	441282	-0	-0	7634628	0	0	8955682	0	0	1321055
DMU8	1	-0	-0	5	-0	-0	4750	-0	-0	293861	-0	-0	4992985	0	0	6163575	0	0	1170590
DMU9	0.92484	-0	-13.111	25.8885	-0	-15942	25100.4	-0	-0	2983670	-0	-0	1.2E+07	1198647	0	1.6E+07	223157	975490	3944659
DMU10	1	-0	-0	15	-0	-0	14225	-0	-0	1137595	-0	-0	1.4E+07	0	0	1.7E+07	0	0	3290818
DMU11	0.96535	-0	-13.362	27.6383	-0	-11788	28655.8	-0	-0	3398353	-0	-0	1.3E+07	614720	0	1.8E+07	135123	479595	4379065
DMU12	1	-0	-0	17	-0	-0	8500	-0	-0	1049979	-0	-0	5279997	0	0	7111393	0	0	1831395
DMU13	1	-0	-0	9	-0	-0	12600	-0	-0	922459	-0	-0	2580992	0	0	3655240	0	0	1074248
DMU14	0.91755	-0	-0	12	-0	-87.695	14218.3	-0	-0	1433298	-0	-0	8233781	873367	0	1.1E+07	133424	740303	2358483
DMU15	0.97383	-0	-0	4	-0	-0	5000	-0	-86137	218091	-0	-0	1723741	57621.2	0	2201592	11296.8	46224.7	477851
DMU16	0.9424	-0	-0	15	-0	-0	15509	-0	-402258	1606972	-0	-0	9169546	682011	0	1.2E+07	121568	560343	2670889
DMU17	0.95329	-0	-2.6605	17.3395	-0	-0	10000	-0	-0	1152758	-0	-0	8447045	502791	0	1.1E+07	88930.8	413860	2317900
DMU18	0.92672	-0	-0	5	-0	-2909.4	9090.61	-0	-70921	729455	-0	-0	2941539	285593	0	3897128	52981.9	232611	955589
DMU19	1	-0	-0	57	-0	-0	37139	-0	-0	3498392	-0	-0	4.3E+07	0	0	5.1E+07	0	0	7823998
DMU20	1	-0	-0	26	-0	-0	8780	-0	-0	2015915	-0	-0	2.5E+07	0	0	2.9E+07	0	0	3917732
DMU21	0.9738	-0	-0	18	-0	-518.56	15981.4	-0	-0	1421956	-0	-0	1.4E+07	465375	0	1.8E+07	83391.6	381985	3564242
DMU22	0.91685	-0	-0	15	-0	-5667.8	15832.2	-0	-0	1715732	-0	-0	8176909	890762	0	1.1E+07	149168	741594	2535507
DMU23	1	-0	-0	14	-0	-0	25044	-0	-0	3018625	-0	-0	5604827	0	0	8140095	0	0	2535269
DMU24	0.95899	-0	-5.5673	28.4327	-0	-13770	22071.7	-0	-0	2397703	-0	-0	1.6E+07	851755	0	2.1E+07	157655	694101	4538510
DMU25	0.96898	-0	-0	12	-0	-10101	8399.31	-0	-0	793470	-0	-0	3737840	156835	0	5055567	37166.4	119668	1317726
DMU26	0.95387	-0	-0	4	-0	-198.6	5309.4	-0	-80054	273532	-0	-0	3594922	206431	0	4474482	32556.9	173875	879560
DMU27	1	-0	-0	3	-0	-0	5550	-0	-0	363754	-0	-0	3269833	0	0	3896832	0	0	626999
DMU28	1	-0	-0	8	-0	-0	4750	-0	-0	492889	-0	-0	1745759	0	0	2322800	0	0	577042
DMU29	1	-0	-0	1	-0	-0	5000	-0	-0	13084	-0	-0	340106	0	0	445028	0	0	104922
DMU30	0.97971	-0	-0	43	-0	-15610	29838	-0	-574785	2800925	-0	-0	3.3E+07	791023	0	3.9E+07	114942	676083	6342250
DMU31	0.9485	-0	-0	20	-0	-1383.8	12922.2	-0	-0	1509205	-0	-0	8200521	555678	0	1.1E+07	110391	445287	2588663
DMU32	1	-0	-0	47	-0	-0	36800	-0	-0	4294486	-0	-0	2.5E+07	0	0	3.2E+07	0	0	7189654
DMU33	1	-0	-0	7	-0	-0	8780	-0	-0	877416	-0	-0	7327602	0	0	8773517	0	0	1445914

Table 5.19. Reference Set for inefficient branches for the year 2014

DMU	VRS Score	Reference Set									
		DMU8	**DMU10**	**DMU12**	**DMU13**	**DMU19**	**DMU20**	**DMU23**	**DMU28**	**DMU29**	**DMU32**
DMU1	0.949093			0.20385				0.04063	0.58817		0.75552
DMU2	0.980016	0.228496		0.21333							
DMU3	0.974518		0.5769	0.4117							0.11395
DMU4	0.906721		0.72495	0.04578				0.22509			0.00417
DMU5	0.950544	0.076338		0.21512	0.03169			0.23071		0.44615	
DMU6	0.951843	0.346341	0.34973	0.25578				0.04815			
DMU9	0.924842			0.23726				0.42405			0.33869
DMU11	0.965348			0.07079				0.52237			0.40685
DMU14	0.917547	0.285898	0.35128	0.07393				0.28889			
DMU15	0.973827	0.188895		0.03129					0.24912	0.5307	
DMU16	0.9424	0.23004	0.45468	0.25078				0.27153			
DMU17	0.953294		0.21883	0.71384			0.05918				0.00815
DMU18	0.926717	0.323617						0.20812		0.46827	
DMU21	0.973795		0.85431	533394				0.00183			0.09047
DMU22	0.916848	0.027544	0.33389	0.30467				0.3339			
DMU24	0.958991	0.027544	0.36614	0.22837				0.3339			
DMU25	0.968978	0.011987		0.59847	0.17206					0.21748	
DMU26	0.953865	0.67257						0.23825		0.30361	
DMU30	0.979714		0.31765			616479					0.06587
DMU31	0.948497		0.08992	0.75254				0.04686			0.11068
Frequency of Count		**11**	**11**	**17**	**2**	**1**	**1**	**14**	**2**	**5**	**9**

The above Table 5.19 lists all the reference sets for the inefficient branches along with its frequency of occurrence. Chen (1997) and Chen and Yeh (1998) used the frequency of reference set to discriminate the branches. The frequency with which an efficient branch shows up in the reference sets of the inefficient branch represents the extent of robustness of that branch relative to other efficient branches. The higher the frequency the higher the robustness of the branch. In other words, a branch that appears with high frequency in the reference set of inefficient branches is likely to be a branch that is efficient with respect to a large number of factors and is probably a good example of an all-round performer or global leader (Kumar and Gulati,2008). The efficient branches that do not appear in the reference set of inefficient banks may not have a common input/output mix and therefore are not suitable examples for inefficient branches to emulate. On the basis of frequency DMU, 12 can be termed highly robust branch and DMU11 and DMU 8 can be termed as a Marginally robust branch (Kumar and Gulati,2008).

5.9.7: Effect of Contextual variables on efficiency scores of PPM- model.

5.9.7A Introduction:

In the current research dependent variables are DEA bootstrapped CRS and VRS scores and the five contextual variables are the total population of the city where the branch is located, capital expenditure on machinery and equipment by the federal government, competition index, number of competition stores and squared number of competition stores. The descriptive statistics of the contextual variables are shown in table 5.20 below.

Table 5.20. Descriptive statistics of contextual variables

Descriptitive Statistics	2014 CRS Score	2014 VRSScore	Cap Expenditure in million $	Population	Competition Index	Number of Competition Stores	Sq of Number of Competition Stores
Mean	0.93135	0.93993	712.09091	366896.69697	1737.69697	5.06061	30.15152
Standard Error	0.00705	0.00389	211.73423	76602.81870	459.29869	0.37924	4.74823
Median	0.93251	0.94189	192.00000	145850.00000	1308.00000	4.00000	16.00000
Mode	#N/A	#N/A	192.00000	383822.00000	1083.00000	4.00000	16.00000
Standard Deviation	0.04048	0.02234	1216.32054	440049.69090	2638.47010	2.17858	27.27650
Sample Variance	0.00164	0.00050	1479435.64773	193643730459.90500	6961524.46780	4.74621	744.00758
Kurtosis	-0.36286	-0.05921	9.82696	1.10830	31.15441	0.36717	1.92237
Skewness	-0.47933	-0.60977	3.13645	1.40664	5.50934	1.05647	1.63372
Range	0.15006	0.08893	5076.00000	1641610.00000	15869.00000	8.00000	98.00000
Minimum	0.84076	0.88735	51.00000	7909.00000	375.00000	2.00000	2.00000
Maximum	0.99081	0.97627	5127.00000	1649519.00000	16244.00000	10.00000	100.00000
Sum	30.73471	31.01785	23499.00000	12107591.00000	57344.00000	167.00000	995.00000
Count	33.00000	33.00000	33.00000	33.00000	33.00000	33.00000	33.00000
Largest(1)	0.99081	0.97627	5127.00000	1649519.00000	16244.00000	10.00000	100.00000
Smallest(1)	0.84076	0.88735	51.00000	7909.00000	375.00000	2.00000	2.00000
Confidence Level(9	0.01435	0.00792	431.28851	156034.83560	935.56082	0.77249	9.67183

Table 5.21. Results of OLS regression

OLS Regression	BOOTSTRAPPED VRS Scores(Dependent Variable)				BOOTSTRAPPED CRS Scores(Dependent Variable)			
Independent Variable	Coefficients	Standard Error	t Stat	P-value	Coefficients	Standard Error	t Stat	P-value
Intercept	0.972543985	0.03121734	31.1539625	1.06037E-22	0.873473852	0.059302	14.7291	1.9988E-14
Cap Expenditure in million $	1.118E-05	5.1328E-06	2.17817086	0.038299236	3.67216E-06	9.75E-06	0.37661	0.70940551
Population	7.5848E-10	1.157E-08	0.06555835	0.948212465	7.49351E-09	2.2E-08	0.34095	0.73577915
Competition Index	-1.9628E-06	2.2814E-06	-0.8603247	0.397185377	2.80869E-06	4.33E-06	0.64806	0.52241823
Number of Competition Stores	-0.01082445	0.01140742	-0.9488953	0.351090477	0.021025332	0.02167	0.97024	0.34053928
Sq of Number of Competition Stores	0.000575111	0.00089382	0.64342867	0.525373783	-0.001948981	0.001698	-1.1478	0.26110514
Multiple R	0.421246376				0.310166206			
R Square	0.177448509				0.096203075			
Adjusted R Square	0.025124159				-0.071166726			
Standard Error	0.02205463				0.041896365			
Observations	33				33			

The five factors population of the city, capital expenditure by federal government on machinery and equipment, number of competition stores, square of the number of competition stores and competition index are the variables that may have an effect on the efficiency scores. The population data of each city where the branch is located and the capital expenditure on machinery and equipment were collected from the Statistics Canada database for the year 2014. The number of competition stores in each city where the branch is located was compiled from the database of the various equipment manufacturers' organization. The competition index HHI is the Herfindahl-Hirschman Index better known as Herfindahl index is a statistical measure of concentration used in economics to measure competitive effects (Nauenberg et al., 1997). HHI accounts for the number of firms in a market as well as concentration and is calculated by squaring the market share of all firms in a market and then summing the squares as follows.

$$\textbf{HHI} = \sum_{i=1}^{n}(\boldsymbol{MS}_{\text{i}})^{2} \quad \ldots\ldots\ldots\ldots\ldots \text{(4.2.2)}$$

$\mathbf{MS_i}$ represents the market share of firm *i* and there are *n* firms in the market. The data of market share was obtained from trade and industry journals.

5.9.7B OLS Regression:

The above five contextual variables were regressed (MSExcel-2016) against the dependent variable bootstrapped CCR scores and VRS scores for the year 2014. The results of the two models are tabulated in the table 5.21 above. From table 5.21, it is found that in the model using VRS scores the coefficients of competition index and number of competition stores have a negative sign. This means that when the competition index and the number of competition stores are high, it brings down the efficiency of the operating branch indicating that there is competition for the business. The coefficients of the other three contextual variables population, capital expenditure and square of the competition stores have a positive sign indicating that they all have a positive influence on the efficiency. Further, when the federal expenditure on machinery and equipment is high the branches have more opportunities to sell equipment and parts. The coefficient of a square number of competitive stores has a positive sign and it is opposite to the negative sign of the number of competitive stores. In other words, the effect of number of competitive stores is an inverted U shape (Ko et al.,2017). This implies that efficiency decreases when the intensity of competition increases up to a point but then efficiency increases after it reaches a threshold level.

In the model that uses CRS scores all the coefficients of the contextual variables have positive sign except the square of the number of competition stores which has a negative sign. This is opposite to the effect on number of competition stores that has a positive sign. This means that when a branch is operating at constant returns to scale efficiency increases with the intensity of competition up to a certain level and then drops after a threshold point is reached. However, the high p- values at 5% level indicates that the coefficient values of the five contextual variables are not significant statistically.

5.9.7C. Tobit Regression:

In this investigation the five contextual variables as mentioned above were regressed with CCR bootstrapped score and VRS score using Tobit regression by using E-Views software and the results of the two models are shown in table 5.22, below.

In both models using bootstrapped CRS and VRS scores, the coefficients of Capital expenditure, population and square number of competition stores have negative coefficients meaning that the efficiency drops when there is increase in capital expenditure, population and the square of the number of competition stores. The negative sign of square of competition stores in conjunction with the positive sign for the number of competition stores can be interpreted as, that efficiency increases with competition up to a point and then drops. The drop-in efficiency score with increase in population and capital expenditure is not explained by the model. The other factors competition index and competition stores have a positive coefficient under both models using CRS and VRS scores. As per Tobit regression, the efficiency drivers are competition index and number of competition stores with p-value being very significant under VRS.

Table 5.22. Results of Tobit regression

Variable	Dependent Variable					
	Bootstrapped CRS Score			Bootstrapped VRS Score		
Independent Variable	Coefficient	z-Statistic	P-Value	Coefficient	z-Statistic	P-Value
CAP_EXPENDITURE_IN_MILLI	-3.88E-05	-1.530467	0.0205	-3.01E-05	-1.028383	0.3038
COMPETITION_INDEX	2.55E-05	2.316559	0	2.04E-05	1.600021	0.1096
NUMBER_OF_COMPETITION_ST	0.334435	29.97683	0.2499	0.329695	25.54595	0
POPULATION	-6.69E-08	-1.150487	0.2499	-9.57E-08	-1.422832	0.1548
SQ_OF_NUMBER_OF_COMPETIT	-0.025421	-15.9513	0	-0.023847	-12.93532	0
Log likelihood	24.86203			20.0549		

5.9.7D. Conclusion

The second stage analysis of DEA was done using both bootstrapped CCR scores and VRS scores of DEA for the year 2014 on the contextual variables using both OLS regression and Tobit regression. Both these regression methods identify the important determinants of the efficiency of heavy duty equipment retailing organizations. It is found from OLS regression competition index and number of competition stores have a negative coefficient for VRS scores implying that increase in these values will decrease the efficiency. Similarly, in CRS scores it was found that the square of number of competition stores has a negative coefficient indicating that they have a negative effect on efficiency whereas all the other factors had a positive coefficient. The mean of CRS and VRS efficiency scores is 0.93135 and 0.93993 respectively. The *p*- value of the contextual variable indicate that the results are not statistically significant.

Similarly, in analysis with Tobit regression coefficients of population, capital expenditure and square of competition stores all have a negative sign with both CRS and VRS scores and the coefficients of other two factors competition index and number of competition stores have a positive sign. However, the *p*-value is zero for competition index and square of competition stores with CRS scores. Similarly, the *p*-value is zero for number of competition stores and square of competition stores with VRS scores. This indicates that the results associated with these factors are statistically significant. However, the other factors population and capital expenditure have a high *p-value* indicating the results are not statistically significant.

These findings have important policy and managerial implications. The competition in the industry is one factor that has major effect on efficiency as it dictates the survival instinct of the organization. This will help improve customer service and in turn help in retention of customers.

5.10 Efficiency Change Over Time (PPM- Model)

5.10.1 Introduction:

The analysis presented till section 5.9 dealt with various DEA models that had data for a single period to calculate efficiency score. In other words, the sections till 5.9 dealt with the uses of DEA under static conditions. The dynamic environment in heavy equipment industry and dealerships may show varying performances over time and this may depend on many external factors such as government regulations, effect of related industries, business cycle to name a few. These dealerships may make gains or losses depending on how they respond to various external business environment and internal influences within the organizations. Therefore, measurement of efficiency under static conditions may be misleading. This section will use two non-parametric DEA models Window analysis and Malmquist index under dynamic (time dependent) situations.

The theoretical aspects of Window analysis and Malmquist productivity index was covered in detail in Chapter 3(Theoretical aspects of DEA). Therefore, in this section analysis of results will be presented for both Window analysis and Malmquist productivity index.

5.10.2 Window Analysis (PPM- Model):

In this research window analysis is carried out on a five-year data of an equipment retailing organization in Canada that has thirty-three retailing branches (DMUs) for the period 2010 to 2014. The formula for calculating the number of data points in Window Analysis as per Cooper et al.,2007 is as below.

$$p = \text{length of window} \, (p \le k)$$
$$w = \text{number of windows}$$
$$k = \text{number of periods}$$
$$\text{is calculated by the formula}$$
$$w = k - p + 1 \quad ....$$
$$\text{In the current research}$$
$$n = 33, k = 5, p = 3, \text{and } w = 3.$$
$$\text{Number of different DMUs} = npw$$
$$= 33x3x3 = 297$$
$$297 \text{ different DMUs}$$
$$\Delta = \text{number of DMUs} = n(p-1)(k-p)$$
$$= 33(5-1)(5-3) = 264$$

In other words, **Δ (delta)** represents additional 264 DMUs that are now available to calculate the change in efficiency scores as compared to the original 33 DMUs. With three inputs and two outputs there are now 264x5= 1320 data entries available to which DEA model can be applied to study the variation in technical efficiency, pure technical efficiency and the scale efficiency. The results of the analysis are tabulated in Tables 5.23,5.24 and 5.25. Table 5.23 shows results of window analysis of CCR (technical efficiency) scores, Table 5.24 depicts results of window analysis of BCC (pure technical efficiency) scores and Table 5.25 indicates results of scale efficiency. In Table 5.25 alongside the scale efficiency scores **C**, **D** and **I**, indicate constant, decreasing and increasing returns to scale. The column views in the result enables us to analyze the stability of results across different data sets and row views helps in determining the trends in efficiency scores within the same data set (Cooper, Seiford and Tone,2007).

A Window analysis of the efficiency scores obtained by the Production Process model (Model 1) that has four inputs number of employees, area of facility, total expenses for the branch, total COGS for the branch and two outputs total sales for the branch and total gross margin for the branch is done with the data for the years 2010-2014.

The three Tables 5.23,5.24 and 5.25 report the results of DEA window analysis with each heavy equipment retailing organization represented as if it were a different DMU in each of the three successive periods as indicated in the first row. The three windows chosen are 2010-2012,2011-2013 and 2012 -2014 and each with the length of window being a three-year period and is similar to the original work of (Charnes, Clark, Cooper and Golany,1985).

Taking DMU1 as an example in Table 5.23, the technical efficiency of heavy equipment retailing organization is analysed using window analysis. The efficiency of DMU1in the first window 2010-2012 is 1.0, 0.74232 and 0.56339. Similarly, in the second window 2011-2013, the efficiency score is 0.89663, 0.87404 and 0.81640 and in the third window 2012-2014 it is 0.87884,0.83185 and0.78192 respectively. The associated mean is also given in the adjacent columns. In the first windows 2010-2012, the efficiency score is dropping from a high of 1 in 2010 to a low of 0.56339 showing a declining trend in efficiency scores. In the second window the trend of efficiency scores is moving from a high of 0.89 to a low of 0.81. In the third window the efficiency score drops from a high of 0.87 in 2012 to a low of 0.78 in 2014, showing a declining trend in efficiency scores. This analysis is across the row of each period. On analyzing the efficiency scores along the column for the three windows it is found that the scores are fluctuating from 58 to 60 in the first two windows somewhat stable but increases to 81 in the third window thus indicating the efficiency scores are not stable.

Similarly considering the same DMU1 as an example to study variations in pure technical efficiency VRS scores can be analysed from Table 5.24. The VRS score in the first window2010-

2012 is found as 1.0,0.82693 and 0.709627 and in the second window 2011-2013 it is found as 0.92991,0.87621 and 0.83963 and in the third window 2012-2014 it is found as 0.878531,0.83963 and 0.78195 respectively. As indicated above the mean is also indicated in the adjacent column. From the above scores it is found, that the VRS scores are declining from a high of 1 to 0.709627 in the first window and similarly in the other two windows VRS score is showing a declining trend. When the scores are analysed along the column the scores are 1,0.91894,0.8480,0.83963 and 0.78195 indicating a drop of efficiency scores from one year to the next.

The Table 5.25 indicates scale efficiency. In other words, it indicates a relationship between efficiency and scale of production. Again, considering the same sample DMU1, it is found from Table 5.25 that the scale efficiency scores in the first window 2010-2012 is 1.0,0.897682 and 0.79392 and all showing decreasing returns to scale. In the second window 2011-2013, the efficiency scores are 0.964209,0.997436 and 0.972696 and all showing a decreasing return to scale. The scale efficiency in the second window is showing a stability. In the third window it is 0.999215,0.990734 and 0.999971again showing the efficiency scores are stable but shows increasing returns to scale in 2014. The trend of scale efficiency in the third window is stable whereas in the first two windows it is not. The scale efficiency table details the properties of each heavy equipment retailing organization at different time and in different windows. Of all the DMUs (297), 27.27% of the DMUs exhibit decreasing returns to scale,20.5% DMUs exhibit constant returns to scale and 45.45% indicate increasing returns to scale. This indicates that production scale is a source of inefficiency in heavy equipment retailing organization.

Table 5.23. Window Analysis of CRS Efficiency Scores

DMU	**Window**	**2010**	**2011**	**2012**	**2013**	**2014**	**Mean**
DMU1	2010-2012	1	0.74232	0.56339			0.7094
	2011-2013		0.89663	0.87404	0.8167		
	2012-2014			0.87784	0.83185	0.78192	
DMU2	2010-2012	1	0.83039	0.7238			0.8590
	2011-2013		0.93698	0.86973	0.83879		
	2012-2014			0.87403	0.8405	0.81759	
DMU3	2010-2012	1	0.72861	0.58919			0.7930
	2011-2013		0.84429	0.78314	0.78694		
	2012-2014			0.78521	0.81893	0.80101	
DMU4	2010-2012	1	0.96147	0.597			0.8296
	2011-2013		0.90278	0.82768	0.78786		
	2012-2014			0.83116	0.78617	0.77245	
DMU5	2010-2012	NA	NA	0.62449			0.7874
	2011-2013		NA	0.83939	0.78552		
	2012-2014			0.85279	0.79992	0.8224	
DMU6	2010-2012	0.8328	1	0.85581			0.9192
	2011-2013		1	0.89214	1		
	2012-2014			0.89373	1	0.79856	
DMU7	2010-2012	0.91261	1	1			0.9660
	2011-2013			1	1	0.91013	
	2012-2014			1	0.91013	0.96122	
DMU8	2010-2012	0.80396	0.886	0.92144			0.9660
	2011-2013			1	0.88915	0.98251	
	2012-2014			0.89153	0.99055	1	
DMU9	2010-2012	0.56823	0.68157	0.62932			0.7483
	2011-2013		0.82157	0.79757	0.83708		
	2012-2014			0.80403	0.84593	0.75	
DMU10	2010-2012	1	1	0.95742			0.9831
	2011-2013		0.97353	0.97137	1		
	2012-2014			0.98697	1	0.95939	
DMU11	2010-2012	0.85414	0.8741	0.84205			0.8615
	2011-2013		0.88317	0.8628	0.86744		
	2012-2014			0.91762	0.93842	0.7143	
DMU12	2010-2012	0.99239	1	0.91919			0.9801
	2011-2013		1	0.97179	0.989		
	2012-2014			0.97526	0.97408	1	

Table 5.23. Window Analysis of CRS Efficiency Scores

DMU13	2010-2012	1	1	1			0.9876
	2011-2013		1	0.96724	1		
	2012-2014			0.98925	1	0.93195	
DMU14	2010-2012	0.67859	0.7213	0.79127			0.7615
	2011-2013		0.80605	0.78605	0.76202		
	2012-2014			0.79127	0.76912	0.7478	
DMU15	2010-2012	NA	NA	NA			0.7712
	2011-2013		NA	NA	0.79859		
	2012-2014			NA	0.80662	0.70864	
DMU16	2010-2012	0.82081	0.91691	0.76974			0.8487
	2011-2013		0.93872	0.85694	0.8488		
	2012-2014			0.85855	0.8488	0.7794	
DMU17	2010-2012	0.77092	0.76501	0.63524			0.8580
	2011-2013		0.89426	0.84774	0.8178		
	2012-2014			0.85022	0.82216	0.82005	
DMU18	2010-2012	0.60761	0.86288	0.71138			0.8258
	2011-2013		0.8968	0.88168	0.9008		
	2012-2014			0.89486	0.9008	0.77584	
DMU19	2010-2012	0.80497	1	1			0.9448
	2011-2013		1	1	0.97939		
	2012-2014			0.97969	0.90146	0.83837	
DMU20	2010-2012	0.82052	0.82446	0.91053			0.8858
	2011-2013		0.91273	0.87062	0.88233		
	2012-2014			0.86927	0.88233	1	
DMU21	2010-2012	0.75425	0.86417	0.8815			0.8874
	2011-2013		0.9426	0.93798	0.89987		
	2012-2014			0.93949	0.90351	0.86402	
DMU22	2010-2012	0.74563	0.79747	0.6568			0.8588
	2011-2013		1	0.93722	0.87276		
	2012-2014			0.95032	0.88246	0.88684	
DMU23	2010-2012	0.7769	1	1			0.9752
	2011-2013		1	1	1		
	2012-2014			1	1	1	
DMU24	2010-2012	0.81379	0.61282	0.68227			0.8088
	2011-2013		0.8699	0.88641	0.84301		
	2012-2014			0.89566	0.84951	0.82652	
DMU25	2010-2012	0.64799	0.71528	0.56305			0.8001
	2011-2013		0.90641	0.83586	0.89685		
	2012-2014			0.85326	0.90481	0.87807	

Table 5.23. Window Analysis of CRS Efficiency Scores

DMU26	2010-2012	0.69666	0.72786	0.74872			0.8170
	2011-2013		0.8932	0.8649	0.85484		
	2012-2014			0.86932	0.85692	0.84133	
DMU27	2010-2012	1	1	1			0.9583
	2011-2013		1	1	0.87081		
	2012-2014			1	0.87735	0.87729	
DMU28	2010-2012	0.6663	0.68643	0.60293			0.8057
	2011-2013		0.89201	0.85977	0.89389		
	2012-2014			0.86281	0.89585	0.89209	
DMU29	2010-2012	NA	NA	NA			1.0000
	2011-2013		NA	NA	NA		
	2012-2014			NA	NA	1	
DMU30	2010-2012	0.60667	0.74319	0.84251			0.8441
	2011-2013		0.86826	0.90722	0.92737		
	2012-2014			0.90722	0.92737	0.86717	
DMU31	2010-2012	0.72405	0.93232	1			0.9258
	2011-2013		0.94097	1	0.95353		
	2012-2014			1	0.95353	0.8285	
DMU32	2010-2012	1	1	1			0.99274
	2011-2013		1	1	1		
	2012-2014			1	1	0.93468	
DMU33	2010-2012	0.71599	0.99224	1			0.9607
	2011-2013		1	0.99685	1		
	2012-2014			0.99946	1	0.94206	

Table 5.24. Window Analysis of VRS Scores

DMU	Window	2010	2011	2012	2013	2014	Mean
DMU1	2010-2012	1	0.82693	0.709627			0.9647
	2011-2013		0.92991	0.876285	0.83963		
	2012-2014		1	0.878531	0.83963	0.78195	
DMU2	2010-2012	1	0.83062	0.723959			0.9926
	2011-2013		1	0.957345	0.92121		
	2012-2014			0.968618	0.92506	0.89299	
DMU3	2010-2012	1	0.73436	0.60992			0.7970
	2011-2013		0.84735	0.783184	0.78823		
	2012-2014			0.789105	0.81986	0.80146	
DMU4	2010-2012	1	1	0.605427			0.8412
	2011-2013		0.94718	0.827716	0.78967		
	2012-2014			0.835712	0.78853	0.77684	
DMU5	2010-2012	NA	NA	0.639492			0.8077
	2011-2013		NA	0.872158	0.82075		
	2012-2014			0.867732	0.81236	0.83426	
DMU6	2010-2012	0.84169	1	0.859016			0.9269
	2011-2013		1	0.910957	1		
	2012-2014			0.918527	1	0.81196	
DMU7	2010-2012	0.96045	1	1			0.9894
	2011-2013		1	1	0.97163		
	2012-2014			1	0.97278	1	
DMU8	2010-2012	1	1	1			1.0000
	2011-2013		1	1	1		
	2012-2014			1	1	1	
DMU9	2010-2012	0.70549	0.74263	0.726582			0.8653
	2011-2013		0.82809	0.813148	0.84157		
	2012-2014			0.813148	0.84748	0.75241	
DMU10	2010-2012	1	1	0.97202			0.9913
	2011-2013		0.97972	0.980003	1		
	2012-2014			0.990804	1	1	
DMU11	2010-2012	0.92505	0.93152	0.901123			0.9281
	2011-2013		0.93152	0.915618	1		
	2012-2014			0.934903	1	0.81372	
DMU12	2010-2012	1	1	0.923258			0.9846
	2011-2013		1	0.973637	0.99514		
	2012-2014			0.981285	0.98828	1	

Table 5.24. Window Analysis of VRS Scores

DMU13	2010-2012	1	1	1			0.9929
	2011-2013		1	0.974038	1		
	2012-2014			0.997023	1	0.96583	
DMU14	2010-2012	0.69145	0.72265	0.607773			0.7475
	2011-2013		0.81785	0.799428	0.76909		
	2012-2014			0.797322	0.77379	0.74903	
DMU15	2010-2012	NA	NA	NA			0.9334
	2011-2013		1	NA	NA		
	2012-2014			NA	0.94671	0.85377	
DMU16	2010-2012	0.82134	0.92949	0.773997			0.8561
	2011-2013		0.94365	0.867062	0.87279		
	2012-2014			0.861495	0.85318	0.78199	
DMU17	2010-2012	0.77142	0.78395	0.652174			0.8162
	2011-2013		0.90687	0.867005	0.83307		
	2012-2014			0.86405	0.82888	0.83883	
DMU18	2010-2012	0.67627	0.90246	0.728998			0.8563
	2011-2013		0.93393	0.924877	0.92382		
	2012-2014			0.910601	0.91236	0.79367	
DMU19	2010-2012	0.86041	1	1			0.9602
	2011-2013		1	1	0.95403		
	2012-2014			1	0.95403	0.87347	
DMU20	2010-2012	0.84369	0.8442	0.912954			0.8951
	2011-2013		0.92228	0.876248	0.88938		
	2012-2014			0.875172	0.89213	1	
DMU21	2010-2012	0.75489	0.86516	0.8826			0.8916
	2011-2013		0.96684	0.940869	0.90058		
	2012-2014			0.940613	0.90378	0.86923	
DMU22	2010-2012	0.76697	0.80165	0.666301			0.8676
	2011-2013		1	0.952399	0.88701		
	2012-2014			0.958082	0.88862	0.88745	
DMU23	2010-2012	0.79936	1	1			0.9777
	2011-2013		1	1	1		
	2012-2014			1	1	1	
DMU24	2010-2012	0.86632	0.67622	0.75136			0.8386
	2011-2013		0.88116	0.906418	0.86657		
	2012-2014			0.906418	0.86657	0.82656	
DMU25	2010-2012	0.658	0.72578	0.573706			0.8141
	2011-2013		0.93006	0.864746	0.90859		
	2012-2014			0.86897	0.91115	0.8863	
DMU26	2010-2012	0.92261	0.95468	0.941827			0.9466
	2011-2013		0.96855	0.943851	0.94992		
	2012-2014			0.962069	0.93609	0.93976	

Table 5.24. Window Analysis of VRS Scores

DMU27	2010-2012	1	1	1			0.9873
	2011-2013		1	1	0.96375		
	2012-2014			1	0.94807	0.97406	
DMU28	2010-2012	1	1	0.995034			0.9985
	2011-2013		1	0.991653	1		
	2012-2014			1	1	1	
DMU29	2010-2012	NA	NA	NA			1.0000
	2011-2013		NA	NA	NA		
	2012-2014			NA	NA	1	
DMU30	2010-2012	0.64737	0.7889	0.903083			0.9002
	2011-2013		0.95517	0.9769	0.9769		
	2012-2014			0.955166	0.97767	0.92137	
DMU31	2010-2012	0.72818	0.9345	1			0.9306
	2011-2013		0.95369	1	0.96135		
	2012-2014			1	0.96135	0.83649	
DMU32	2010-2012	0.74564	1	1			0.9675
	2011-2013		1	1	1		
	2012-2014			1	1	0.96271	
DMU33	2010-2012	0.75481	1	1			0.9666
	2011-2013		1	1	1		
	2012-2014			1	1	0.94514	

Table 5.25. Window Analysis of Scale Efficiency

DMU	Window	2010	2011	2012	2013	2014
DMU1	2010-2012	**1 (C)**	**0.897682**	**0.79392**		
	2011-2013		**0.964209**	**0.997436**	**0.972696**	
	2012-2014			**0.999215**	**0.990734**	**0.999971**
DMU2	2010-2012	**1(C)**	**0.999728(D)**	**0.999782(D)**		
	2011-2013		**0.936983(I)**	**0.908476(I)**	**0.910528(I)**	
	2012-2014			**0.90235(I)**	**0.908588(I)**	**0.915563(I)**
DMU3	2010-2012	**1(C)**	**0.992177(D)**	**0.966005(D)**		
	2011-2013		**0.996385(D)**	**0.999948(D)**	**0.998353(D)**	
	2012-2014			**0.99507(I)**	**0.998867(I)**	**0.999443(I)**
DMU4	2010-2012	**1(C)**	**0.96147(D)**	**0.986083 (D)**		
	2011-2013		**0.953119(D)**	**0.99996(I)**	**0.997705(D)**	
	2012-2014			**0.994552(I)**	**0.997015(I)**	**0.994356**
DMU5	2010-2012	**NA**	**NA**	**0.976542(I)**		
	2011-2013		**NA**	**0.962424(I)**	**0.957074(I)**	
	2012-2014			**0.982777(I)**	**0.984687(I)**	**0.985784(I)**
DMU6	2010-2012	**0.989448(I)**	**1(C)**	**0.996273(I)**		
	2011-2013		**1(C)**	**0.979346(I)**	**1(C)**	
	2012-2014			**0.973003(I)**	**1(C)**	**0.983497(I)**
DMU7	2010-2012	**0.950185(I)**	**1(C)**	**1(C)**		
	2011-2013		**1(C)**	**1(C)**	**0.936703(I)**	
	2012-2014			**1(C)**	**0.935594(I)**	**0.961217(I)**
DMU8	2010-2012	**0.803961(I)**	**0.886002**	**0.921444(I)**		
	2011-2013		**1(C)**	**0.889151(I)**	**0.982506(I)**	
	2012-2014			**0.891525(I)**	**0.990552(I)**	**1(C)**
DMU9	2010-2012	**0.805447(D)**	**0.917778(D)**	**0.866133(D)**		
	2011-2013		**0.992119(D)**	**0.980843(D)**	**0.994673(D)**	
	2012-2014			**0.988788(D)**	**0.998172(I)**	**0.996804(I)**
DMU10	2010-2012	**1(C)**	**1(C)**	**0.984983(D)**		
	2011-2013		**0.993688(D)**	**0.991191(D)**	**1(C)**	
	2012-2014			**0.996126(I)**	**1(C)**	**0.959387(D)**
DMU11	2010-2012	**0.923355(D)**	**0.938356(D)**	**0.934445(D)**		
	2011-2013		**0.948092(D)**	**0.942313(D)**	**0.867442(D)**	
	2012-2014			**0.98151(D)**	**0.93842(D)**	**0.877819(D)**

Table 5.25. Window Analysis of Scale Efficiency

DMU12	2010-2012	**0.992389(D)**	**1(C)**	**0.995598(D)**		
	2011-2013		**1(C)**	**0.998106(I)**	**0.993834(I)**	
	2012-2014			**0.993864(I)**	**0.985636(I)**	**1(C)**
DMU13	2010-2012	**1(C)**	**1(C)**	**1(C)**		
	2011-2013		**1(C)**	**0.993023(D)**	**1(C)**	
	2012-2014			**0.992201(D)**	**1(C)**	**0.964925(I)**
DMU14	2010-2012	**0.981398(I)**	**0.998136(I)**	**0.999168(D)**		
	2011-2013		**0.98558(I)**	**0.98327(I)**	**0.99082(I)**	
	2012-2014			**0.992405(I)**	**0.993974(I)**	**0.998356(I)**
DMU15	2010-2012	**NA**	**NA**	**NA**		
	2011-2013		**NA**	**NA**	**0.798587(I)**	
	2012-2014			**NA**	**0.852023(I)**	**0.830018(I)**
DMU16	2010-2012	**0.999353(I)**	**0.986464(I)**	**0.9945(I)**		
	2011-2013		**0.994775(D)**	**0.988331(I)**	**0.972512(I)**	
	2012-2014			**0.996578(I)**	**0.994869(I)**	**0.996684(I)**
DMU17	2010-2012	**0.999348(D)**	**0.97584(D)**	**0.974033(D)**		
	2011-2013		**0.986096(I)**	**0.977779(I)**	**0.981673(I)**	
	2012-2014			**0.983992(I)**	**0.991891(I)**	**0.977613(I)**
DMU18	2010-2012	**0.898463(I)**	**0.956148(I)**	**0.97583(I)**		
	2011-2013		**0.960245(I)**	**0.953294(I)**	**0.975082(I)**	
	2012-2014			**0.982709(I)**	**0.987331(I)**	**0.977539(I)**
DMU19	2010-2012	**0.935568(D)**	**1(C)**	**1(C)**		
	2011-2013		**1(C)**	**0.979394(D)**	**0.951764(D)**	
	2012-2014			**0.97969(D)**	**0.9449(D)**	**0.959818(D)**
DMU20	2010-2012	**0.972535(D)**	**0.976623(D)**	**0.997349(I)**		
	2011-2013		**0.989647(I)**	**0.993573(I)**	**0.992071(I)**	
	2012-2014			**0.993259(I)**	**0.989007(I)**	**1(C)**
DMU21	2010-2012	**0.999147(D)**	**0.998848(D)**	**0.998756(D)**		
	2011-2013		**0.974925(I)**	**0.996928(I)**	**0.999207(D)**	
	2012-2014			**0.998801(I)**	**0.999696(I)**	**0.994011(I)**
DMU22	2010-2012	**0.972182(I)**	**0.994786(I)**	**0.985744(I)**		
	2011-2013		**1(C)**	**0.984059(I)**	**0.983939(I)**	
	2012-2014			**0.991897(I)**	**0.993071(I)**	**0.999313(I)**
DMU23	2010-2012	**0.971903**	**1(C)**	**1(C)**		
	2011-2013		**1(C)**	**1(C)**	**1(C)**	
	2012-2014			**1(C)**	**1(C)**	**1(C)**

Table 5.25. Window Analysis of Scale Efficiency

DMU24	2010-2012	**0.939358(D)**	**0.906243(D)**	**0.908051(D)**		
	2011-2013		**0.98722(D)**	**0.977928(D)**	**0.972804(D)**	
	2012-2014			**0.988126(D)**	**0.980311(D)**	**0.999951(I)**
DMU25	2010-2012	**0.984799(I)**	**0.985527(I)**	**0.981434(I)**		
	2011-2013		**0.97457(I)**	**0.966589(I)**	**0.987076(I)**	
	2012-2014			**0.981921(I)**	**0.993046(I)**	**0.990716(I)**
DMU26	2010-2012	**0.755098(I)**	**0.762413(I)**	**0.794961(I)**		
	2011-2013		**0.922206(I)**	**0.916348(I)**	**0.899906(I)**	
	2012-2014			**0.903589(I)**	**0.915424(I)**	**0.89526(I)**
DMU27	2010-2012	**1(C)**	**1(C)**	**1(C)**		
	2011-2013		**1(C)**	**1(C)**	**0.903561(I)**	
	2012-2014			**1(C)**	**0.925401(I)**	**0.900654(I)**
DMU28	2010-2012	**0.666302(I)**	**0.686429(I)**	**0.605942(I)**		
	2011-2013		**0.892006(I)**	**0.867002(I)**	**0.89389(I)**	
	2012-2014			**0.862814(I)**	**0.895854(I)**	**0.892094(I)**
DMU29	2010-2012	**NA**	**NA**	**NA**		
	2011-2013		**NA**	**NA**	**NA**	
	2012-2014			**NA**	**NA**	**1(C)**
DMU30	2010-2012	**0.937137(D)**	**0.94206(D)**	**0.932922(D)**		
	2011-2013		**0.961352(D)**	**0.949806(D)**	**0.949302(D)**	
	2012-2014			**0.949806(D)**	**0.948551(D)**	**0.941173(D)**
DMU31	2010-2012	**0.99434(I)**	**0.997668(I)**	**1(C)**		
	2011-2013		**1(C)**	**1(C)**	**0.991867(D)**	
	2012-2014			**1(C)**	**0.991867(D)**	**0.990454(I)**
DMU32	2010-2012	**0.975306(D)**	**1(C)**	**1(C)**		
	2011-2013		**1(C)**	**1(C)**	**1(C)**	
	2012-2014			**1(C)**	**1(C)**	**0.970892(D)**
DMU33	2010-2012	**0.948572(I)**	**0.992239(I)**	**1(C)**		
	2011-2013		**1(C)**	**0.996852(I)**	**1(C)**	
	2012-2014			**0.999461(I)**	**1(C)**	**0.996735(I)**

(Note: **C** indicates constant returns to scale, **D** indicates decreasing returns to scale, **I** indicate, increasing returns to scale**)**

Similar analysis can be extended to other DMUs and a meaningful analysis of trends and stability of efficiency scores can be studied. This variation in the scores reflects simultaneously the absolute performance of the heavy equipment retailing organization over time and the relative performance of the heavy equipment retailing organization in comparison to the branches in the same sample.

5.10.3 Malmquist Productivity Index(PPM-Model):

In the current research Malmquist index (catch-up), CRS efficiency change, and Technology change are calculated for the year 2010 to 2014. The data of Malmquist index (catch up) is given in Table 5.26, CRS efficiency change is given in Table 5.27 and technological change is given in Table 5.28.

From Table 5.26, it is evident that DMU1's (branch 1) MI are 1.240212,0.677551, 1.007033 and 1.028553 in the year 2011, 2012,2013 and 2014 respectively. The MI that represents the overall efficiency measure can be decomposed in to two mutually exclusive components. One measuring the change in technical efficiency (catch up effect) and the other measuring the change in technology (innovation).

From Table 5.27, the change in efficiency scores (catch up) for the years 2011-2014 are 0.970605, 0.917905, 0.96676 and 1.017468. For the year 2011 it is less than 1 indicating that the efficiency has decreased as compared to 2010, in 2012 it is less than 1 indicating that the efficiency has decreased as compared to 2011, in 2013 it is less than 1 and hence efficiency has decreased from 2012 and in 2014 it is greater than 1 indicating that efficiency has increased from 2013.

From Table 5.28 the change in frontier shift for the years 2011 to 2014 are 1.27772, 0.73815, 1.041657 and 1.010895 respectively. This can be verified by computing the MI as a product of catch up and frontier shift for the DMU 1 for 2011.

For DMU1, M_1= 0.970605(Catch up) X 1.27772(Frontier shift) =1.240(the figure can be verified from the MI score in table 5.26). Similarly, the MI, catch up and Frontier shift can be analyzed for other DMUs (branches of heavy equipment retailing organization) and efficiency variation analyzed and improved.

Table 5.26. Malmquist Productivity Index 2010-2014

DMU	2011 MI(t-1, t)	2012 MI(t-1, t)	2013MI(t-1, t)	2014MI(t-1, t)
DMU1	1.240212	0.677551	1.007033	1.028553
DMU2	0.95076	0.830447	1.255898	0.879939
DMU3	0.780042	0.833632	1.177688	1.135889
DMU4	1.379364	0.47634	0.904868	1.422126
DMU5	NA		0.931229	1.112406
DMU6	1.185214	0.899472	1.496753	0.674335
DMU7	1.633989	0.904307	1.224962	1.645486
DMU8	1	1	1	1
DMU9	1.092178	0.945382	0.866408	0.84112
DMU10	1.298582	0.817759	1.738207	0.753104
DMU11	1.32101	0.848512	0.921845	0.807003
DMU12	1.057376	0.885231	1.020273	1.028198
DMU13	1.289424	1.104039	0.750979	0.766645
DMU14	1.043274	0.935582	0.977805	1.125504
DMU15	NA	NA	NA	1
DMU16	1.18817	0.727092	0.963502	1.026022
DMU17	0.81872	0.856864	1.237135	1.265006
DMU18	1.313015	0.816298	1.109575	0.79902
DMU19	2.182052	1.036905	0.939502	0.836396
DMU20	0.934693	1.634896	0.841931	2.934278
DMU21	1.197643	1.125199	1.016961	1.166382
DMU22	1.118023	0.771271	0.909259	1.288833
DMU23	1	1	0.852051	1.255689
DMU24	0.767797	1.121833	1.015563	1.017997
DMU25	1.09275	0.775123	1.377713	0.903342
DMU26	1.156423	1.15901	1.348159	0.55342
DMU27	1.447074	0.69105	1	1
DMU28	1.836865	0.541702	1.004991	2.113209
DMU29	NA	NA	NA	NA
DMU30	1.300286	1.434259	1.247904	0.884862
DMU31	2.184815	1.410851	0.847879	0.354723
DMU32	1.650397	1.031194	1.251764	0.694583
DMU33	1.807508	1.072465	1.393922	0.653387

Table 5.27.Technical Efficiency change (Catch up) 2010-2014

DMU	2011 EC(t-1, t)	2012 EC(t-1, t)	2013EC(t-1, t)	2014EC(t-1, t)
DMU1	0.970605	0.917905	0.96676	1.017468
DMU2	1	1	0.937751	1.036466
DMU3	0.888431	0.945771	1.023957	1.069692
DMU4	1	0.850202	0.930693	1.077964
DMU5			0.924677	1.045423
DMU6	1.040398	0.929541	1.0758	0.896437
DMU7	1	1	1	1
DMU8	1	1	1	1
DMU9	1.030153	0.943812	1.084509	0.93413
DMU10	1	1	1	1
DMU11	0.932498	1.023974	1.047281	0.859623
DMU12	1	1	1	1
DMU13	1	1	1	1
DMU14	1.1163	0.993031	0.978057	1.010602
DMU15				0.8898
DMU16	0.961712	0.907233	1.051998	0.92743
DMU17	0.922753	0.999096	0.940947	1.09902
DMU18	1.373777	0.978799	0.979792	0.912443
DMU19	1	1	1	1
DMU20	1.023991	0.982825	0.981818	1.1086
DMU21	1.237111	0.97951	0.974512	1.076727
DMU22	1.270394	0.979009	0.92721	1.045786
DMU23	1	1	1	1
DMU24	0.910961	1.010805	0.941974	1.062283
DMU25	1.360997	0.97247	1.02403	1.043617
DMU26	1	0.983164	0.973244	1.032634
DMU27	1	1	1	1
DMU28	1	1	1	1
DMU29				
DMU30	1.255404	1.029293	1.036289	1
DMU31	1.082343	1	1	0.916652
DMU32	1.096404	1	1	1
DMU33	1.198446	1	1	1

Table 5.28.Technological Change (Frontier Shift) 2010-2014

DMU	2011 TC(t-1, t)	2012TC(t-1, t)	2013TC(t-1, t)	2014TC(t-1, t)
DMU1	1.277772	0.73815	1.041657	1.010895
DMU2	0.95076	0.830447	1.339265	0.84898
DMU3	0.878	0.881431	1.150134	1.061884
DMU4	1.379364	0.560266	0.972251	1.319271
DMU5	NA	NA	1.007085	1.064073
DMU6	1.139194	0.967652	1.391293	0.752239
DMU7	1.633989	0.904307	1.224962	1.645486
DMU8	1	1	1	1
DMU9	1.060209	1.001664	0.798894	0.900432
DMU10	1.298582	0.817759	1.738207	0.753104
DMU11	1.416635	0.828646	0.880227	0.938787
DMU12	1.057376	0.885231	1.020273	1.028198
DMU13	1.289424	1.104039	0.750979	0.766645
DMU14	0.934582	0.942147	0.999742	1.113696
DMU15	NA	NA	NA	1.123848
DMU16	1.235474	0.801438	0.915878	1.106306
DMU17	0.887258	0.857639	1.314777	1.151031
DMU18	0.95577	0.83398	1.13246	0.875693
DMU19	2.182052	1.036905	0.939502	0.836396
DMU20	0.912794	1.663466	0.857523	2.646831
DMU21	0.968097	1.148736	1.043559	1.083266
DMU22	0.88006	0.787809	0.98064	1.232406
DMU23	1	1	0.852051	1.255689
DMU24	0.842843	1.109841	1.078122	0.958311
DMU25	0.802904	0.797066	1.345383	0.865588
DMU26	1.156423	1.178857	1.385222	0.53593
DMU27	1.447074	0.69105	1	1
DMU28	1.836865	0.541702	1.004991	2.113209
DMU29	NA	NA	NA	NA
DMU30	1.035751	1.393441	1.204205	0.884862
DMU31	2.018598	1.410851	0.847879	0.386977
DMU32	1.505283	1.031194	1.251764	0.694583
DMU33	1.50821	1.072465	1.393922	0.653387

In summary the Malmquist DEA calculates efficiency for the following output oriented VRS models (please ref fig in Ch3.Ozcan,2014).

1) Calculating the frontier in time period **t+1** and comparing efficiency scores of heavy equipment retailing organization in time period **t+1**.
2) Calculating the frontier in period **t** and comparing efficiency scores of heavy equipment retailing organizations at period **t**.
3) Comparing the efficiency scores of periods **t+1** to the frontier at period **t.**
4) Comparing the efficiency scores of time period t, to the frontier at time period **t+1**.

Therefore, efficiency component of the index measures changes in technical efficiency from time period t to t+1.In other words it measures how the heavy equipment retailing organizations that are examined (thirty-three of them) have managed to catch up to the frontier. The technical component of the index measures changes in the production frontier (a shift in best practice technology) from period t to t+1.If the values of the Malmquist Index and its components are greater than 1 equal to 1 or less than 1, they indicate progress, no progress or regress respectively (Caves et al.,1982, Fare et al.,1994).

5.10.4 Conclusion:

The objective of this section was to estimate the technical efficiency and efficiency change in the heavy equipment retailing organization during the period 2010 to 2014. For the estimation, Data Envelopment Analysis was applied and used both Window analysis and Malmquist Index on the data of the heavy equipment retailing organizations in Canada. The output oriented CCR model was used to calculate the efficiency. It was found from the Window analysis by interpreting the scores across the window, there is no pattern emerging out of scores and keeps fluctuating and therefore does not show any specific trend. In the case of analysis of scores along the column there is some stability between two periods and again there is a fluctuation among other periods and therefore indicates a fluctuating efficiency scores. The average MI scores for all heavy equipment retailing organization ranges from 0.80 to 1.21 of which 12 units show a MI score greater than one and 21 units show a score less than 1 indicating that there is scope for improving efficiency. Similarly, 11 units had a catch-up score of greater than 1 indicating that there is increase in efficiency and 21 units had a score less than 1 indicating that there is decrease in efficiency. Again, this indicates there is scope for improving efficiency among twenty-one retailing organizations. Similarly, fifteen units had a frontier shift score of greater than 1 indicating that efficiency increased between 2010 to 2014 and seventeen heavy equipment retailing organizations had a frontier shift score of less than one thereby providing an opportunity to improve efficiency of these units.

5.11: Network Data Envelopment Analysis (PPM- Model)

5.11.1 Introduction:

So far, the models reviewed in DEA to measure efficiency have considered the DMU as a complete system but ignoring the structure within the system. In other words, the DMU is treated as a black box (Fare & Grosskopf, 2000). In this section we will explore the efficiency of the internal structure of DMU using Network DEA(NDEA). Each DMU in a heavy equipment retailing organization has sales, service and parts divisions in their operations. The efficiency of these three divisions will be analyzed using NDEA.

There are various types of structures in Network systems and every study on network DEA is associated with a structure that is very specific to the business organization. A network model for measuring efficiency is developed based on the needs of the business organization for practical applications. The theoretical aspects of NDEA and various network structures have been reviewed in-depth in Chapter 3 on theoretical aspects of DEA.

In the current research efficiency of DMU has been studied using the following two Network DEA models.

1)Network DEA: Using Parallel Structure. 2)Network DEA: Two stage structure

5.11.2 Network DEA Parallel Model

The model is applied to a heavy equipment retailing organization in Canada using data for the period 2014 that has thirty-three business units (retailing organizations). The data for the year 2014 was chosen as the data was available for all thirty- three DMUs. Each of these retailing unit is called a branch. Each branch sells heavy equipment, services them in their workshop using a large inventory of parts stocked in the branch. In other words, there are three subordinated divisions in each branch and they are sales service and parts divisions and each of them operate in parallel as independent business units as profit centers within a branch. Therefore, each branch can be termed as a DMU and each division sales, service and parts can be termed as Sub-DMU operating in parallel. Kao's NDEA model with parallel structure can be applied to such a business system. As per Kao's model one of the assumption is that each input /output of the DMU is the sum its subordinated sub-DMUs.

Therefore, for the study the Production Process model in the black box approach is used and network DEA model is applied to the production process model. The inputs and outputs used are the same that were used for the production process model. The four inputs used are number of employees, area of facility, total expenses for the branch and total COGS for the branch and the two outputs used are total sales for the branch and total gross margin for the branch. The sum of above inputs and outputs for sales, service and parts individually is equal to the total inputs and outputs for the branch. This assumption is made by Kao (2014) in the parallel model

The following Table 5.29, shows the results of efficiency measurement of the equipment retailing organization using the parallel network DEA model of Kao (2014). The third column in the table shows efficiency score, fourth column shows the inefficiency slack, the fifth column shows the inefficiency score (1-efficiency score) and the sixth column shows the branch efficiency score. Each of the sub –DMU i.e. sales, service and parts can be treated as independent DMUs to calculate their efficiency by using conventional CCR model. In the table A1 indicates sales division, B1 indicates service division and C1 indicates parts division within DMU 1and so is the case for other branches.

For DMU1, the inefficiency slacks of division A1, B1 and C1 are 0.1463, 0.029264 and 0.042428. The CCR efficiency of DMU1 is 0.821023 and the sub –DMU efficiency score of A1, B1 and C1 are 0.829659, 0.712441 and 0.835142.This indicates that although DMU 1 has an efficiency score of 82.10%, Sub DMU A1 sales has an efficiency of 82.96%,Sub DMUB1 service has an efficiency of 71.24% and DMUC1 parts has an efficiency of 83.51%.In other words sub-DMU sales and parts have a higher efficiency than the branch efficiency whereas the service efficiency has a much lower efficiency as compared to the branch efficiency. The level of inefficiency is indicated by the slack. As you scroll the results of each DMU and sub –DMU, it is found that Sub DMU C8 parts, Sub DMU B21serivce, Sub DMU C24 parts, Sub DMU B25 service and Sub DMU A33 sales have an efficiency score of 1 indicating that they are efficient whereas all other sub DMUs are inefficient. However, none of the DMU is found efficient. The maximum value of inefficiency slack is 0.190738 0f sub DMUA9 and the minimum is zero. The minimum value of Sub DMU efficiency score is 0.332032 of sub DMU B24 and the maximum value is 1.

Similarly, the maximum value of CCR score of DMU10 is 0.957729 and the minimum value is 0.768980 of DMU9.In other words inefficiency ranges from 4.23% to 23.11%.

Table 5.29. Inefficiency Slack, DMU efficiency score and Sub-DMU efficiency scores

DMU	SubDMU	Eff Score Sub DMU	Inefficiency Slack Sub DMU	Inefficiency Score Sub DMU	CCR Branch Efficiency	CCR Branch Inefficiency Slack	DMU	SubDMU	Eff Score Sub DMU	Inefficiency Slack Sub DMU	Inefficiency Score Sub DMU	CCR Branch Efficiency	CCR Branch Inefficiency Slack
DMU1	A1(Sales)	0.829659	0.1463	0.170341	0.821023	0.217992	DMU8	A8(Sales)	0.908878	0.058527	0.091122	0.899574	0.111637
DMU1	B1(Service)	0.712441	0.029264	0.287559			DMU8	B8(Service)	0.643954	0.05311	0.356046		
DMU1	C1(Parts)	0.835142	0.042428	0.164858			DMU8	C8(Parts)	1	0	0		
DMU2	A2(Sales)	0.711403	0.235864	0.288597	0.794147	0.259212	DMU9	A9(Sales)	0.783176	0.190738	0.216824	0.76898	0.300423
DMU2	B2(Service)	0.910179	0.012418	0.089821			DMU9	B9(Service)	0.615154	0.05375	0.384846		
DMU2	C2(Parts)	0.964007	0.01093	0.035993			DMU9	C9(Parts)	0.800991	0.055935	0.199009		
DMU3	A3(Sales)	0.916793	0.062539	0.083207	0.891118	0.122186	DMU10	A10(Sales)	0.962173	0.020843	0.037827	0.957729	0.044137
DMU3	B3(Service)	0.785694	0.023523	0.214306			DMU10	B10(Service	0.855097	0.007005	0.144903		
DMU3	C3(Parts)	0.861494	0.036124	0.138506			DMU10	C10(Parts)	0.963378	0.016289	0.036622		
DMU4	A4(Sales)	0.836805	0.14868	0.163195	0.81036	0.234019	DMU11	A11(Sales)	0.738115	0.165501	0.261885	0.783404	0.27648
DMU4	B4(Service)	0.529927	0.046273	0.470073			DMU11	B11(Service	0.641123	0.057999	0.358877		
DMU4	C4(Parts)	0.826006	0.039066	0.173994			DMU11	C11(Parts)	0.890289	0.05298	0.109711		
DMU5	A5(Sales)	0.815187	0.13657	0.184813	0.815086	0.226864	DMU12	A12(Sales)	0.843701	0.079332	0.156299	0.871524	0.147415
DMU5	B5(Service)	0.787277	0.036196	0.212723			DMU12	B12(Service	0.821332	0.023866	0.178668		
DMU5	C5(Parts)	0.829745	0.054097	0.170255			DMU12	C12(Parts)	0.912662	0.044217	0.087338		
DMU6	A6(Sales)	0.936124	0.05311	0.063876	0.900054	0.111045	DMU13	A13(Sales)	0.83171	0.085785	0.16829	0.829499	0.205547
DMU6	B6(Service)	0.726415	0.018479	0.273585			DMU13	B13(Service	0.692907	0.067542	0.307093		
DMU6	C6(Parts)	0.813932	0.039456	0.186068			DMU13	C13(Parts)	0.890262	0.05222	0.109738		
DMU7	A7(Sales)	0.945604	0.043699	0.054396	0.926535	0.07929	DMU14	A14(Sales)	0.836084	0.141419	0.163916	0.793485	0.260263
DMU7	B7(Service)	0.827888	0.014098	0.172112			DMU14	B14(Service	0.482201	0.051857	0.517799		
DMU7	C7(Parts)	0.889227	0.021493	0.110773			DMU14	C14(Parts)	0.77473	0.066987	0.22527		

A1= Sales Division; **B1**= Service Division; **C1**= Parts Division, **DMU** =Branch

Table 5.29. Inefficiency Slack, DMU efficiency score and Sub-DMU efficiency scores.

DMU	SubDMU	Eff Score Sub DMU	Inefficiency Slack Sub DMU	Inefficiency Score Sub DMU	CCR Branch Efficiency	CCR Branch Inefficiency Slack	DMU	SubDMU	Eff Score Sub DMU	Inefficiency Slack Sub DMU	Inefficiency Score Sub DMU	CCR Branch Efficiency	CCR Branch Inefficiency Slack
DMU15	A15(Sales)	0.744188	0.187082	0.255812	0.770079	0.298568	DMU22	A22(Sales)	0.879407	0.10975	0.120593	0.840408	0.189898
DMU15	B15(Service	0.835806	0.013789	0.164194			DMU22	B22(Service	0.792014	0.023565	0.207986		
DMU15	C15(Parts)	0.797839	0.097698	0.202161			DMU22	C22(Parts)	0.660187	0.056583	0.339813		
DMU16	A16(Sales)	0.879989	0.096474	0.120011	0.863343	0.158289	DMU23	A23(Sales)	0.892483	0.056364	0.107517	0.89582	0.116295
DMU16	B16(Service	0.736349	0.021047	0.263651			DMU23	B23(Service	0.736528	0.055558	0.263472		
DMU16	C16(Parts)	0.851529	0.040768	0.148471			DMU23	C23(Parts)	0.98853	0.004372	0.01147		
DMU17	A17(Sales)	0.896924	0.083835	0.103076	0.854947	0.169664	DMU24	A24(Sales)	0.791986	0.191861	0.208014	0.770529	0.29781
DMU17	B17(Service	0.716554	0.032054	0.283446			DMU24	B24(Service	0.332032	0.105949	0.667968		
DMU17	C17(Parts)	0.778932	0.053775	0.221068			DMU24	C24(Parts)	1	0	0		
DMU18	A18(Sales)	0.794172	0.186595	0.205828	0.785585	0.272937	DMU25	A25(Sales)	0.794528	0.154521	0.205472	0.863207	0.15847
DMU18	B18(Service	0.543691	0.047417	0.456309			DMU25	B25(Service	1	0	0		
DMU18	C18(Parts)	0.851692	0.038925	0.148308			DMU25	C25(Parts)	0.984895	0.003949	0.015105		
DMU19	A19(Sales)	0.984218	0.012249	0.015782	0.933398	0.071354	DMU26	A26(Sales)	0.911664	0.077688	0.088336	0.88246	0.133196
DMU19	B19(Service	0.681316	0.021847	0.318684			DMU26	B26(Service	0.566868	0.037658	0.433132		
DMU19	C19(Parts)	0.835632	0.037258	0.164368			DMU26	C26(Parts)	0.892977	0.01785	0.107023		
DMU20	A20(Sales)	0.988091	0.010371	0.011909	0.910164	0.098703	DMU27	A27(Sales)	0.927628	0.063847	0.072372	0.852989	0.172348
DMU20	B20(Service	0.428855	0.057337	0.571145			DMU27	B27(Service	0.552918	0.048653	0.447082		
DMU20	C20(Parts)	0.756817	0.030996	0.243183			DMU27	C27(Parts)	0.669943	0.059848	0.330057		
DMU21	A21(Sales)	0.959284	0.032356	0.040716	0.927606	0.078044	DMU28	A28(Sales)	0.742222	0.201266	0.257778	0.784159	0.275251
DMU21	B21(Service	1	0	0			DMU28	B28(Service	0.675369	0.059095	0.324631		
DMU21	C21(Parts)	0.782938	0.045688	0.217062			DMU28	C28(Parts)	0.952342	0.014891	0.047658		

A1= Sales Division; **B1**= Service Division; **C1**= Parts Division, **DMU** =Branch

Table 5.29. Inefficiency Slack, DMU efficiency score and Sub-DMU efficiency scores.

DMU	SubDMU	Eff Score Sub DMU	Inefficiency Slack Sub DMU	Inefficiency Score Sub DMU	CCR Branch Efficiency	CCR Branch Inefficiency Slack
DMU29	A29(Sales)	0.74253	0.199642	0.25747	0.719287	0.390266
DMU29	B29(Service)	0.141687	0.149816	0.858313		
DMU29	C29(Parts)	0.907322	0.040808	0.092678		
DMU30	A30(Sales)	0.931124	0.062113	0.068876	0.877535	0.139556
DMU30	B30(Service)	0.511166	0.05178	0.488834		
DMU30	C30(Parts)	0.80533	0.025663	0.19467		
DMU31	A31(Sales)	0.847053	0.126652	0.152947	0.818053	0.222415
DMU31	B31(Service)	0.532425	0.081716	0.467575		
DMU31	C31(Parts)	0.936023	0.014047	0.063977		
DMU32	A32(Sales)	0.886518	0.089865	0.113482	0.846132	0.181849
DMU32	B32(Service)	0.580432	0.088325	0.419568		
DMU32	C32(Parts)	0.979612	0.003658	0.020388		
DMU33	A33(Sales)	1	0	0	0.905935	0.103832
DMU33	B33(Service)	0.546796	0.064207	0.453204		
DMU33	C33(Parts)	0.635567	0.039625	0.364433		

A1= Sales Division; **B1**= Service Division; **C1**= Parts Division, **DMU** =Branch

5.11.3 Network DEA Two Stage Process:

As per the relational model of Kao& Hwang (2008), the system efficiency is product of the efficiencies of the two sub-processes. The two sub- processes are selling and generating profit. The overall efficiency is the product of the two sub- processes branch sales and generation of profit margin. The inputs for the relational model are number of employees, area of facility, total department expenses and total COGS of the branch. The intermediate product is branch sales and the output is profit margin that happens in the second stage process.

The efficiency score of the system including the efficiency of the stages using the relational model of Kao & Hwang is shown in Table 5.30. From table 5.30, for DMU1, the system efficiency score is 0.55788 and the first stage score is 0.930246 and second stage score is 0.599713. The product of the two-stage efficiency score is **0.599713x0.930246= 0.55788**, the system efficiency score. There is only one DMU23 that has a score of one in all stages.

The low level of system efficiency can be interpreted as below. The mean of first stage efficiency is low at 0.600729 and the intermediate product sales has a mean of 0.9606 and the mean of second stage efficiency of generating profit margin is0.62366. This indicates that although sales are made, it is difficult to maintain margin. The system efficiency can be increased by increasing profit margin of sales from equipment, service and parts.

Table 5.30. Two Stage (Series) Process DEA scores.

DMU	Score	Score_Stage1	Score_Stage2
DMU1	0.55788	0.930246	0.599713
DMU2	0.65698	0.945015	0.695202
DMU3	0.5843	0.965043	0.605463
DMU4	0.37521	0.9029	0.415564
DMU5	0.72117	0.950534	0.758701
DMU6	0.51612	0.951304	0.54254
DMU7	0.47362	1	0.473617
DMU8	0.60979	1	0.609786
DMU9	0.53666	0.897789	0.597758
DMU10	0.6229	0.99902	0.623515
DMU11	0.66269	0.938974	0.70576
DMU12	0.82686	1	0.826862
DMU13	0.94361	1	0.943613
DMU14	0.44964	0.916691	0.490505
DMU15	0.5947	0.944756	0.629473
DMU16	0.539	0.941799	0.572312
DMU17	0.53956	0.950098	0.567897
DMU18	0.54875	0.921278	0.595643
DMU19	0.47363	0.963665	0.491493
DMU20	0.43276	1	0.432757
DMU21	0.55894	0.971495	0.57534
DMU22	0.49141	0.913958	0.537674
DMU23	1	1	1
DMU24	0.56154	0.944889	0.594291
DMU25	0.73439	0.965188	0.760875
DMU26	0.4807	0.949302	0.506376
DMU27	0.51661	1	0.516607
DMU28	0.77071	0.966245	0.797629
DMU29	0.75698	1	0.75698
DMU30	0.43329	0.92872	0.466547
DMU31	0.60293	0.94526	0.637845
DMU32	0.72233	0.998663	0.7233
DMU33	0.52841	0.998608	0.529144

First Stage= Factors of production; Second Stage= Gross Margin.

5.11.4 Conclusion:

In this section two network DEA models are used to study the efficiency of a heavy equipment retailing organization in Canada. In the first model the branch was considered to have three divisions sales, service and parts and all of them operating independently and in parallel. The total of the individual division's, sales, service and parts, is the input /output of the branch. In other words, the sum of the input/output of the divisions is the system's input /output. The conventional DEA model treats such a system as a black-box without considering the internal structure. However, by using Kao's, 2014 parallel network DEA model, each division in the branch was treated as an independent DMU in measuring relative efficiency. The model helps in decomposing the inefficiency slack of the system in to the inefficiency slack of the individual production units. This helps decision maker to make improvements in units that are less efficient.

The efficiency score calculated by parallel model are smaller than the one calculated by the conventional model due to stronger constraints in the parallel model. Therefore, only few DMUs will be efficient in the parallel model. This increases the discriminating power in performance evaluation. It is found that from the conventional model none of the branches are efficient and the score of the individual units is also less than the score of the branches. It is also found that the inefficiency slack of the branch can be decomposed into inefficiency slack of the individual divisions.

The operations of the branch were treated to be a series system with inputs as number of employees, area of facility, total department expenses and total COGS of the branch. The intermediate product is branch sales and the outputs are the profit margin and is the second stage process. The intermediate first stage is generation of sales and therefore sales becomes the intermediate product. To find the efficiency of such a system relational two- stage model of Kao&

Hwang, 2008 was used to find the efficiency of the two intermediate stages. The product of the efficiency of the two stages is the system efficiency. The two-stage system can be viewed as a series system.

The efficiency of the heavy equipment retailing organization is studied using both parallel and series network model. The two models give two diverse ways of measuring efficiency. In parallel model inefficiency slack was used to understand inefficiency whereas in series model efficiency scores of sub processes were used to measure efficiency. One of the assumptions of the relationship model is that all the outputs of one sub process must be the input of the next sub process. The relationship may not hold good if any input of the sub-process is not the output of the preceding process or an output of a sub process that is not an input to the next sub process.

5.12: Outliers in Data Envelopment Analysis (PPM- Model)

5.12.1: Introduction:

The theoretical aspects of outliers, various methods that are used in the literature are dealt in detail in Chapter 3(Theoretical aspects of DEA). Of the various methods used the following two methods will be used in this research to detect and remove outliers as they are easier to use with the available software and well accepted in the literature.

1)Super efficiency approach of Banker and Chang,2006. 2)The scalar method of Tran, Shively and Preckel,2008

5.12.2: Super -Efficiency approach to detect outliers (Banker and Chang,2006):

Banker and Chang (2006) suggested using a screen based on the super efficiency score to identify those observations that are more likely to be contaminated with noise. This is done by eliminating from the sample those observations with super-efficiency scores higher than a preselected screen.

Of the thirty-three DMUs, 13 DMUs are found efficient and they are DMU7,8,10, 12,13,19, 20,23,27,28,29,32 and 33(Table5.31). According to Banker and Chang (2006) there is no framework for selecting the screen levels. The authors used levels 1, 1.2, 1.6 and 2.0 as four screen levels in their paper.

The efficiency scores of the DMUs are in the range of 1.048585 to 3.9837. The mean of these thirteen efficiency scores works out to 1.4962. If we apply a screen level of 1.6 then the DMUs with scores above 1.6 would be DMU8 and DMU20. Therefore, DMU8 and DMU 20 are outliers. The efficiency scores for the rest of the DMUs are evaluated again and it is called BG-SE estimates and they are given in the following table. The dropped DMUs would be 8, and 20 as per the above decision rule.

Table 5.31.VRS Super Efficiency (FPA) scores (PPM-Model) - 2014 Data

DMU	**Pure Technical Efficiency Score(VRS) FPA Method**	**DMU**	**Pure Technical Efficiency Score(VRS) FPA Method**
DMU1	0.949093	DMU18	0.926717
DMU2	0.980016	DMU19	1.436189
DMU3	0.974518	DMU20	2.182778
DMU4	0.906721	DMU21	0.973795
DMU5	0.950544	DMU22	0.916848
DMU6	0.951843	DMU23	1.336403
DMU7	1.097122	DMU24	0.958991
DMU8	3.9837	DMU25	0.968978
DMU9	0.924842	DMU26	0.953865
DMU10	1.249168	DMU27	1.179321
DMU11	0.965348	DMU28	1.30765
DMU12	1.216633	DMU29	1
DMU13	1.048585	DMU30	0.979714
DMU14	0.917547	DMU31	0.948497
DMU15	0.973827	DMU32	1.361682
DMU16	0.9424	DMU33	1.051372
DMU17	0.953294		

Table 5.32. Banker-Gifford Super Efficiency Scores.

DMU	VRS Super Eff Score	DMU	VRS Super Eff Score
DMU1	0.949093	DMU17	0.959323
DMU2	1.169878	DMU18	0.962385
DMU3	0.974518	DMU19	1.570693
DMU4	0.906721	DMU21	0.973795
DMU5	0.965512	DMU22	0.916891
DMU6	0.958656	DMU23	1.336403
DMU7	2.098307	DMU24	0.958991
DMU9	0.924842	DMU25	0.978736
DMU10	1.294318	DMU26	1.090844
DMU11	0.965348	DMU27	1
DMU12	1.23456	DMU30	0.979714
DMU13	1.210453	DMU31	0.948497
DMU14	0.922348	DMU32	1.361682
DMU15	1	DMU33	1.063909
DMU16	0.942781		

The revised estimated super-efficiency scores (Table 5.32) are now less than the screen level after eliminating the outliers except DMU 7. This may be analyzed further.

5.12.3: Tran et al., 's method of detecting outliers in DEA:

Tran et al.,2010 suggested an easy and effective method to detect super-efficient outliers. The lambda λ_j in CRS model and VRS model represents the weight assigned to the j th DMU to construct a virtually efficient DMU for evaluating DMU_0. To find the efficiency scores of all j DMUs, the corresponding model must be solved j times generating jxj matrix. The resulting λ values containing all λ's can be organized as follows.

The DMUs that perform significantly better than the peer DMUs are considered outliers as they have high number of occurrences and have high value of cumulative weight. The value of C_j and S_j, (j=1,2,3…n) should be calculated after running a model and value of C_j and S_j, with a certain higher threshold can be identified as outlier and then removed from the data set. The selection of threshold is subjective and is not discussed in the literature. The process stops once a desired degree of convergence in the weights has been reached (Tran et al.,2010). Jun Wang (2017) in his PhD thesis suggests use of median plus 2x standard deviation as the threshold. Any DMU with both number of occurrences and cumulative weight higher than median plus 2x standard deviation can be considered significantly larger than vast majority and therefore can be identified as an outlier (Jun Wang, 2017)

Table 5.33. Tran’s Method of Finding Outliers

DMU	**Score**	**Benchmark(Lambda)**	**Sum Lambda**
DMU20	2.036087	DMU7(1.594045)	1.594045
DMU29	2.01309	DMU8(0.044524)	0.44524
DMU8	1.332947	DMU10(0.141050); DMU29(0.222973); DMU7(0.295698)	0.659721
DMU23	1.315397	DMU13(1.307373); DMU8(0.446728)	1.754101
DMU12	1.166419	DMU23(0.038409); DMU32(0.204839)	0.588929
DMU7	1.072082	DMU20(0.068259); DMU8(1.033408)	1.101667
DMU13	1.046769	DMU12(0.164120); DMU23(0.244089); DMU29(1.018406)	1.426615
DMU27	1.037421	DMU33(0.402217); DMU8(0.036896)	0.439113

Of the thirty-three DMUs, only the above eight DMUs are efficient under output-oriented CRS model (Table 5.33). From the sum of lambdas, DMU23 and DMU20 have the highest sum and therefore can be considered as outlier. Similarly, DMU 23 occurs twice in the reference set and therefore can be considered as outlier as per Tran’s method. Therefore, as per Tran’s method DMU 23 and DMU 20 are outliers. As per Banker and Chang (2006), the outliers are DMU8 and DMU20. Both these methods together show, that DMU8, DMU20 and DMU 23 as outliers.

5.14.4 Conclusion:

From the above two methods both employing the super-efficiency model, outliers are detected in two different ways. In VRS oriented output model using Banker and Chang method it was found that DMU20 was found to be an outlier whereas with CCR model using Tran's method it was found that DMU23 and DMU20 are outliers. Both these methods identify DMU20 as an outlier and DMU 23 is identified as an outlier only by Tran's method**.**

5.15: Branch Profit Maximization Model (Model 2).

This is the second DEA model that is developed to analyze efficiency from the perspective of profit maximization. The inputs for this model are number of employees, area of facility, COGS of equipment sales, COGS of parts sales and COGS of Service sales and outputs are sales revenue from equipment, parts and service. These inputs were used in study of retail sector and automotive retail by Joe et al., 2009, Narasimhan et al., 2005, Donthu and Yoo, (1998) and Chen (2011). Similarly, the outputs were used in study of retail sector and automotive retail by Thomas et al., (1998), Moreno et al., (2006), Narasimhan et al., (2005) to mention a few. The model used to maximize profit is output oriented VRS and CRS model. In profit maximization the LP model is solved for maximization of profit. This means how much more profit can be generated by using the current level of inputs. Further to increase the discriminating power of the model weight restriction model used and details of the same are explained below.

Inputs (5): Number of employees, Area of Facility, COGS of Equipment Sales, COGS of Service, COGS-of-Parts.

Outputs (3): Total Equipment Sales Revenue, Parts Sales Revenue, Total Service Revenue

When the above number of factors are used it meets the basic rule of degree of freedom as mentioned in the chapter on DEA theoretical aspects, “As proposed by Cooper et al. (2007) a general rule for minimum number of DMUs (n) is that it should exceed the greater of the product of the input (m) and output(s) variables or three times the sum of the number of input (m) and output (s) variables.

$$n \geq \max\{m * s, 3(m+s)\}$$ ”

5.15.1 CCR and BCC Models:

The number of DMUs is 33 and three times the sum of inputs and outputs is 24 and greater than 33 meets the above condition. However, when CRS and VRS models are run to find the efficiency scores the number of DMUs shown as efficient is very high. The efficiency scores are given in table 5.34 and 5.35 for CRS and VRS models. It is found from the table5.35 that under CRS model with the above 5 inputs and 3 outputs, the model shows 25,20,22,22 and 19 DMUs as efficient in the period 2014,2013,2012,2011 and 2010 respectively. It is also found from table 5.35 that under VRS models with the same 5 inputs and 3 outputs used as above, the VRS model shows 31,27,26,26 and 25 DMUs as efficient in the period 2014,2013,2012,2011 and 2010 respectively. The number of branches shown as efficient are very high as compared to the number of DMUs under study indicating the discriminating power is very low.

Therefore, the number of inputs and outputs were altered and new efficiency scores under CRS and VRS scores were found using the data for the period 2014. The number of inputs were maintained at 5 and outputs varied as total sales for the branch and total gross margin for the branch instead of sales revenue for equipment, service and parts. The CRS efficiency scores are given in table 5.34 and VRS efficiency scores are given in table 5.35. It is found from CRS scores that the number of efficient DMUs decrease from 25 to 13 and 14 when the number of outputs is reduced from 3 to 2 and the output mix is changed from individual sales of equipment, service and parts to total branch sales and total gross margin for the branch.

However, the objective is to find how to maximize the revenue from equipment sales, service sales and parts sales. Therefore, to attain better discrimination weight restrictions are imposed on the model and weight restriction model is used and details are given in the next section.

5.15.2: Weight Restrictions in DEA– Profit Maximization Model:

In the above analysis, basic DEA models that are used to evaluate efficiency, no judgement has been made about the importance of one input versus another and it was assumed that all the outputs had the same importance. However, in real life the importance of various inputs and outputs varies. We can define the following ratios with upper and lower bound restrictions to impose restrictions on input or output weights

$$L_{ik} \leq \frac{v_i}{v_k} \leq U_{ik} \quad where\, i = 1, 2, 3....m$$

v_i and v_k represent the weights for two different inputs and $L_{i,k}$ and $U_{i,k}$ denotes the lower and upper bound on this ratio respectively. This indicates that many such ratios can be calculated, and their lower and upper bounds can be determined. There are three outputs as in the present research, there can be 3! (n!) ratios that is six ratios can be calculated. However, the manager should use prudent judgement and practical vision in proper selection of ratios so that policy and managerial implications can be tested appropriately. Finally, weight restrictions can also be used between input and outputs.

In the current research five different models of weight restrictions have been analyzed for efficiency score using data for the period 2014 as it had a maximum number of 25 DMUs units on the frontier in the CRS model and 31 DMUs on the frontier in VRS model. The ratio of two inputs parts sales to service sales, and three inputs to outputs viz., COGS of equipment sales to revenue of equipment sales, COGS of parts sales to revenue of parts sales and COGS of service sales to revenue of service sales, were used in these models and weight restrictions imposed on them. In the models the weight restrictions were used based on the thumb rule practiced in the industry

The thumb rule in the industry is that ratio of parts sales to service sales is 1:2, gross margin on parts sales is a maximum of 40%, margin on service sales is a maximum 0f 70% and margin on equipment sales is 8%. Based on these thumb rules weight restrictions were used in the analysis and the results are as below in Table 5.38.

Table 5.34. Profit Maximization Model CRS Scores (2010-2014)

DMU	2014 Technical Efficiency Score(CRS)5I, 3O	2013Technical Efficiency Score(CRS)	2012Technical Efficiency Score(CRS)	2011Technical Efficiency Score(CRS)	2010Technical Efficiency Score(CRS)
DMU1	0.990697	0.993318	1	1	0.99679
DMU2	1	1	1	1	1
DMU3	0.988331	0.991208	0.990229	1	0.983762
DMU4	1	0.990755	1	0.936969	0.910549
DMU5	1	0.952255	0.990246	1	NA
DMU6	1	1	0.998027	1	1
DMU7	1	1	1	1	1
DMU8	1	1	1	1	1
DMU9	1	0.979314	1	1	0.96458
DMU10	1	1	1	1	1
DMU11	0.989419	1	0.989024	0.971449	0.971189
DMU12	1	1	1	1	1
DMU13	1	1	1	0.994245	1
DMU14	1	0.998916	1	0.982894	0.942354
DMU15	0.979182	0.960616	NA	NA	NA
DMU16	0.986675	1	0.998838	1	1
DMU17	0.997619	0.989412	1	1	1
DMU18	0.985652	1	0.997219	1	1
DMU19	1	0.992422	1	1	1
DMU20	1	1	1	1	1
DMU21	1	0.98666	1	0.988084	0.985055
DMU22	1	1	1	1	1
DMU23	1	1	1	1	1
DMU24	1	1	1	0.980439	1
DMU25	1	1	0.974241	0.983342	0.983396
DMU26	1	0.997577	0.991515	0.981164	0.993565
DMU27	1	1	1	1	1
DMU28	1	1	1	1	1
DMU29	1	NA	NA	NA	NA
DMU30	0.992018	0.960385	0.95691	0.933596	0.948609
DMU31	1	1	1	1	0.980045
DMU32	1	1	1	1	1
DMU33	1	1	1	1	1

Table 5.35. Profit Maximization Model VRS Scores (2010-2014)

DMU	**2014Pure Technical Efficiency Score(VRS)**	**2013Pure Technical Efficiency Score(VRS)**	**2012 Pure Technical Efficiency Score(VRS)**	**2011Pure Technical Efficiency Score(VRS)**	**2010Pure Technical Efficiency Score(VRS)**
DMU1	1	1	1	1	1
DMU2	1	1	1	1	1
DMU3	0.991033	1	0.998208	1	1
DMU4	1	0.995736	1	1	1
DMU5	1	0.954387	0.998582	1	NA
DMU6	1	1	1	1	1
DMU7	1	1	1	1	1
DMU8	1	1	1	1	1
DMU9	1	1	1	1	1
DMU10	1	1	1	1	1
DMU11	1	1	1	1	1
DMU12	1	1	1	1	1
DMU13	1	1	1	1	1
DMU14	1	0.999698	1	0.986047	0.945793
DMU15	1	1	NA	NA	NA
DMU16	0.990511	1	0.999737	1	1
DMU17	1	0.996265	1	1	1
DMU18	1	1	1	1	1
DMU19	1	1	1	1	1
DMU20	1	1	1	1	1
DMU21	1	0.998264	1	0.988297	0.985117
DMU22	1	1	1	1	1
DMU23	1	1	1	1	1
DMU24	1	1	1	0.990418	1
DMU25	1	1	0.985608	0.98407	0.98766
DMU26	1	1	1	0.995856	1
DMU27	1	1	1	1	1
DMU28	1	1	1	1	1
DMU29	1	NA	NA	NA	NA
DMU30	1	1	0.989984	0.979536	0.974619
DMU31	1	1	1	1	0.983724
DMU32	1	1	1	1	1
DMU33	1	1	1	1	1

Table 5.36. Profit Maximization Model CRS Efficiency Scores 2014

DMU	Technical Efficiency Score(CRS)5I,3O	Technical Efficiency Score(CRS)5I,3O Eqp sales replaced by Total Sales as Input rest same	Score(CRS) Same inputs,2 outputs Total Branch Sales,Total GM for the branch	Technical Efficiency Score(CRS)6 inputs,2 Outputs.Extra input Total Branch Expenses
DMU1	0.990697	0.989593	0.977342	0.977342
DMU2	1	1	0.971906	0.971906
DMU3	0.988331	0.988573	0.969453	0.988018
DMU4	1	1	0.961119	0.961119
DMU5	1	0.999267	0.976565	0.976565
DMU6	1	1	1	1
DMU7	1	1	0.977942	1
DMU8	1	1	1	1
DMU9	1	1	0.991238	0.991238
DMU10	1	1	1	1
DMU11	0.989419	0.995351	0.952526	0.956736
DMU12	1	1	1	1
DMU13	1	1	1	1
DMU14	1	1	0.990023	0.990023
DMU15	0.979182	0.978492	0.948975	0.955663
DMU16	0.986675	0.991349	0.977277	0.977277
DMU17	0.997619	0.997117	0.977723	0.97829
DMU18	0.985652	0.993592	0.983603	0.983603
DMU19	1	1	0.975543	0.988001
DMU20	1	1	1	1
DMU21	1	1	1	1
DMU22	1	1	1	1
DMU23	1	1	1	1
DMU24	1	1	0.971921	0.973578
DMU25	1	1	0.98084	0.993845
DMU26	1	1	0.991978	0.993641
DMU27	1	1	1	1
DMU28	1	1	0.992661	0.995507
DMU29	1	1	1	1
DMU30	0.992018	0.998297	0.955833	0.980293
DMU31	1	1	0.960165	0.961866
DMU32	1	1	1	1
DMU33	1	1	1	1

Table 5.37. Profit Maximization Model VRS Efficiency Scores 2014

DMU	**Pure Technical Efficiency Score(VRS)5I, 3O**	**PureTechnical Efficiency Score(VRS)5I,3O Eqp sales replaced by Total Sales as Input rest same**	**PureTechnical Efficiency Score(vRS) Same inputs,2 outputs Total Branch Sales,Total GM for the branch**	**Pure Technical Efficiency Score(VRS)6 inputs,2 Outputs.Extra input Total Branch Expenses**
DMU1	1	1	0.998994	0.998994
DMU2	1	1	1	1
DMU3	0.991033	0.991259	0.969969	0.990366
DMU4	1	1	0.96312	0.96312
DMU5	1	1	0.989357	0.989357
DMU6	1	1	1	1
DMU7	1	1	1	1
DMU8	1	1	1	1
DMU9	1	1	1	1
DMU10	1	1	1	1
DMU11	1	1	1	1
DMU12	1	1	1	1
DMU13	1	1	1	1
DMU14	1	1	1	1
DMU15	1	1	1	1
DMU16	0.990511	0.994007	0.979722	0.979722
DMU17	1	1	0.981428	0.983144
DMU18	1	1	0.986088	0.986088
DMU19	1	1	1	1
DMU20	1	1	1	1
DMU21	1	1	1	1
DMU22	1	1	1	1
DMU23	1	1	1	1
DMU24	1	1	0.978159	0.979997
DMU25	1	1	0.990777	0.995774
DMU26	1	1	1	1
DMU27	1	1	1	1
DMU28	1	1	1	1
DMU29	1	1	1	1
DMU30	1	1	1	1
DMU31	1	1	0.961164	0.962281
DMU32	1	1	1	1
DMU33	1	1	1	1

Table 5.38. Weight Restrictions Models 2014 Data.

Weight Restrictions---->			**Weight Restrictions Type1Parts Sales to Service Sales Model 1**		**Weight Restriction Type2 Equipment sales to COGS Eqp sales Model 2**		**Weight Restriction Type2 Parts sales to COGS Parts sales Model 3**		**Weight Restriction Type2 Service sales to COGS Service sales Model 4**		**Weight Restriction Type2 All Input/Output restrictions Model 5**	
DMU	**2014 Technical Efficiency Score(CRS)**	**2014Pure Technical Efficiency Score(VRS)**	**2014Technical Efficiency Score(CRS)**	**2014 Pure Technical Efficiency Score(VRS)**	**Technical Efficiency Score(CRS)**	**Pure Technical Efficiency Score(VRS)**	**Technical Efficiency Score(CRS)**	**Pure Technical Efficiency Score(VRS)**	**Technical Efficiency Score(CRS)**	**Pure Technical Efficiency Score(VRS)**	**Technical Efficiency Score(CRS)**	**Pure Technical Efficiency Score(VRS)**
DMU1	0.990697	1	0.978164	1	0.990449	1	0.972923	1	0.990697	1	0.862615	0.95532
DMU2	1	1	1	1	1	1	1	1	1	1	1	1
DMU3	0.988331	0.991033	0.985319	0.985966	0.988172	0.990738	0.970062	0.970062	0.988331	0.991033	0.921269	0.9256
DMU4	1	1	1	1	0.998436	1	1	1	1	1	0.945648	0.945688
DMU5	1	1	1	1	1	1	0.993778	1	1	1	0.993729	1
DMU6	1	1	1	1	1	1	1	1	1	1	1	1
DMU7	1	1	1	1	1	1	0.994578	1	1	1	0.969595	1
DMU8	1	1	1	1	1	1	1	1	1	1	1	1
DMU9	1	1	1	1	1	1	1	1	1	1	1	1
DMU10	1	1	1	1	1	1	1	1	1	1	1	1
DMU11	0.989419	1	0.977458	1	0.988655	1	0.973693	1	0.989419	1	0.973065	1
DMU12	1	1	1	1	1	1	1	1	1	1	1	1
DMU13	1	1	1	1	1	1	1	1	1	1	1	1
DMU14	1	1	1	1	1	1	1	1	1	1	0.981695	0.994873
DMU15	0.979182	1	0.97021	1	0.979091	1	0.96758	1	0.978057	1	0.818588	0.981614
DMU16	0.986675	0.990511	0.979479	0.982044	0.980122	0.983822	0.985961	0.987511	0.986121	0.990511	0.857469	0.863256
DMU17	0.997619	1	0.99496	0.997312	0.996417	1	0.99049	0.991548	0.997619	1	0.957974	0.964988
DMU18	0.985652	1	0.983801	0.9912	0.985613	1	0.98001	0.988746	0.985652	1	0.82663	0.84499
DMU19	1	1	0.992796	1	1	1	1	1	1	1	0.812331	0.972523
DMU20	1	1	1	1	1	1	1	1	1	1	1	1
DMU21	1	1	1	1	1	1	1	1	1	1	1	1
DMU22	1	1	1	1	1	1	1	1	1	1	1	1
DMU23	1	1	1	1	1	1	1	1	1	1	1	1
DMU24	1	1	0.986696	0.993202	1	1	1	1	1	1	0.798868	0.815125
DMU25	1	1	1	1	1	1	1	1	1	1	1	1
DMU26	1	1	1	1	1	1	1	1	1	1	0.869976	0.975264
DMU27	1	1	1	1	1	1	1	1	1	1	0.950046	1
DMU28	1	1	1	1	1	1	1	1	1	1	1	1
DMU29	1	1	1	1	1	1	1	1	1	1	0.94339	1
DMU30	0.992018	1	0.992018	1	0.902571	0.954535	0.991681	1	0.985775	1	0.70589	0.80347
DMU31	1	1	1	1	0.914755	0.916403	1	1	0.999664	1	0.830749	0.830835
DMU32	1	1	1	1	1	1	1	1	1	1	1	1
DMU33	1	1	1	1	1	1	1	1	1	1	1	1

The above four weight restrictions were used using the thumb rule and the weight restriction models were run individually as Model 1,2,3 and 4. In model 5 all the weight restrictions were applied simultaneously. The summary of efficient and inefficient DMUs (Branches) is given in table 5.39.

Table 5.39. Summary of Weight Restrictions Models 2014 Data

	Basic Models		Model 1		Model 2		Model 3		Model 4		Model 5	
DMUs	CRS	VRS	CRS	VRS	CRS	VRS	CRS	VRS	CRS	VRS	CRS	VRS
Efficient Units	25	31	23	29	23	29	23	28	24	31	15	20
Inefficient Units	8	2	10	4	10	4	10	5	9	2	18	13

It is found from the above table that as the weight restriction is imposed one by one the number of efficient units increased in CRS model from 25 to 23 and to 15 when all the weight restrictions were applied simultaneously in Model 5. Similarly, in VRS models the number of efficient units dropped from 31 to 20 when all the weight restrictions were applied simultaneously in Model 5. In other words, imposing weight restrictions increases the discriminatory power of the model. The mean of efficiency dropped from 99.72% to 93.99% a drop of 5.73% using CRS model and mean of efficiency dropped from 99.94% to 96.58% a drop of 3.36% using VRS model weight restriction models. In summary weight restriction models increases the discriminating power. Therefore, one can conclude that using all weight restrictions together gives a greater opportunity to improve efficiency of branches.

5.16: Branch Expenses Minimization Model (Model 3)

This is the third DEA model that is developed to analyze efficiency from the perspective of minimization of expenses. The inputs for this model are department expenses, depreciation and amortization, and total COGS for the branch which is sum of COGS of equipment sales, COGS of rental sales, COGS of parts sales and COGS of Service sales. These inputs were used in a study of fourteen general merchandizers in USA by Joo, Nixon and Stoeberl (2011). Similarly, the output used in this study was sum of revenue from equipment sale, rental sales, service sales and branch sales termed as total branch sales. The model used to minimize expenses is input oriented VRS and CRS model. In expense minimization the LP model is solved for minimization of expenses. This means how to minimize the current level of expenses to maintain the current level of sales revenue with three inputs and one output.

Inputs: Total COGS for the branch, departmental expenses, Depreciation and Amortization. **Outpu**t: Total Branch Sales.

5.16.1: CCR and BCC models

The efficiency scores for the above minimization problem under both CRS and VRS is given in table 5.40 and 5.41 respectively. In the CRS model 6 DMUs 12,3,15,23,27 and 29 were found 100% efficient in 2014,6 DMUs 6,8,10,13,15, and 23 were found efficient in 2013,3 DMUs 23,27 and 31 were found efficient in 2012,4 DMUs 7,13,19 and 23 were found efficient in 2011 and 6 DMUs 7,8,10,13,23,and 33 were found efficient in 2010.In other words under CRS model 20%,12.9%,9.67%,18.75% and 18.18%,were found efficient during the period 2010-2014 indicating that there is ample opportunity for the other branches to minimize expenses.

Similarly under VRS model 15 DMUs(7,8,10,12,13,15,19,20,21,23,27,29,30,32 and 33) in 2014,12 DMUs(6,8,10,11,13,15,19,23,27,30,32 and 33) in 2013,8DMUs (7,13,19, 23, 27,31,32,and 33) in 2012, 8 DMUs (5,7,8,13,19,23,32 and 33) in 2011 and 11 DMUs (7,8,10,11, 12, 13,19,23,24,32 and 33 in 2010 were found efficient. In other words, 36.6%,25.8%,25.8%, 37.5% and 45.45% in the period 2010-2014 respectively are found to be efficient under VRS model indicating again that there is scope for reduction of expenses under VRS model.

In table 5.42, scale efficiency of these branches is given. It is found that there are 6,4,3,6 and 6 DMUs scale efficient in the period 2010-2014 respectively. Similarly,2,2,13 and 1

DMUs are having increasing returns to scale in the period2010,2011,2013 and 2014 respectively indicating that the branches can increase their expenses. There are 22,25,28,13 and 26 DMUs in the period 2010-2014 respectively that have decreasing returns to scale indicating that the expenses can be minimized by downsizing the branch operations. The inefficiency between CRS model and VRS model is due to the different scales used by the two models. CRS model uses proportional increases and decreases of input and output variables for computing efficiency scores whereas BCC model applies a variable return to scale.

It has to be noted in the analysis that only three inputs and one output was used in the analysis and this meets the basic rule of degree of freedom as mentioned in the chapter on DEA theoretical aspects, “As proposed by Cooper et al. (2007) a general rule for minimum number of DMUs (n) is that it should exceed the greater of the product of the input (m) and output(s) variables or three times the sum of the number of input (m) and output (s) variables.

$$n \geq \max\{m * s, 3(m+s)\}$$ ”

In this model 3 inputs and one output times three is 12 which is way below the total number of DMUs 33 and therefore models exhibit a high discriminatory power in analysis.

Table 5.40. Input Oriented CRS Scores Expense Minimization Model (2010-2014)

DMU	**2014Technical Efficiency Score(CRS)**	**2013Technical Efficiency Score(CRS)**	**2012Technical Efficiency Score(CRS)**	**2011Technical Efficiency Score(CRS)**	**2010 Technical Efficiency Score(CRS)**
DMU1	0.907665	0.917896	0.770711	0.811602	0.842795
DMU2	0.930432	0.904839	0.803586	0.775158	0.861042
DMU3	0.925089	0.966658	0.766861	0.774779	0.928998
DMU4	0.856461	0.904301	0.760546	0.693404	0.761352
DMU5	0.932842	0.851724	0.748342	0.760054	NA
DMU6	0.896806	1	0.772287	0.815194	0.921796
DMU7	0.892143	0.984051	0.807912	1	1
DMU8	0.943141	1	0.85431	0.803516	1
DMU9	0.890797	0.913598	0.731751	0.730363	0.863395
DMU10	0.937728	1	0.874088	0.956844	1
DMU11	0.929785	0.997567	0.781548	0.915619	0.986307
DMU12	1	0.986245	0.801949	0.888385	0.993664
DMU13	1	1	0.941526	1	1
DMU14	0.89129	0.945712	0.743698	0.789641	0.824314
DMU15	1	1	NA	NA	NA
DMU16	0.895138	0.980939	0.819156	0.935876	0.934265
DMU17	0.908986	0.916333	0.774208	0.764933	0.910417
DMU18	0.906035	0.892035	0.754638	0.826404	0.837089
DMU19	0.892955	0.956853	0.965787	1	0.954087
DMU20	0.87408	0.919479	0.732713	0.779672	0.824176
DMU21	0.91742	0.963862	0.840768	0.864437	0.88687
DMU22	0.879357	0.849301	0.742555	0.753422	0.804126
DMU23	1	1	1	1	1
DMU24	0.91232	0.91592	0.728624	0.722107	0.910029
DMU25	0.960496	0.951549	0.750844	0.738177	0.864399
DMU26	0.893972	0.915888	0.716646	0.760998	0.86798
DMU27	1	0.987119	1	0.740578	0.878237
DMU28	0.956723	0.91	0.761628	0.725417	0.853151
DMU29	1	NA	NA	NA	NA
DMU30	0.879435	0.939444	0.84942	0.757416	0.832271
DMU31	0.919449	0.934366	1	0.883815	0.860975
DMU32	0.953445	0.985282	0.838306	0.876776	0.937776
DMU33	0.913797	0.993494	0.903734	0.916443	1

Table 5.41. Input Oriented VRS Scores Expense Minimization Model (2010-2014)

DMU	2014 Pure Technical Efficiency Score(VRS)	2013 Pure Technical Efficiency Score(VRS)	2012 Pure Technical Efficiency Score(VRS)	2011Pure Technical Efficiency Score(VRS)	Pure Technical Efficiency Score(VRS)
DMU1	0.949122	0.930824	0.973745	0.965736	0.987211
DMU2	0.938895	0.923009	0.951415	0.904731	0.931272
DMU3	0.97463	0.96726	0.950971	0.95727	0.982396
DMU4	0.896599	0.921711	0.946247	0.839981	0.871036
DMU5	0.935311	0.863612	0.901966	1	NA
DMU6	0.932412	1	0.935709	0.978338	0.96925
DMU7	1	0.99028	1	1	1
DMU8	1	1	0.941758	1	1
DMU9	0.930261	0.926504	0.927042	0.930685	0.990359
DMU10	1	1	0.996398	0.992479	1
DMU11	0.966456	1	0.989264	0.986382	1
DMU12	1	0.991234	0.989139	0.971336	1
DMU13	1	1	1	1	1
DMU14	0.956866	0.954542	0.958758	0.916046	0.887815
DMU15	1	1	NA	NA	NA
DMU16	0.925149	0.983835	0.939801	0.969031	0.93866
DMU17	0.950749	0.920867	0.945185	0.93273	0.946563
DMU18	0.906477	0.901621	0.909568	0.865384	0.851912
DMU19	1	1	1	1	1
DMU20	1	0.919971	0.917081	0.944933	0.964437
DMU21	1	0.970878	0.960581	0.940667	0.907233
DMU22	0.901363	0.849993	0.910559	0.919164	0.856553
DMU23	1	1	1	1	1
DMU24	0.958563	0.920985	0.919801	0.915203	1
DMU25	0.971089	0.958785	0.903843	0.8833	0.923645
DMU26	0.936741	0.946076	0.85619	0.821219	0.871838
DMU27	1	1	1	0.812661	0.974162
DMU28	0.959753	0.956581	0.873756	0.810749	0.861712
DMU29	1	NA	NA	NA	NA
DMU30	1	1	0.959963	0.878899	0.923326
DMU31	0.945286	0.949307	1	0.914782	0.898503
DMU32	1	1	1	1	1
DMU33	1	1	1	1	1

Table 5.42. Input Oriented Scale Efficiency Scores Expense Minimization Model (2010-2014)

DMU	2014 Scale Efficiency Score	RTS	2013 Scale Efficiency Score	RTS	2012 Scale Efficiency Score	RTS	2011 Scale Efficiency Score	RTS	2010Scale Efficiency Score	RTS
DMU1	0.956321	Decreasing	0.98611	Decreasing	0.791492	Decreasing	0.840397	Decreasing	0.853713	Decreasing
DMU2	0.990986	Decreasing	0.980315	Increasing	0.844622	Decreasing	0.856782	Decreasing	0.924587	Decreasing
DMU3	0.94917	Decreasing	0.999377	Increasing	0.806398	Decreasing	0.809363	Decreasing	0.945644	Decreasing
DMU4	0.955233	Decreasing	0.981111	Decreasing	0.80375	Decreasing	0.825499	Decreasing	0.874077	Decreasing
DMU5	0.99736	Decreasing	0.986235	Increasing	0.829679	Decreasing	0.760054	Increasing	NA	NA
DMU6	0.961813	Decreasing	1	Constant	0.82535	Decreasing	0.833244	Decreasing	0.95104	Decreasing
DMU7	0.892143	Decreasing	0.993709	Increasing	0.807912	Decreasing	1	Constant	1	Constant
DMU8	0.943141	Decreasing	1	Constant	0.907143	Decreasing	0.803516	Increasing	1	Constant
DMU9	0.957577	Decreasing	0.98607	Decreasing	0.78934	Decreasing	0.784758	Decreasing	0.8718	Decreasing
DMU10	0.937728	Decreasing	1	Constant	0.877247	Decreasing	0.964095	Decreasing	1	Constant
DMU11	0.962056	Decreasing	0.997567	Decreasing	0.79003	Decreasing	0.92826	Decreasing	0.986307	Decreasing
DMU12	1	Constant	0.994966	Increasing	0.810755	Decreasing	0.914601	Decreasing	0.993664	Decreasing
DMU13	1	Constant	1	Constant	0.941526	Decreasing	1	Constant	1	Constant
DMU14	0.931468	Decreasing	0.99075	Decreasing	0.775689	Decreasing	0.86201	Decreasing	0.928475	Decreasing
DMU15	1	Constant	1	Constant	NA	NA	NA	NA	NA	NA
DMU16	0.967561	Decreasing	0.997056	Increasing	0.871627	Decreasing	0.965785	Decreasing	0.995318	Decreasing
DMU17	0.956074	Decreasing	0.995077	Increasing	0.819107	Decreasing	0.820101	Decreasing	0.961814	Decreasing
DMU18	0.999512	Increasing	0.989367	Increasing	0.829666	Decreasing	0.954956	Decreasing	0.9826	Decreasing
DMU19	0.892955	Decreasing	0.956853	Decreasing	0.965787	Decreasing	1	Constant	0.954087	Decreasing
DMU20	0.87408	Decreasing	0.999465	Decreasing	0.798961	Decreasing	0.825108	Decreasing	0.854567	Decreasing
DMU21	0.91742	Decreasing	0.992774	Decreasing	0.87527	Decreasing	0.918962	Decreasing	0.977554	Decreasing
DMU22	0.975586	Decreasing	0.999186	Increasing	0.815494	Decreasing	0.819682	Decreasing	0.938793	Decreasing
DMU23	1	Constant	1	Constant	1	Constant	1	Constant	1	Constant
DMU24	0.951758	Decreasing	0.994501	Decreasing	0.792153	Decreasing	0.789013	Decreasing	0.910029	Decreasing
DMU25	0.989091	Decreasing	0.992452	Increasing	0.830725	Decreasing	0.835704	Decreasing	0.935856	Decreasing
DMU26	0.954343	Decreasing	0.968092	Increasing	0.837017	Decreasing	0.926669	Decreasing	0.995575	Increasing
DMU27	1	Constant	0.987119	Increasing	1	Constant	0.911299	Decreasing	0.901531	Increasing
DMU28	0.996843	Decreasing	0.951305	Increasing	0.871671	Decreasing	0.894749	Decreasing	0.990064	Decreasing
DMU29	1	Constant	NA	NA	NA	NA	NA	NA	NA	NA
DMU30	0.879435	Decreasing	0.939444	Decreasing	0.884847	Decreasing	0.861779	Decreasing	0.901384	Decreasing
DMU31	0.972668	Decreasing	0.984262	Decreasing	1	Constant	0.966149	Decreasing	0.958232	Decreasing
DMU32	0.953445	Decreasing	0.985282	Decreasing	0.838306	Decreasing	0.876776	Decreasing	0.937776	Decreasing
DMU33	0.913797	Decreasing	0.993494	Decreasing	0.903734	Decreasing	0.916443	Decreasing	1	Constant

5.17: Branch Assets Maximization Model (Model 4).

In this fourth DEA model an attempt is made to analyze efficiency of revenues over total assets from the perspective of maximization of assets. The inputs for this model are current assets, fixed assets, other assets and total COGS for the branch which is sum of COGS of equipment sales, COGS of rental sales, COGS of parts sales and COGS of Service sales. These inputs were used in a study of fourteen general merchandizers in USA by Joo, Nixon and Stoeberl (2011). Similarly, the output used in this study is sum of revenue from equipment sales, rental sales, service sales and branch sales termed as total branch sales. The model used to maximize assets is output oriented VRS and CRS models. In asset maximization, the LP model is solved for maximizing the sales of the branch with the existing level of assets. This means how to maximize the sales utilizing the existing assets more efficiently using four inputs and one output.

Inputs: Total COGS of the branch, Current Assets, Fixed Assets, Other Assets

Output: Total branch Sales (Equipment +Rentals+ Parts +Service) Revenues

5.17.1.CCR and BCC Models:

The efficiency scores of the model are given in table 5.43,5.44 and 5.45 for CRS, VRS and Scale efficiency. In the CRS model 2 DMUs 20 and 23 were found 100% efficient in 2014,4 DMUs 6,13,21 and 23 were found efficient in 2013,6 DMUs 12,13,19,21,23 and 32 were found efficient in 2012,5 DMUs 4,6,12,16 and 23 were found efficient in 2011 and 4 DMUs 8,12,16 and 23 were found efficient in 2010. In other words, under CRS model 13.33%,16.12%,19.35%,12.5% and 6.06%, were found efficient during the period 2010-2014 indicating that there is ample opportunity for the other branches to maximize assets.

Similarly, under VRS model 5 DMUs (19,20,23,29, and 32) in 2014, 8 DMUs (6,11,13,15,19,21,23 and 32) in 2013,5 DMUs (12,13,19, 21 and 23) in 2012, 8 DMUs (4,5,6,12,6,19,23 and 32) in 2011 and 7 DMUs (8,11, 12,16,19,23 and 32) in 2010 were found efficient. In other words, 23.33%,25.0%,16.2%, 25% and 15.15% in the period 2010-2014 respectively are found to be efficient under VRS model indicating again that there is scope for maximization of assets under VRS model.

In table 5.45, scale efficiency scores of the branches are given. It is found that there are 20,20,17,21 and11 DMUs scale efficient in the period 2010-2014 respectively. This indicates that these branches are utilizing their assets efficiently.Similarly, 1,5,1 and 12 DMUs are having increasing returns to scale in the period 2011,2013 and 2014 respectively indicating that the branches can decrease utilization of assets. There are 10,10,9,10 and 10 DMUs in the period 2010-2014 respectively that have decreasing returns to scale indicating that the assets can be maximized by downsizing the branch operations. The inefficiency between CRS model and VRS model is due to the different scales used by the two models. CRS model uses proportional increases and decreases of input and output variables for computing efficiency scores whereas BCC model applies a non-linear scale.

It has to be noted in the analysis that only four inputs and one output was used in the analysis and this meets the basic rule of degree of freedom as mentioned in the chapter on DEA theoretical aspects, “As proposed by Cooper et al. (2007) a general rule for minimum number of DMUs (n) is that it should exceed the greater of the product of the input (m) and output(s) variables or three times the sum of the number of input (m) and output (s) variables.

$$n \geq \max\{m * s, 3(m+s)\}$$ ”

In this model 4 inputs and one output times three is 15 which is way below the total number of DMUs 33 and therefore models exhibit a high discriminatory power in analysis.

5.17.2: Conclusions:

In this chapter the number of branches (DMUs) that are to be studied were chosen and the data collected for the study for the period 2010-2014. Then various possible inputs and outputs were identified for analyzing the efficiency of the heavy equipment retailing organization under study. These inputs and outputs were drawn from the lists of inputs and outputs used in various research papers found in the literature review of the retail sector and automotive industry. It was also justified how the financial parameters can be used as factors of production for the study by decomposing the return on investment(ROI) and return on assets(ROA) using DuPont method. Then appropriate variables from among all the available variables were chosen for the study using the stepwise approach method.

Then four DEA models, production process model, profit maximization model, expense minimization model and assets maximization models were developed to study the efficiency of the heavy equipment retailing organization. Appropriate inputs and outputs that were relevant for these models were chosen to conform to the degree of freedom requirement as proposed by Cooper et al.,2007, to maintain the discriminatory power of the model. "As proposed by Cooper et al., (2007) a general rule for minimum number of DMUs (n) is that it should exceed the greater of the product of the input (m) and output(s) variables or three times the sum of the number of input (m) and output (s) variables.

$$n \geq \max\{m*s, 3(m+s)\}$$ "

The first model developed was the production process model with factors of production as number of employees, area of facility, total COGS of the branch, total departmental expenses as four inputs and total sales revenue and total gross margin as two outputs. An in-depth analysis of this model was done using various extended models of DEA, to find the consistency of results and to increase the discriminating power of the model. The DEA models used were CRS, VRS, Super-Efficiency and Cross efficiency model. The efficiency scores were then bootstrapped with 2000 replications to find out the bias in the scores.

The corrected efficiency scores were then regressed with five environmental variables to find if the contextual variables have any influence on the efficiency scores. Then window analysis and Malmquist index were used to study the efficiency change over time. Then the two methods viz. super-efficiency model and Tran's method were used to find if there are any outlier DMUs in the study. Lastly Network DEA parallel and series structure was used to find the efficiency of the internal structure of the DMU that has sales, service and parts operations. A detailed analysis has been provided at the completion of each model.

Table 5.43. Output Oriented CRS Scores Asset Maximization Model (2010-2014)

DMU	**2014Technical Efficiency Score(CRS)**	**2013 Technical Efficiency Score(CRS)**	**2012Technical Efficiency Score(CRS)**	**2011Technical Efficiency Score(CRS)**	**2010Technical Efficiency Score(CRS)**
DMU1	0.846694	0.856984	0.951558	0.893453	0.929981
DMU2	0.878835	0.850381	0.963006	0.929192	0.947145
DMU3	0.848563	0.775918	0.887272	0.793275	0.857047
DMU4	0.914092	0.939258	0.983258	1	0.994505
DMU5	0.901593	0.826758	0.915843	0.760054	NA
DMU6	0.902074	1	0.97003	1	0.982924
DMU7	0.868348	0.899815	0.972741	0.945854	0.970087
DMU8	0.849973	0.880559	0.930892	0.793474	1
DMU9	0.84606	0.84715	0.917235	0.881875	0.921801
DMU10	0.854483	0.852762	0.93559	0.830993	0.863041
DMU11	0.882538	0.910401	0.975872	0.90779	0.943937
DMU12	0.927371	0.955699	1	1	1
DMU13	0.975129	1	1	0.955851	0.919673
DMU14	0.887165	0.894807	0.953048	0.927928	0.876081
DMU15	0.856406	0.70994	NA	NA	NA
DMU16	0.933215	0.95233	0.971862	1	1
DMU17	0.917627	0.883756	0.957173	0.962313	0.950264
DMU18	0.845376	0.829547	0.923128	0.855101	0.877145
DMU19	0.921173	0.950101	1	0.933771	0.942388
DMU20	1	0.951617	0.971678	0.967989	0.978597
DMU21	0.997462	1	1	0.962901	0.921688
DMU22	0.901556	0.85992	0.923522	0.950914	0.876716
DMU23	1	1	1	1	1
DMU24	0.844939	0.825736	0.899321	0.840064	0.891752
DMU25	0.902393	0.885567	0.917689	0.909317	0.940004
DMU26	0.817472	0.779861	0.872509	0.850334	0.871178
DMU27	0.820576	0.784791	0.853825	0.810958	0.827281
DMU28	0.916137	0.781605	0.891817	0.844808	0.886898
DMU29	0.900961	NA	NA	NA	NA
DMU30	0.866337	0.875215	0.918032	0.837254	0.85004
DMU31	0.859242	0.813727	0.87917	0.799518	0.788008
DMU32	0.896235	0.96754	1	0.943605	0.913577
DMU33	0.878693	0.959693	0.964043	0.938348	0.914127

Table 5.44. Output Oriented VRS Scores Asset Maximization Model (2010-2014)

DMU	2014Pure Technical Efficiency Score(VRS)	2013Pure Technical Efficiency Score(VRS)	2012Pure Technical Efficiency Score(VRS)	2011Pure Technical Efficiency Score(VRS)	2010Pure Technical Efficiency Score(VRS)
DMU1	0.944447	0.936882	0.974071	0.970136	0.987462
DMU2	0.882953	0.850381	0.963006	0.929192	0.947145
DMU3	0.908352	0.927378	0.952419	0.959397	0.982969
DMU4	0.914092	0.939258	0.984377	1	0.994505
DMU5	0.906473	0.826758	0.915843	1	NA
DMU6	0.902074	1	0.971252	1	0.982924
DMU7	0.868348	0.899815	0.972741	0.945854	0.970087
DMU8	0.85064	0.880559	0.930892	0.793474	1
DMU9	0.916325	0.930277	0.927724	0.94214	0.990579
DMU10	0.93366	0.931832	0.975726	0.920376	0.971024
DMU11	0.962948	1	0.989367	0.986068	1
DMU12	0.927737	0.955699	1	1	1
DMU13	0.982493	1	1	0.955851	0.919673
DMU14	0.887165	0.894807	0.954412	0.927928	0.876081
DMU15	0.868923	1	NA	NA	NA
DMU16	0.933215	0.95233	0.972523	1	1
DMU17	0.917627	0.883756	0.957173	0.962313	0.950264
DMU18	0.850302	0.829547	0.923128	0.855101	0.877145
DMU19	1	1	1	1	1
DMU20	1	0.951617	0.971947	0.967989	0.978597
DMU21	0.997462	1	1	0.962901	0.921688
DMU22	0.901556	0.85992	0.923522	0.950914	0.876716
DMU23	1	1	1	1	1
DMU24	0.932785	0.904205	0.920805	0.917336	0.945213
DMU25	0.905286	0.885567	0.917689	0.909317	0.940004
DMU26	0.820407	0.779861	0.872509	0.850334	0.871178
DMU27	0.824342	0.784791	0.853825	0.810958	0.827281
DMU28	0.92928	0.781605	0.891817	0.844808	0.886898
DMU29	1	NA	NA	NA	NA
DMU30	0.952401	0.921099	0.921839	0.898021	0.92533
DMU31	0.900231	0.888204	0.880297	0.8508	0.902314
DMU32	1	1	1	1	1
DMU33	0.878693	0.959693	0.964793	0.938348	0.914127

Table 5.45. Output Oriented Scale Eff Scores Asset Maximization Model (2010-2014)

DMU	**2014Scale Efficiency Score**	**RTS**	**2013Scale Efficiency Score**	**RTS**	**2012Scale Efficiency Score**	**RTS**	**2011Scale Efficiency Score**	**RTS**	**2010 Scale Efficiency Score**	**RTS**
DMU1	0.896497	Decreasing	0.914719	Decreasing	0.976888	Decreasing	0.920957	Decreasing	0.941789	Decreasing
DMU2	0.995336	Increasing	1	Constant	1	Constant	1	Constant	1	Constant
DMU3	0.934178	Decreasing	0.836679	Decreasing	0.931599	Decreasing	0.826848	Decreasing	0.871896	Decreasing
DMU4	1	Constant	1	Constant	0.998863	Increasing	1	Constant	1	Constant
DMU5	0.994616	Increasing	1	Constant	1	Constant	0.760054	Increasing	NA	NA
DMU6	1	Constant	1	Constant	0.998742	Increasing	1	Constant	1	Constant
DMU7	1	Constant	1	Constant	1	Constant	1	Constant	1	Constant
DMU8	0.999216	Increasing	1	Constant	1	Constant	1	Constant	1	Constant
DMU9	0.923319	Decreasing	0.910642	Decreasing	0.988694	Decreasing	0.936034	Decreasing	0.930568	Decreasing
DMU10	0.915197	Decreasing	0.915145	Decreasing	0.958866	Decreasing	0.902884	Decreasing	0.888794	Decreasing
DMU11	0.916496	Decreasing	0.910401	Decreasing	0.98636	Decreasing	0.920616	Decreasing	0.943937	Decreasing
DMU12	0.999606	Increasing	1	Constant	1	Constant	1	Constant	1	Constant
DMU13	0.992504	Increasing	1	Constant	1	Constant	1	Constant	1	Constant
DMU14	1	Constant	1	Constant	0.99857	Increasing	1	Constant	1	Constant
DMU15	0.985594	Increasing	0.70994	Increasing	NA	NA	NA	NA	NA	NA
DMU16	1	Constant	1	Constant	0.99932	Increasing	1	Constant	1	Constant
DMU17	1	Constant	1	Constant	1	Constant	1	Constant	1	Constant
DMU18	0.994207	Increasing	1	Constant	1	Constant	1	Constant	1	Constant
DMU19	0.921173	Decreasing	0.950101	Decreasing	1	Constant	0.933771	Decreasing	0.942388	Decreasing
DMU20	1	Constant	1	Constant	0.999724	Increasing	1	Constant	1	Constant
DMU21	1	Constant	1	Constant	1	Constant	1	Constant	1	Constant
DMU22	1	Constant	1	Constant	1	Constant	1	Constant	1	Constant
DMU23	1	Constant	1	Constant	1	Constant	1	Constant	1	Constant
DMU24	0.905825	Decreasing	0.913217	Decreasing	0.976669	Decreasing	0.915764	Decreasing	0.943441	Decreasing
DMU25	0.996804	Increasing	1	Constant	1	Constant	1	Constant	1	Constant
DMU26	0.996423	Increasing	1	Constant	1	Constant	1	Constant	1	Constant
DMU27	0.995431	Increasing	1	Constant	1	Constant	1	Constant	1	Constant
DMU28	0.985857	Increasing	1	Constant	1	Constant	1	Constant	1	Constant
DMU29	0.900961	Increasing	NA	NA	NA	NA	NA	NA	NA	NA
DMU30	0.909635	Decreasing	0.950185	Decreasing	0.995871	Decreasing	0.932332	Decreasing	0.918634	Decreasing
DMU31	0.954469	Decreasing	0.916148	Decreasing	0.998719	Decreasing	0.939725	Decreasing	0.873319	Decreasing
DMU32	0.896235	Decreasing	0.96754	Decreasing	1	Constant	0.943605	Decreasing	0.913577	Decreasing
DMU33	1	Constant	1	Constant	0.999223	Increasing	1	Constant	1	Constant

The second model developed was the profit maximization model with five inputs as number of employees, area of facility, COGS of sales, COGS of service, COGS of parts and four outputs revenue from sales, service and parts and total gross margin. The objective of this model was to maximize revenue with the current level of inputs. The basic DEA models CRS and VRS were run but found the discriminatory power was too low with 5 inputs and two outputs. Therefore, to obtain the best results some inputs and outputs were consolidated and reduced to make the model give better results. Although this yielded some results, to get a better discriminatory power, weight restrictions were imposed on the model using the weight restriction model and this gave desired results.

The third model used was expense minimization model and this model used had three inputs variables that were related to expenses and one output related to revenues. The objective of this model was to find how to reduce current expenses while maximizing sales. The DEA models used was the basic CRS and VRS models and these models were used to also discuss about scale efficiency.

The last model used was the asset maximization model and the objective of the model was to maximize the current assets to increase sales. The four input variables were all related assets to fixed assets, current assets and other assets and total COGS of the branch and the output was sales revenue. The DEA models used was the basic CRS and VRS models and these models were used to also discuss about scale efficiency.

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Chapter VI: Discussions

6.1 Introduction:

The objective of this chapter is to summarize and give concluding remarks about this research. The eight research questions that were set out in Chapter 4 will be addressed and discussed in this chapter from the results of using four models in chapter 5.

The research addresses the inadequacies in the current methods of measuring performance of heavy equipment dealerships and proposes a DEA based approach for measuring the efficiency of heavy equipment dealerships (one single measure of performance based on multiple inputs and outputs,). The eight research questions set out in Chapter 1, and findings from the analysis in Chapter 5 are summarized and presented below.

Since the original work on Data Envelopment Analysis(DEA) in 1978 by Charnes, Cooper and Rhodes, till today spanning a period of forty years there has been a rapid growth in studies related to DEA. DEA has become a widespread analytical tool for evaluating the relative efficiency of comparable organizations (Banker,2004). There are about 15,000 DEA related articles published in various journals (Paradi et al.,2017). DEA, is a widely researched topic and a mathematical tool for measuring efficiency or performance and has received great attention from various fields of management science. The most popular application areas are energy, industry, banking, education and health care including hospitals (Emrouznejad and Yang,2018).

The five major application areas of DEA are banking, education, transportation, health care and Agriculture and Farm(Liu,2013). Among the different industries where DEA was used, there were only 28 published research papers up until 2013 (Li,2013), in related industry like Automobiles. From the literature review on DEA carried out by Emrouznejad &Yang 2018, Liu

2013, and the author's own search in leading journals for application in heavy equipment dealerships, it was found that there was no publication of research papers in which DEA was used to measure the efficiency of heavy equipment retailing organizations.

Therefore, this research is a first of its kind that uses DEA to find efficiency and performance of heavy equipment dealerships. The objective of the research is to use DEA in performance measurement of a heavy equipment dealership in Canada that overcomes the shortcomings of the various methods that are currently used in measuring the performance. The sophisticated linear programing tool that DEA uses enables heavy equipment dealerships to benchmark and identify best practices that are not otherwise visible via other methodologies that are currently used in the heavy equipment dealerships.

This section uses various models used in the black box approach of DEA and are discussed as below.

In the current research Data Envelopment analysis is used to study the performance of a heavy equipment dealership that has a network of thirty-three branches spread from the East to the West coast of Canada. The data used for the research is the audited data from the financial statements of the company for the period 2010 to 2014. In addition to the financial data, other data needed for the research was obtained from the company's administration department. The inputs and outputs for the research was selected from the judgmental screening process of Golany and Roll (1989) with a brain storming session with experts in the dealership and the various inputs and outputs used in similar study in retail automotive applications and other retail applications such as grocery stores, supermarket, restaurant chains etc. as listed in Chapter 2 on literature review. Of the many inputs and outputs available for the research the most appropriate variables were selected as suggested in the literature based on the requirement of the model using stepwise approach of

Wagner and Shimshak (2007). Thereafter, the choice of returns to scale was determined using Kolmogrov-Smirnov test and the orientation of the model was selected based on maximization of outputs for three models and minimization of inputs for one model.

Having thus established the appropriate variables of the model the efficiency was found using the Black Box approach. In the Black Box approach in addition to the basic CCR and BCC models various other models such as multiplier restrictions, super efficiency and cross efficiency models were also used to improve the discriminating power of the analysis. Once these results were obtained the efficiency scores were validated using Bootstrap DEA.

An architecture that lists all the models that were used in the Black Box approach was drawn as a basis for reference in chapter 4 on research methodology. In the analysis four models namely production process maximization model (outputs maximized), profit maximization model (output maximized),expense minimization model(input minimization) and asset maximization model(outputs maximized).Of the four models production process maximization (PPM) model was analyzed in depth using the discriminatory models of super-efficiency, cross efficiency, using bootstrap DEA to find bias in efficiency scores, time series analysis using window analysis and Malmquist productivity index, using contextual variables to study effect on efficiency scores and detecting outliers if any in the analysis. Profit maximization model used the basic models and weight restriction models and the other two models expense minimization and asset maximization used only the basic models for brevity.

The discussions below will be based on how each of the above four models have been able to address each of the research questions.

6.2: Branch Production Process Maximization Model:

This is the first model used in the analysis with four inputs and two outputs of the DMU. The inputs used are number of employees, area of facility, total departmental expenses and COGS of the branch and the outputs used are total branch sales revenue and total gross margin. Using these inputs and outputs the following DEA applications were used in the analysis and will discuss how each of these models have been able to address each of the research question.

6.2.1: Basic Models:

The CRS model of Charnes Cooper and Rhodes (1978) was used to find the technical efficiency of the thirty-three DMUs during the period from 2010 to 2014. It was found that 6 DMUs in 2010,11 DMUs in 2011,9 DMUs in 2012,7 DMUs in 2013 and 8 DMUs in 2014 had a technical efficiency score of 1 indicating they are efficient and the others that had scores less than 1 and indicating they are inefficient. The trends of CRS and VRS efficiency change for each DMU over the five-year period is shown in the Table 6.1 and Table 6.2 with spark lines graph respectively. Similarly Figure 6.1and Figure 6.2 show the graph of CRS scores for the period 2014 and for the period 2010-2014. The Figure 6.3 and 6.4 show the graph of VRS scores for the period 2014 and for the period 2010-2014.

The number of branches that are inefficient that are with scores less than 1, 24 in 2010,

21 in 2011,22 in 2012,25 in 2013 and 25 in 2014 can be seen from Table 6.1. This means that there still exists an opportunity to increase gross margin and sales revenue with the same level of inputs of number of employees, area of facility, total expenses and total COGS of the branch. However, these results are under the assumption of constant returns to scale where it means that for every increase in inputs there is a direct increase in outputs. However, this does not happen in practical

applications such as heavy equipment dealerships as there are scale efficiencies involved in operations. Therefore, to know the causes of the overall technical inefficiency in the branches, the measure of technical efficiency is decomposed in to two components namely pure technical efficiency and scale efficiency using BCC model of Banker, Charnes and Copper (1984) that addresses variable returns to scale. The graphs in Figures 6.1 to 6.4 gives a visual representation of the CRS and VRS scores for the period 2010-2014 and for the period 2014 individually for both CRS and VRS scores.

The BCC model was used to decompose the technical efficiency into pure technical efficiency and scale efficiency that addresses the variable returns to scale. It is important to note that in contrast to technical efficiency measure, the pure technical efficiency(PTE) is devoid of scale effect. Inefficiency from pure technical efficiency score results from managerial sub-performance. In other words, PTE efficiency score also indicates the ability of the manager to convert the resources of the branch in to outputs i.e. sales revenue and improve gross margin. The BCC model was run to estimate the frontier for all branches for each year separately.

Table 6.1. Trend of CRS Scores (2010-2014)

DMU	**2014Technical Efficiency Score(CRS)**	**2013Technical Efficiency Score(CRS)**	**2012Technical Efficiency Score(CRS)**	**2011Technical Efficiency Score(CRS)**	**2010Technical Efficiency Score(CRS)**	**Trend CRS Scores PP Model**
DMU1	0.930246	0.909605	0.931374	0.892344	0.878887	
DMU2	0.945015	0.93342	0.997941	0.982162	0.974435	
DMU3	0.965043	0.965099	0.950019	0.958615	0.959912	
DMU4	0.9029	0.916736	0.938968	0.988051	0.962324	
DMU5	0.950534	0.856296	0.931752	0.760054	0.986504	
DMU6	0.951304	1	0.957465	1	0.997952	
DMU7	1	0.993479	1	1	1	
DMU8	1	0.992927	0.986958	1	0.902284	
DMU9	0.897789	0.913078	0.914275	0.900003	1	
DMU10	0.99902	1	1	1	0.979374	
DMU11	0.938974	0.997149	0.982101	0.973535	1	
DMU12	1	1	1	1	1	
DMU13	1	1	1	1	0.874374	
DMU14	0.916691	0.880852	0.9239	0.908951	0.933018	
DMU15	0.944756	0.932221	0.946444	0.970717	0.938022	
DMU16	0.941799	0.979694	0.960434	0.959842	0.8942	
DMU17	0.950098	0.915935	0.936257	0.927245	0.960531	
DMU18	0.921278	0.89202	1	1	0.971762	
DMU19	0.963665	0.968675	0.975476	0.98117	0.904202	
DMU20	1	0.974648	0.965074	0.955843	0.871228	
DMU21	0.971495	0.962607	0.908628	0.921262	1	
DMU22	0.913958	0.847937	1	1	0.934469	
DMU23	1	1	0.911565	0.891073	0.930432	
DMU24	0.944889	0.915095	0.911387	0.894267	0.906407	
DMU25	0.965188	0.948104	0.916269	0.899284	1	
DMU26	0.949302	0.914337	1	1	0.934732	
DMU27	1	0.9474	0.944022	0.910519	0.863465	
DMU28	0.966245	0.936525	0.934245	0.869369	0.900804	
DMU29	1	0.938573	1	1	0.96646	
DMU30	0.92872	1	1	1	0.964236	
DMU31	0.94526	0.98728	0.990772	0.993787	NA	
DMU32	0.998663	1	NA	NA	NA	
DMU33	0.998608	NA	NA	NA	NA	

Table 6.2. Trend of VRS Scores:

DMU	**2014Pure Technical Efficiency Score(VRS)**	**2013Pure Technical Efficiency Score(VRS)**	**2012 Pure Technical Efficiency Score(VRS)**	**2011 Pure Technical Efficiency Score(VRS)**	**2010Pure Technical Efficiency Score(VRS)**	**Trend of VRS Scores PP Model**
DMU1	0.949093	0.936882	0.974071	0.970136	0.987462	
DMU2	0.980016	0.970611	1	1	0.976322	
DMU3	0.974518	0.965116	0.957558	0.970153	0.991311	
DMU4	0.906721	0.93337	0.949942	1	1	
DMU5	0.950544	0.875524	0.948191	1	0.986519	
DMU6	0.951843	1	0.966077	1	1	
DMU7	1	1	1	1	1	
DMU8	1	1	1	1	0.990579	
DMU9	0.924842	0.930347	0.927724	0.94214	1	
DMU10	1	1	1	1	1	
DMU11	0.965348	1	0.989367	0.986068	1	
DMU12	1	1	1	1	1	
DMU13	1	1	1	1	0.875815	
DMU14	0.917547	0.883851	0.930313	0.911928	0.943952	
DMU15	0.973827	1	0.947707	0.974064	0.950182	
DMU16	0.9424	0.982196	0.96062	0.960442	0.894508	
DMU17	0.953294	0.920335	0.949247	0.928613	1	
DMU18	0.926717	0.905725	1	1	0.981346	
DMU19	1	1	0.977769	0.983972	0.91851	
DMU20	1	0.975476	0.966566	0.956838	0.879032	
DMU21	0.973795	0.964022	0.917062	0.930282	1	
DMU22	0.916848	0.857559	1	1	1	
DMU23	1	1	0.920805	0.917336	0.940305	
DMU24	0.958991	0.921516	0.9151	0.902041	0.929111	
DMU25	0.968978	0.954077	0.953981	0.932615	1	
DMU26	0.953865	0.945376	1	1	1	
DMU27	1	1	1	1	0.92533	
DMU28	1	1	0.944973	0.898021	0.910909	
DMU29	1	1	1	1	1	
DMU30	0.979714	1	1	1	0.966613	
DMU31	0.948497	1	0.996238	1		
DMU32	1	1	0	0	0	
DMU33	1	0	0	0	0	

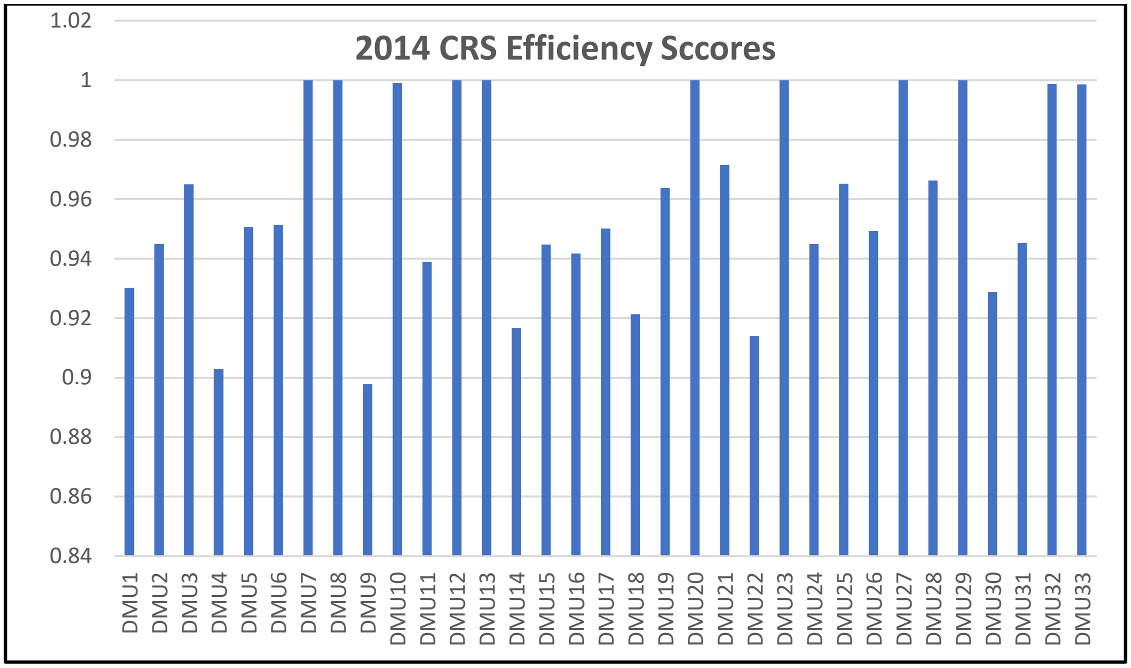

Figure 6.1. CRS Efficiency scores for 2014

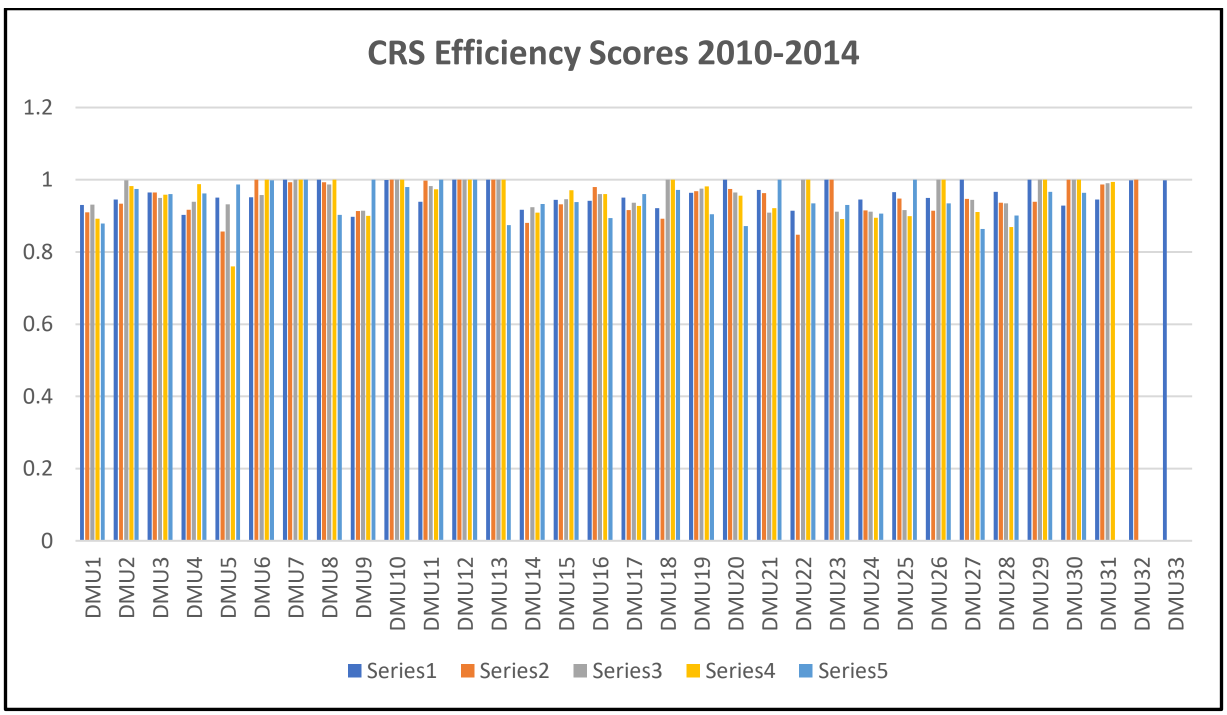

Figure 6.2. CRS Efficiency Scores 2010-2014

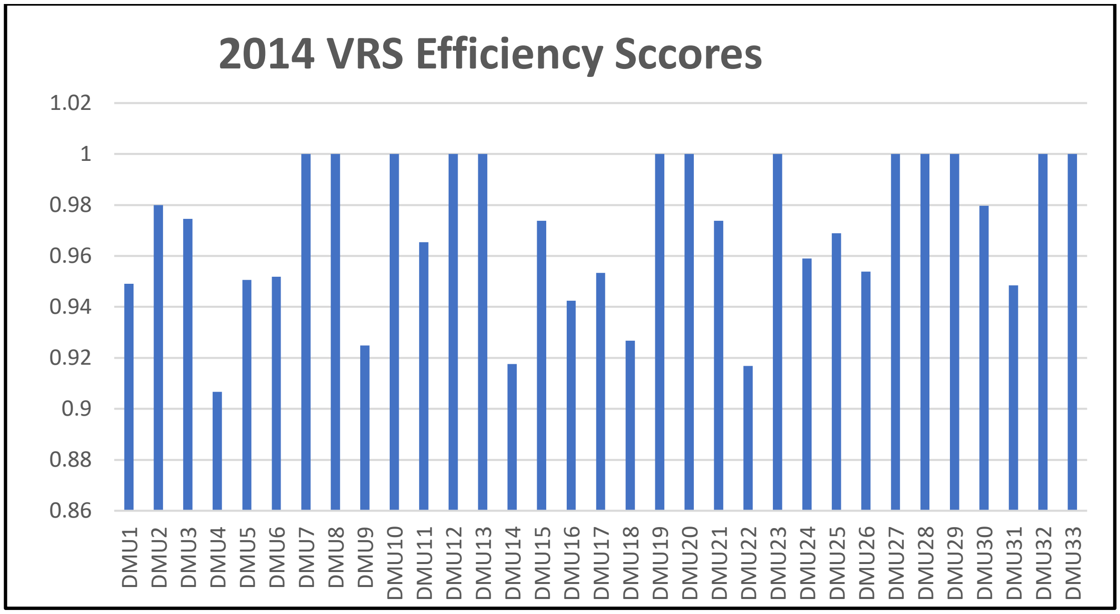

Figure 6.3.VRS Efficiency scores for 2014

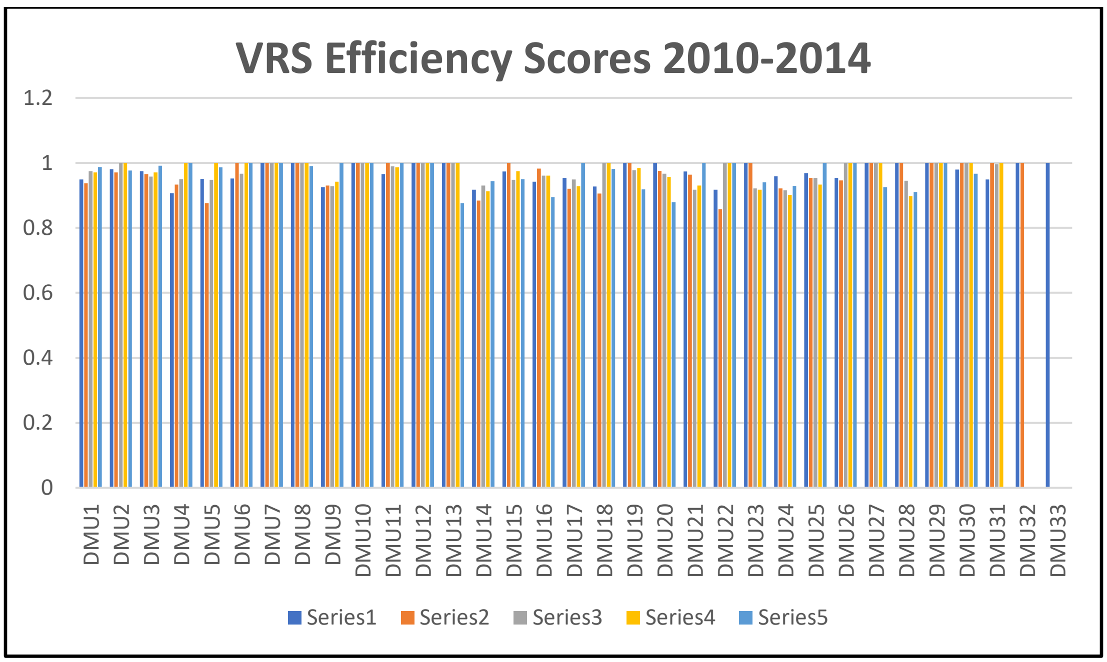

Figure 6.4. VRS Efficiency Scores 2010-2014

It was found using the BCC model that there are 13 efficient DMUs in 2010,16 in 2011,12 in 2012,16 in 2013 and 13 in 2014.In other words the number of branches that were found inefficient during the period 2010-2014 using BCC model is 17,15,19,16 and 20 respectively. In other words, these branches can become efficient.

There are 6,11,9,7 and 8 DMUs in 2010 to 2014 that are scale efficient with a score of 1. This indicates that all these branches are operating at the most productive scale size(MPSS). The scale inefficiency variations, increasing returns to scale, decreasing returns to scale or constant returns to scale is shown in Table 6.3. It is found that many DMUs are operating very close to scale efficiency and there are a few that can be made scale efficient. Therefore, using BCC model, the technical efficiency can be decomposed to pure technical efficiency and scale efficiency.

While decomposing the technical efficiency into pure technical efficiency and scale efficiency it is found that there are 5,13,10,9 and 8 DMUs with increasing scale efficiency (when inputs are increased by **m**, output increases by more than **m**) in the period 2010-2014 respectively and 20,12,13,11 and 16 DMUs with decreasing scale efficiency (when inputs are increased by **m**, then output increases by less than **m**). This indicates that there is an opportunity for the management to adjust the operations to reduce costs and maximize profits. When a branch is operating under increasing scale efficiency it means they are operating below their optimal scale size and therefore they can increase the scale of operations by expanding. When a branch is operating under decreasing scale efficiency this means that they are operating above the optimal size and therefore downsizing is the most strategic option. This is as per the theory of the firms where one of the objectives of the firm is to operate at MPSS (most productive scale size) i.e., under CRS in order to minimize costs and maximize revenue.

Table 6.3. Scale Efficiency Variations 2010-2014

DMU	2014Scale Efficiency Score	2013Scale Efficiency Score	2012 Scale Efficiency Score	2011 Scale Efficiency Score	2010 Scale Efficiency Score	Trend of ScaleEfficiency
DMU1	0.980142	0.970885	0.956166	0.919814	0.890047	
DMU2	0.964285	0.961683	0.997941	0.982162	0.998068	
DMU3	0.990278	0.999983	0.992127	0.988107	0.968325	
DMU4	0.995785	0.982179	0.988447	0.988051	0.962324	
DMU5	0.99999	0.978038	0.982663	0.760054	0.999985	
DMU6	0.999435	1	0.991085	1	0.997952	
DMU7	1	0.993479	1	1	1	
DMU8	1	0.992927	0.986958	1	0.910865	
DMU9	0.970748	0.981439	0.985503	0.955275	1	
DMU10	0.99902	1	1	1	0.979374	
DMU11	0.972679	0.997149	0.992656	0.98729	1	
DMU12	1	1	1	1	1	
DMU13	1	1	1	1	0.998355	
DMU14	0.999067	0.996607	0.993106	0.996736	0.988417	
DMU15	0.970147	0.932221	0.998667	0.996564	0.987203	
DMU16	0.999362	0.997453	0.999807	0.999376	0.999656	
DMU17	0.996648	0.995219	0.986315	0.998527	0.960531	
DMU18	0.994131	0.984868	1	1	0.990234	
DMU19	0.963665	0.968675	0.997654	0.997153	0.984422	
DMU20	1	0.999151	0.998457	0.99896	0.991121	
DMU21	0.997638	0.998533	0.990804	0.990304	1	
DMU22	0.996848	0.98878	1	1	0.934469	
DMU23	1	1	0.989965	0.97137	0.9895	
DMU24	0.985295	0.993033	0.995942	0.991382	0.975564	
DMU25	0.996088	0.99374	0.960469	0.96426	1	
DMU26	0.995217	0.967167	1	1	0.934732	
DMU27	1	0.9474	0.944022	0.910519	0.933142	
DMU28	0.966245	0.936525	0.988648	0.968094	0.988907	
DMU29	1	0.938573	1	1	0.96646	
DMU30	0.94795	1	1	1	0.997541	
DMU31	0.996587	0.98728	0.994513	0.993787		
DMU32	0.998663	1	NA	NA	NA	
DMU33	0.998608	NA	NA	NA	NA	

In the short run firms may operate in the zone of IRS or DRS but in the long run the branches will move towards CRS by becoming larger or smaller to survive in the competitive market. This process may involve change in the operating strategy of the firm by scaling up or scaling down in size. The management may use this information to determine the size of the branch at a given time in business cycle of the industry/firm.

In summary DEA has identified a group of optimally performing dealership branches that are defined as efficient by assigning them a score of "1". These efficient branches are then used to create an efficiency frontier or data envelope against which all other dealership branches are compared. In other words, dealerships that require relatively more weighted inputs to produce more weighted outputs than do dealerships on the efficiency frontier are considered technically inefficient.

It is found that as per theory when using BCC model there are more efficient firms as compared to using CCR model. This happens as the stringent condition of constant return to scale in CCR model is relaxed and a variable return to scale is used in BCC model.

In Figures 6.5, graph using data for the year 2014, production frontier is drawn for both CRS and VRS models using the input Total COGS and output as Gross Margin. The black line being the CRS frontier and the dotted line is the VRS Frontier. The inefficient DMUs are the ones that are points in lavender color. We can also note from the graph the segments of the BCC frontier that are either increasing or decreasing. There are some DMUs, DMU 12 at a point where CRS and VRS are tangent to each other indicating that these DMUs are both CCR and BCC efficient and that these DMUs 's returns are constant. Hence such a DMU would be considered having the optimal scale size.

The above analysis using both CCR and BCC models have answered the first research question *"To construct a model to measure the performance and relative efficiency of a heavy equipment dealer in Canada treating it as a Black box without considering its internal structure.*"

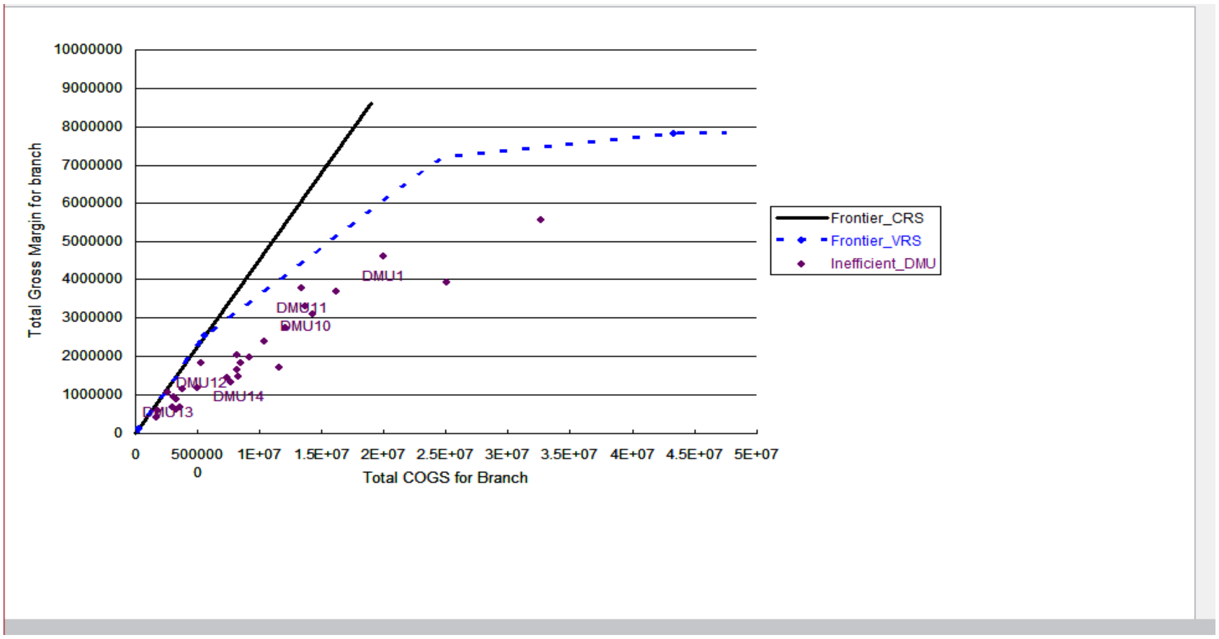

Figure 6.5. CRS and VRS Frontier for 2014

6.2.2 Super-Efficiency Models:

It was found that using the standard CCR and BCC models, many DMUs are shown as efficient with a score of 1. However, it is important to know which DMU out of these efficient DMUs is the most efficient DMU. Such an identification of DMU will help in determining the best performing DMU and also in benchmarking. This has been achieved in the research using the Super-Efficiency model of Andersen and Peterson (1993). In other words, super-efficiency models can identify super-efficient DMUs.

In the research the “Super efficiency model” was run and the scores were found under both CRS and VRS and tabulated in table 5.11and 5.12 along with scores from CRS and VRS models. The super-efficiency model was run for all the data for the period 2010- 2014 under both constant returns to scale and variable returns to scale.

On analyzing the scores for the data for 2014 under output oriented constant returns to scale, it is found that there are notable differences between the scores of the DMUs that were efficient under constant returns to scale. The branches of the dealership DMUs7,8,12,13,20,23,27 and 29 had a score of 1 under the CRS model but under super-efficiency model these DMUs have an efficiency score greater than 1. It is to be noted that the inefficiency scores of all the other branches remained the same under the super-efficiency model. The scores that are greater than one for these DMUs can now differentiate these branches based on the scores that are greater than one and these DMUs can be ranked. Of these seven DMUs the scores can be sorted from highest to lowest as 2.01309(DMU29),1.037421(DMU27),1.315397(DMU23),2.036087(DMU20),1.046769 (DMU13), 1.166419 (DMU12), 1.332947(DMU8) and 1.072082(DMU7). This implies that branch 20 is the most super-efficient branch among the eight branches and gets the first rank and called global leader of all the branches and the other branches follow ranks based on score. Similar interpretations can be done for other periods.

Similarly, under VRS output oriented super efficiency model, for the year 2014, there are 13 DMUs that are efficient and are having scores more than one under VRS. These scores are 1.051372(DMU33),1.361682(DMU32),1.249168(DMU10),1.436189(DMU19)1.336403(DMU2 3),2.182778(DMU20),1.179321(DMU27),1.216633(DMU12),1.048585(DMU13), and 1.097122 (DMU7). Of these three DMUs 8,28 and 29 were infeasible DMU 20 has the highest score under VRS model and ranked first and all other branches can be ranked accordingly based on the model. The inefficiency scores of other DMUs remain unaffected or remain the same in super-efficiency evaluation. Similar interpretations can be done for other periods.

The Super efficiency model provides more insight to the equipment dealership management with respect to benchmarking of their facilities. Therefore, many branches cannot claim that they are showing best performance as evaluated under CRS and VRS models. Therefore, Super-efficient models provide further improvements in performance evaluations and setting better targets for the managers.

It is found that under VRS super efficiency evaluation for the year 2014, DMU,28 and 29 had a score that was infeasible. In the words the LP problem for this DMU under super efficiency was unbounded. However, under the regular VRS model the efficiency score of these DMUs was 1. The infeasibility of DMUs means that the outputs of these DMU have no reference point for output variations. Graphically the output is such that in the frontier the output point is lying on a line parallel to the output axis. Therefore, FPA approach was used to resolve the infeasibility and the scores obtained for these DMUs was higher than one. The problem of infeasibility is an active area of research in DEA.

6.2.3 Cross Efficiency Models:

Efficiency is a simple ratio of weighted sum of outputs over the weighted sum of inputs and in order to maximize efficiency score, DEA selects the best set of weights for each DMU based on a set of constraints. These constraints are that no weight is negative, and the resulting ratio is not greater than one and we can term this as simple efficiency.

When DEA provides the best set of weights for a particular DMU and that the same set of weights is used for all other DMUs in the group, the resulting score is called the Cross-efficiency score of each other DMUs as viewed by the original DMU.

The Cross-efficiency score was calculated in the research for the data of all the five years from 2010 to 2014 and is given in table 5.14. The cross-efficiency score for each branch was calculated for each year. The score indicates how each branch rates itself with its best selected weights and also calculates the averaged peer appraisal by each DMU or branch. The DMUs that are termed efficient in the CCR model with score 1 have a score less than 1 in the cross-efficiency model when evaluated by peers. This indicates that when the branches are evaluated by other branches that are peers the efficiency further evaluated stringently and therefore the efficiency score drops down further.

The Maverick Index was calculated for each DMU for the years 2010-2014.When a DMU is maverick it indicates it operates far from other DMUs. The maverick index was scaled up by ten for easy comparison.

In the year 2014 DMU 29 and DMU 28 are operating with high maverick index of 4.91 and 3.87 respectively as compared to others that have a maverick index ranging from 0.34(DMU8) to 3.17(DMU9). Similarly, the year 2013 has a maverick index of 3.11 for DMU23 and the maverick index ranges from 0.50(DMU6) to 2.64(DMU15). Therefore, using the maverick index, one can identify the DMUs that has high index and also rank the DMUs (Doyle and Green,1994).

DMUs that has the greatest relative increment when shifting from peer efficiency to simple efficiency are called Mavericks. If a DMU has a higher Maverick index it means that it operates far from the rest of the DMUs. If a DMU has a high simple efficiency and a low peer efficiency, then it has a high maverick indicator indicating that it is a Maverick DMU. Therefore, Maverick index helps us to identify those DMUs that are operating differently and those that are all round performers.

The cross efficiency average scores represent a true peer assessment as performance of each branch is assessed using the weights of other branches thereby representing all-round performance of branches. This cross efficiency is a useful method of post DEA analysis validation of results (Hollingsworth and Wildman,2002).

6.2.4 Bootstrap DEA:

The observed input and output data in DEA may be subject to various measurement errors. Therefore, one of the criticisms of DEA is that it assumes that the distance between observation and frontier reflects inefficiency. However, this is not true as the distance of the DMU from the frontier consists of both efficiency and noise. Simar and Wilson (1998,2000) proposed a bootstrap method that enables construction of a confidence interval for DEA efficiency scores. The Simar and Wilson method of 1998 is used in the research where a simulation of a true sampling distribution is employed by using outputs from DEA. This way a new data set is generated, and DEA scores are re-estimated using this data set. The bootstrap procedure is to collect the efficiency scores obtained from actual data and then randomly sample with replacement from this collection of data to construct a pseudo -data of efficiency scores for the branches. These artificial data are associated with the efficiency scores for another round of DEA. This procedure is repeated 200 times, it generates a large number of efficiency scores for each branch. In bootstrap the mean and variance of each of the empirical distribution of efficiency score is analyzed. It was found that the average bias for the five year period ranged from a minimum of 0.73% to a maximum of 5.28% for CRS scores and from a minimum of 0.72% to a maximum of 3.93% for VRS scores. The error in the CRS and VRS scores were corrected by using the bias.

The above three analysis namely super-efficiency, cross efficiency and bootstrap DEA are post DEA analysis. Super efficiency helps in ranking all the efficient units that have a score of 1, cross efficiency validates the score by peer evaluation in addition to self-evaluation and the bootstrap DEA finds the bias in the data and thereby helps in correcting the scores. Therefore, all these three analyses not only answer the first research question of finding the efficiency scores but also helps in post analysis of the scores. The production process model is maximizing the outputs of sales revenue and gross margins and therefore answers the third question of maximizing efficiency.

6.2.5: Envelopment Models:

The envelopment model of DEA as described in Chapter 3 on theoretical aspects of DEA, is the dual of the primal or the multiplier model.This was used to find the proportionate movement and slack improvement value and consequently find the Target efficient value. In the analysis in section 5.1.10, it was found how DMU 1, can become efficient by making four adjustments to its score and operate on the efficient frontier. The, first type of adjustment is known as radial adjustment while the second, third and fourth adjustments are known as slack adjustments. Such targets that need to be set up are given by solving the envelopment models. The solution of the envelopment model has given such targets for all the inefficient units to become efficient units by precisely telling what has to be done. One of the shortcomings of DEA is that it does not tell the process of using these targets to become efficient(AvKiran,2006).

The solution of the envelopment model of the production process model answers question 5 *"How to improve the efficiency of inefficient units so that they also become efficient"* by setting targets with the help of slacks.

6.2.6: Network Data Envelopment Analysis(NDEA):

The Network DEA (NDEA) approach has been used in the research in the production process model using two methods. The basic assumption in using Kao's 2009 parallel model is that the sum of inputs of three sub DMUs (equipment, parts and service) is equal to the total inputs of the whole unit (branch) and the sum of the outputs of sub DMU (equipment, parts and service) is equal to the output of the whole branch.

In the first method the structure is assumed to be operating in parallel and the efficiency has been calculated. While using CCR model it was found that some DMUs are efficient whereas using NDEA with parallel structure, no DMU was found to be efficient. However, some sub DMUs were found to be efficient under NDEA. The inefficiency of a DMU and sub DMU were calculated in terms of slack.

In the second method, the structure of the DMU is considered as a two-stage process. The first stage is the input stage, the second stage is the intermediate stage and the third stage is the output stage. The inputs are the same as in production process model namely number of employees, area of facility, total departmental expenses and total COGS of the branch. These inputs are used to create the intermediate product sales and the second stage is generation of profit margin. The two sub processes are first stage use of resources (inputs) and the intermediate product is generation of sales revenue. The overall efficiency of the system is the product of resources (inputs) and the efficiency of sales generation. It was found in this method that overall system efficiency is much lower than efficiency of intermediate stage and the second stage. The intermediate stage efficiency is high, however the efficiency of generating margin is low and therefore the product of these two efficiencies is bringing down the overall efficiency.

This analysis has helped in answering two questions namely question 2, "*Measure the performance and relative efficiency of heavy equipment dealer considering the internal structure of each of its branches using Network Data Envelopment Analysis" and Q6 "Compare the efficiency between the two approaches of Black Box and Network DEA approach"*.

6.2.7: Contextual Variables:

The environmental variables also called the contextual variables may influence the efficiency scores of the equipment branches. The contextual variables chosen were:1) Federal capital expenditure in millions of dollars,2) Population of the city where branch is located, 3) Competition index,4) Number of competition stores and 5) Square of competition stores. The data for the federal expenditure and population of the city were collected from Statistics Canada whereas the number of competition stores and market share of the equipment manufacturer was collected from trade journals. The Herfindahl-Hirschman Index(HHI) competition index was calculated using the market share and is an indication of the competition index.

The two methods used to find the effect of contextual variables on the efficiency score were the Ordinary Least Square (OLS) method and the Tobit regression method.

Under both these methods the efficiency scores were regressed against the efficiency scores found by both VRS and CRS models and corrected for bias from bootstrapped DEA. It was found that these contextual variables had varying degree of influence on the efficiency scores. It was found that there was a variation in the behavior of the environmental variables. Under OLS, in case of VRS scores the effect of capital expenditure showed a statistically significant result on efficiency scores whereas all other factors did not show a statistically significant result. In case of VRS scores competition index and number of competition stores had a negative coefficient whereas other three had positive coefficient. In case of CRS scores all had a positive coefficient except the square of competition stores. However, all the variables had a p-value higher than 0.05 indicating that the effect is not statistically significant?

Under Tobit regression with both CRS and VRS scores, the coefficients of Capital expenditure, population and square number of competition stores have negative coefficients meaning that the efficiency drops when there is increase in capital expenditure, population and the square of the number of competition stores. The other two variables competition index and number of competition stores have a positive sign indicating the effect of environmental variables on efficiency scores. However, the p-value of competition index and square of number of competition stores using CRS scores is less than 0.05 indicating they are statistically significant and similarly using VRS scores it was found that number of competition stores and square of number of competition stores had a p-value less than 0.05 indicating a statistically significant result.

Comparing both OLS and Tobit regression, it is found that Tobit regression gives a statistically significant result as compared to OLS regression.

The second stage analysis regressing efficiency scores with environmental variables answers the question "*To find the effect of environmental (contextual) variables on the efficiency score"*.

6.2.8: Change in Efficiency over time (PPM- Model):

The research investigated if there are variations in efficiency scores over time for the period 2010-2014 using two methods called Window analysis and Malmquist Index. In Window analysis the scores along the row indicates the trend of the efficiency scores of the DMUs and the column view indicates the stability of the efficiency score of the DMUs.

Malmquist Productivity Index(MPI) method enabled to find the frontier shift that occurs due to technological change between two periods. MI also helped to calculate the catch up between two periods and finally Malmquist index is calculated by the product of technical change and efficiency change.

A Window analysis and MPI of the efficiency scores obtained by the Production Process model (Model 1) that has four inputs number of employees, area of facility, total expenses for the branch, total COGS for the branch and two outputs total sales for the branch and total gross margin for the branch is done with the data for the years 2010-2014.

On analyzing the CRS and VRS efficiency scores along the column for the three windows it is found that the scores are fluctuating and then somewhat stable in other windows. This variation for each branch indicates that the efficiency scores fluctuate for the same inputs and outputs for different periods and the source for this fluctuation needs to be investigated. Similarly, it was found that of the total DMUs (297), 27.27% of the DMUs exhibited decreasing returns to scale,20.5% DMUs exhibited constant returns to scale and 45.45% indicated increasing returns to scale. In other words, it indicates that 20.5% of the branches operate at the most productive scale size. This indicates that production scale is a source of inefficiency in heavy equipment retailing organization.

. The Malmquist productivity index represents the overall efficiency measure and can be decomposed into two mutually exclusive components. One measuring the change in technical efficiency (catch up effect) and the other measuring the change in technology (innovation). It is found from the analysis that there is a change in efficiency due to both catches up effect and change in technology.

This analysis of variation in efficiency over time answers the question *“To study the variations in efficiency scores over time using Window Analysis and Malmquist Index”.*

6.2.9: Detection of Outliers in PPM- Model:

From among the efficiency scores of DMU, there could be some DMUs that may have a very high-efficiency score in relation to the other DMUs and are termed as outliers. Therefore, one of the objectives was to find if there are any outliers. A literature review of various methods used to find out outliers in DEA was made and details are given in Chapter 2. Of these methods two methods the super -efficiency approach of Banker and Chang (2006) and the second scalar method of Tran, Shively, and Preckel (2008) were used as they are popularly used in literature.

It was found using the Banker and Chang method that DMU20 was found to be an outlier whereas, using Tran's method it was found that DMU23 and DMU20 are outliers. Both methods identify DMU20 as an outlier and DMU 23 is identified as an outlier only by Tran's method. Therefore, DMU20 can be excluded from other analyses and further investigated for causes in such high efficiency. This analysis answers the question *"To find out if there are any DMUs that are outliers"*.

In conclusion, Production Process Model has been able to answer all the research questions as found above. However, three more models were also used to find the efficiency from a different perspective. The three other models used were to see efficiency from the perspective of profit maximization, asset utilization and minimization of expenses. However, in all these models, analyses were restricted to only the basic models except for profit maximization where the extended DEA model weight restrictions were used.

6.2.10: Reliability and Validity of DEA Models:

Researchers who use quantitative research or use a positivist paradigm use experimental techniques and quantitative methods to verify hypothesis (Guba and Lincoln,1994). The positivist approach to research focusses on patterns, generalizations, methods, procedures, measurements and cause and effect relationships (Lincoln et al.,2011, Denscombe,2010). Reliability and Validity are the classic criteria of an excellent quality research (Eriksson and Kovalainen,2012). In a quantitative study, validity is defined as the extent to which a concept is accurately measured, and reliability relates to the consistency of a measure (Heale and Twycross,2015). Reliability is the consistency of measurement over time or stability of measurement over a variety of conditions(Drost,2011). As per Drost 2011, and Heale and Twycross (2015) the following figure indicates various reliability and validity measures in a research.

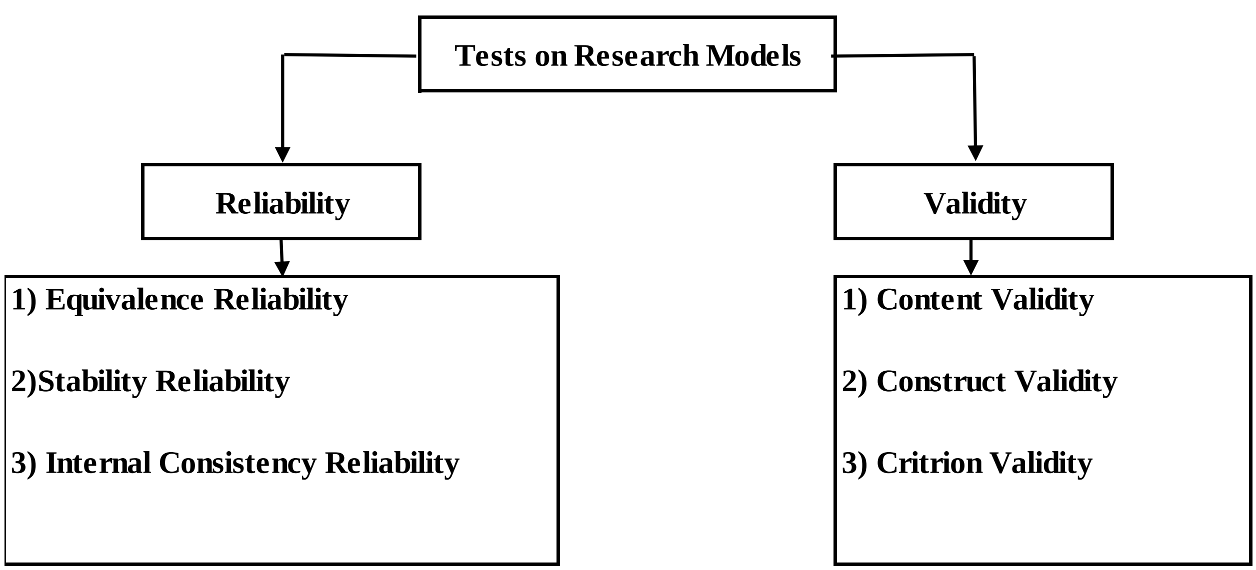

Figure 6.6. Types of Reliability and Validity

Correlation coefficient termed reliability coefficient is a measure of association and is often used to estimate reliability(Drost,2011). The reliability is the correlation between two or more variables that measure the same thing. There are three types of reliability as shown in the figure above (Drost,2011). The equivalency reliability relates to the consistency of measurement amongst multiple users, stability reliability explains the consistency of an instrument with repeated testing or measurement over time and internal consistency refers to consistency within the instrument and how well it measures (Drost 2011, Heale and Twycross,2015).

Similarly, validity is concerned with the meaningfulness of research components (Drost,2011). There are three types of Validity: content validity, construct validity and criterion-related validity (Parkin and Hollingsworth,1997: Heale and Twycross,2015). The content validity will answer the question "*does the DEA score represent the concept of efficiency*". The construct validity will answer the question "*does DEA score behave as the concepts underlying efficiency would suggest?"*. The criterion-related validity will answer the question – *"does the DEA score concur with or predict other measures of efficiency?" (*Parkin and Hollingsworth,1997*)*.

Robustness(Stability) of DEA Results:

Robustness or stability or sensitivity analysis of DEA scores has taken varied approaches in DEA literature (Cooper et al.,2007). DEA model may give misleading results either due to misspecification of the model due to the omission of an important factor in the model or due to inadequate data or misspecification of returns to scale (Pedraja et al.,1999). There is a range of issues related to the application of DEA procedure with respect to the homogeneity of units under assessment, selection of inputs and outputs and weights to these factors (Dyson et al.,2001). It is important to study the stability(robustness) of the efficient frontier due to perturbations in data(AvKiran,2007). Some of the stability tests include i) deleting variables and its impact on efficient frontier membership ii) number of variables in relation to sample size and dimensionality iii) deleting variables and its impact on rankings and iv) testing sensitivity scores using super efficiency models (AvKiran,2007). Simar and Wilson (1998) also proposed Bootstrapped DEA models to test the robustness of the DEA scores. In this research Bootstrapped DEA will be used to test the robustness of DEA scores and the model.

The table 6.4 and 6.6 presents the summary results of the original DEA CCR scores and bootstrapped DEA CCR scores and original DEA BCC and bootstrapped DEA BCC scores for the period 2010-2014. Table 6.5 and Table 6.6 presents the Spearman's rank correlation coefficient for CCR and BCC scores respectively at significance level 5%.

Table 6.4. Original DEA CCR and Bootstrap DEA CCR scores

	Original DEA CRS Scores			**Bootstrap DEA CRS Scores**				**Confidence Interval 5%**	
Year	**Mean**	**S.Dev**	**Min**	**Mean**	**Bias**	**S.Dev**	**Min**	**LB**	**UB**
2010	**88.71**	**4.18**	**86.34**	**91.94**	**2.71**	**1.97**	**85.22**	**89.48**	**96.25**
2011	**89.5**	**4.13**	**86.34**	**92.62**	**2.65**	**2.41**	**88.21**	**90.16**	**98.93**
2012	**90.34**	**4.09**	**90.86**	**94.22**	**1.95**	**1.48**	**90.51**	**92.42**	**97.23**
2013	**92.18**	**2.98**	**84.79**	**92.84**	**2.21**	**1.75**	**89.91**	**90.81**	**97.01**
2014	**96.06**	**5.6**	**89.77**	**93.99**	**2.07**	**1.52**	**89.35**	**92.08**	**97.3**
Average	**91.35**	**4.19**	**87.62**	**93.12**	**2.31**	**1.82**	**88.64**	**90.99**	**97.34**

Table 6.5. Spearman Rank Correlation Coefficient of CCR scores

YEAR	CRSBootstrapDEA-2010	CRSBootstrapDEA-2011	CRSBootstrapDEA-2012	CRSBootstrapDEA-2013	CRSBootstrapDEA-2014
DEA2010	0.986**				
DEA2011		0.991**			
DEA2012			0.988**		
DEA2013				0.977**	
DEA2014					0.961**

** Significance at 5%

Table 6.6. Spearman Rank Correlation Coefficient of BCC scores

YEAR	VRSBootstrapDEA-2010	VRSBootstrapDEA-2011	VRSBootstrapDEA-2012	VRSBootstrapDEA-2013	VRSBootstrapDEA-2014
DEA2010	0.995**				
DEA2011		0.989**			
DEA2012			0.993**		
DEA2013				0.997**	
DEA2014					0.979**

Table 6.7. Original DEA BCC and Bootstrap DEA BCC scores

	Original DEA VRS Scores			Bootstrap DEA VRS Scores				Confidence Interval	
Year	Mean	S.Dev	Min	Mean	Bias	S.Dev	Min	LB	UB
2010	90.77	4.26	87.58	94.65	2.16	2.25	87.09	92.61	1.00
2011	91.41	4.14	87.58	95.23	2.07	2.1	89.34	93.27	1.00
2012	91.19	4.12	89.8	95.17	1.90	1.64	91.79	93.38	98.84
2013	93.70	3.01	85.75	94.14	2.49	2.47	89.53	91.78	1.00
2014	97.02	5.1	90.67	94.9	2.12	1.78	91.54	92.9	98.86
Average	92.81	4.126	88.27	94.81	2.14	2.04	89.85	92.78	99.54

There are four factors that distinguish the above empirical results. Firstly, the average estimate of the bootstrap CCR efficiency score was 93.12% as compared to the average original CCR score of 91.35%. Similarly, the average estimate of the BCC score is 94.81% as compared to the original DEA BCC score of 92.818%. Secondly, the average minimum value of the original DEA CCR score is 87.62% and BCC score is 88.27%. After using the bootstrap method and adjusting for bias the average minimum bootstrap efficiency for CCR and BBC score is 88.64% and 89.85% respectively. Thirdly, the average bias which is the difference between the original DEA efficiency sore and bootstrap efficiency score for both CCR and BCC score is 2.31% and 2.14%, both in the range of 2%. Fourthly, the important point to be noted is that the average DEA efficiency score for each branch is included in the 95% confidence interval of the bootstrap efficiency score. This emphasizes the importance of confidence interval for measuring the efficiency score of each branch.

Parkin and Hollingsworth (1997) used the Spearman Rank Correlation coefficient to compare the results of the models over a period. The results of Spearman Rank correlation coefficient for the CCR and BCC scores for the period 2010-2014 are shown in table 6.5 and 6.6 above. This helps to know the length of time an inefficient branch has remained in that state. It is found from the table that the rank correlation of efficiency scores between each pair of yearly observations is in the range of 0.96 to 0.99. This is a large significantly positive value. The results in the above two tables indicate that there is no significant difference between the original DEA efficiency score and the bootstrap efficiency score. This also indicates that the original DEA efficiency estimates are robust and consistent.

In this research the above analysis also addresses *equivalence reliability* (verifying by relating two sets of test scores to one another to estimate the extent of relationship or association), tested by finding the Spearman correlation coefficient between original DEA scores and bootstrap DEA scores.

In this research, the internal consistency (the degree to which tests or procedures evaluate the same characteristic) is tested by using DEA CCR/BCC models for the period 2010-2014 that measures consistently the technical efficiency, pure technical efficiency, and scale efficiency.

Validity Tests:

As described above positivism as a research methodology involves quantitative analysis of various types and research conclusions based on certain mathematical process (Yee and Khin,2010). As per Parkin and Hollingsworth,1997, internal validity addresses the question if the methods alter the results.

In the present research, internal validity is tested by comparing the results that are obtained using a different selection of inputs and outputs using both outputs oriented CCR and BCC models.

Table 6.8. Models used to test internal validity

	Factors	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9	Model 10	Model 11
Inputs	Total Cost of Goods Sold(COGS)	*		*	*	*	*	*	*	*	*	*
	Number of Staff	*	*		*	*	*	*		*		
	Total Area of Facility	*	*	*		*	*	*			*	
	Total Department Expenses	*	*		*		*	*	*			
Outputs	Total Branch Sales	*	*	*	*	*		*	*	*	*	*
	Total Gross Magin of the Branch	*	*	*	*	*	*		*	*	*	

The production process model is taken as the base model and ten other models were tested by dropping one input at a time, one output at a time and by various combination of inputs and outputs to establish internal validity. The CCR and BCC efficiency scores along with scale efficiency are given in table 6.9.

It is found from the table 6.9 in model 2 when the input Cost of goods sold was dropped, the drop in CCR and BCC efficiency score was significant as compared to the base production process model with four inputs and two outputs. The drop in CCR and BCC scores were 30.63% and 23.57% respectively. This indicates that COGS is a very important input that can significantly alter the shape of the frontier. Similarly dropping total branch sales as output in model 6, the drop in CCR and BCC scores were 18.9% and 13.23%. This is also a significant drop in efficiency score as compared to the basic production process model. Similarly, in the model just retaining one input as COGS and one output as total branch sales it was found that

CCR and BCC scores dropped by 10.78% and 6.56% indicating again that the efficiency scores vary with the selection of inputs and outputs.

One of the limitations of DEA is that it does not indicate how to choose the inputs and outputs for the analysis. This is one of the pitfalls of including factors discriminately (Dyson et al.,2001).

Parkin and Hollingsworth (1997) adapted Spearman's rank order correlations to determine the stability of efficiency score estimates over time. The Spearman rank order correlation coefficient was calculated between each year for CCR scores and the results are presented in the table 6.10. This table indicates that the rank correlation between each pair of years is positive and statistically significant. It is found from the table 6.10 that the Spearman rank correlation coefficient is high and statistically significant between adjacent years.

Table 6.9.CCR and BCC scores of various models

Efficiency Scores	Model 1 All Inputs and Outputs	Model 2 Dropped COGS	Model 3 Dropped Total Staff	Model 4 Dropped Area of Facility	Model 5 Dropped Expenses	Model 6 Dropped Total Branch Sales
CCR Scores	0.9606497	0.66639	0.94771	0.95047	0.94612	0.7791
BCC Scores	0.9702242	0.74152	0.96131	0.96593	0.96423	0.84184
Scale Efficiency Scores	0.990137	0.89202	0.98583	0.98411	0.98131	0.92778

Mean Efficiency Scores	Model 7 Dropped Total Gross Margin of the Branch	Model 8 Dropped Total Staff and Area of Facility	Model 9 Dropped Area and Expenses	Model 10 Dropped Expenses and Staff	Model 11Dropped Area of Facility,Number of Staff,Total Expenses and Gross Margin	
CCR Scores	0.96064	0.91864	0.92454	0.92454	0.85704	
BCC Scores	0.97022	0.9599	0.9501	0.9501	0.90655	
Scale Efficiency Scores	0.99104	0.96129	0.97313	0.97313	0.94675	

Table 6.10. Spearman Rank Correlation Coefficient CCR Scores

Year	2010	2011	2012	2013	2014
2010	1	0.228	0.023	0.061	0.111
2011		1	0.757(0.000)	0.406	0.108
2012			1	0.693(0.000)	0.218
2013				1	.502(0.017)

but the correlation coefficient drops as the time increases. In other words, it implies that the change in the relative performance of the branches between each pair of years is stable.

Like any other technique, DEA has its own limitations. DEA is computationally intensive (Agha Ali,1990) as each of the models proposed for performing DEA analysis requires the solution of **n** linear programs where **n** is the number of DMUs. Cooper, Seiford &Tone,2007 suggest trying different models if you cannot identify the characteristics of the production frontier in the preliminary survey. It may be risky to rely only on one model. If the application has important consequences it is wise to try different models and methods and compare the results to arrive at a definitive conclusion. " The basic models, CCR and BCC have limitations in that both these models show more than one DMU as efficient. Therefore, to discriminate, between the efficient units, extended models, Super efficiency, cross efficiency and weight restrictions have been used in the research. However, DEA is a fair and equitable methodology and you will have to prove it (Paradi et al.,2017). I will have to support my argument by articulating how the limitations of basic models are supported by extended DEA models. Therefore, there is a need for comprehensive analysis. Adequate care will be taken to avoid errors.

From the various reliability and validity analysis conducted above, it is found that DEA analysis meets diverse types of validity and reliability tests despite its limitations.

6.3: Branch Profit Maximization Model:

The objective of the model is to maximize profits. The inputs (5) used for this

model is number of employees, the area of facility, COGS of equipment sales, COGS of parts sales and COGS of Service sales. The outputs (3) used in the study were revenue from Equipment, Service, and Parts sales. However, when CRS and VRS models were run to find the efficiency scores with eight factors as above the number of DMUs shown as efficient were very high. Therefore, the revenue from sales from individual operations equipment, service, and parts were replaced by total branch sales and total gross margin for the branch. The number of efficient units dropped significantly by just dropping one output.

The objective is to find how to maximize the revenue from equipment sales, service sales, and parts sales. Therefore, to attain better discrimination weight restrictions are imposed on the model. The ratio of two inputs parts sales to service sales, and three inputs to outputs viz., COGS of equipment sales to revenue of equipment sales, COGS of parts sales to revenue of parts sales and COGS of service sales to revenue of service sales, were used in these models to impose weight restrictions imposed on the ratios based on the thumb rules practiced in the industry. All these weight restrictions were used individually, and one model was run combining all the weight restrictions together. It was found that when all the weight restrictions were used together the discrimination power of the analyses increased.

Using CRS model with the above 5 inputs and 3 outputs, the model shows 25,20,22,22 and 19 DMUs as efficient in the period 2014,2013,2012,2011 and 2010 respectively and under VRS models with the same 5 inputs and 3 outputs used as above, the VRS model shows 31,27,26,26 and 25 DMUs as efficient in the period 2014,2013,2012,2011 and 2010 respectively. The weight restriction model was applied to the data of period 2014 and the number of efficient units dropped to 15 from 25 under CRS model and 31 to20 under VRS model when all weight restrictions were used together. Therefore, the profit maximization model answers the research question of finding efficiency scores.

6.4: Branch Expense Minimization Model:

This is the third model used to study the efficiency of heavy equipment branches from the minimization of expenses perspective. The inputs (3) used for this model are department expenses, depreciation and amortization, and total COGS for the branch which is the sum of COGS of equipment sales, COGS of rental sales, COGS of parts sales and COGS of Service sales and the only output used was the total branch sales. With four factors for the analysis of an input-oriented model to minimize expenses were run. It was found that there were 6,4,3,6 and 6 DMUs found efficient under CRS model and 11,8,8,12 and 15 DMUs efficient under VRS model for the period 2010 to 2014 respectively.

It is pertinent to note that with less number of factors the discrimination power of the model is high showing less number of DMUs as efficient.

6.5: Branch Asset Maximization Model:

In this DEA model analyses is made to find if assets are utilized effectively by each branch and therefore output-oriented model is used. The inputs (4) for this model are current assets, fixed assets, other assets and total COGS for the branch which is the sum of COGS of equipment sales, COGS of rental sales, COGS of parts sales and COGS of Service sales. Similarly, the output (1) used in this study is the sum of revenue from equipment sales, rental sales, service sales and branch sales termed as total branch sales.

It is found that 4,5,6,4 and 2 DMUs were found efficient under CRS model and 7,8,5,8 and 5 DMUs were found efficient under VRS model in the period 2010 to 2014 respectively.

It is to be noted again that with less number of factors used in the analysis, there is a more discriminatory power of the analysis showing less number of DMUs as efficient.

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Chapter VII: Conclusions, Limitations and Future Directions of Research

7.1: Introduction:

Heavy equipment manufacturers retail their equipment through a network of dealers that are independently owned and operated businesses with exclusive geographical territories. Considering the large investment that is involved in the retailing of the heavy equipment, most of the dealers (retailing organizations) want to operate the dealerships efficiently to make profits and sustain operations in a constantly changing business environment. Therefore, the objective of the research was to use a new performance measurement system that will help in assessing the relative efficiency of the branches, improve the efficiency of the inefficient branches, compare the efficiency of the branches, understand the factors that has bearing on the efficiency of the branches, find if there is any change in efficiency over time and how efficiency of departments in the branch have relation to the overall efficiency of the branch.

The purpose of the study was to develop models and methods that are more appropriate and suitable than the conventional methods of measuring performance and that can fulfill the objective of the research.

It was found that the theory and methodology of Data Envelopment Analysis have been widely used to measure efficiency in the public and private sectors. In the public-sector DEA has been widely used in schools, universities, hospitals and other government agencies such as police, fisheries to mention a few. In the private sector, DEA has been used to measure efficiency in automobile dealerships, in supply chain performance, logistics etc. However, DEA was never used to study efficiency in heavy equipment retail organizations. In this research DEA, a linear programming technique has been used to measure the performance of heavy equipment retailing organizations with the purpose of finding efficiency in the production process of a branch, improve the profitability of the branch, increase asset utilization of the branch and to minimize the expenses of the branch. Even though DEA has some pitfalls, it is still widely used by academicians and practitioners to measure efficiency (Dyson et al.,2001).

7.2. Overview of the Research:

The thesis has focused on how to measure the performance of a heavy equipment retailing organization using DEA and thus create a superior performance measuring system. In this endeavor, four DEA models were used to measure efficiency namely Branch Production Process model, Branch profit maximization model, Branch expense minimization model and Branch Asset maximization model.

To develop this model, the author extensively reviewed the literature in the application of DEA to the automotive industry and other retail sectors such as superstores, grocery stores, restaurant chain and wine stores to mention a few. While reviewing the literature various inputs and outputs that were used for the study were considered and a broad list of inputs and outputs that had similar attributes were listed that can be used for measuring the performance of heavy equipment retailing organization. The inputs and outputs drawn were a mix of monetary and other factors such as a number of employees and area of the facility. The list of factors were narrowed down using a stepwise approach and only the most appropriate factors were chosen for each model that satisfied the degree of freedom formula **$n \geq \max\{mxs, 3(m+s)\}$**, where m=number of inputs, s=number of outputs and n= number of DMUs as proposed by Cooper et al., (2007). The return to the scale of the model was determined as a variable return to scale-based b on the Kolmogorov-Smirnov test. Then the orientation of the model was chosen as output orientation for three models as the production process, profit and assets were to be maximized and for one model orientation chosen was input orientation as this model involved expense minimization.

Having set the parameters for running the model, the above mentioned four models namely Branch Production Process model, Branch profit maximization model, Branch expense minimization model and Branch Asset maximization model were run with the appropriate factors. The analysis was done as per the framework and architecture of the research as detailed in chapter4.

Of the four models, Production process model was analyzed in detail using secondary data for the period 2010-2014. First the basic models CRS and VRS models were run to know the technical efficiency and pure technical efficiency. The technical efficiency was divided by pure technical efficiency to obtain the scale efficiency. While using these two models technical, pure technical and scale efficiency were determined, it was found that both these models showed more than one DMU as efficient with a score of 1. Therefore, to rank these DMUs in the order of efficiency, super-efficiency model was used that gives an efficiency score of more than 1 for each DMU while the inefficiency score remained the same. Similarly, to evaluate the DMUs by the peer DMUs cross efficiency model was used and this was used to calculate the maverick Index. The maverick index indicates how a DMU is far away from other DMUs in the analysis. The analysis above calculated the efficiency scores; however, these scores may have random errors. To calculate the bias and determine the confidence interval, Bootstrap-DEA was used. In Bootstrap-DEA, 2000 replications were done for the CRS and VRS scores and the bias and confidence intervals were found.

To test the effect of environmental variables on efficiency scores both OLS and Tobit regressions were used. The CRS and VRS efficiency scores were regressed against the five environmental factors competition index, number of competition stores, square of competition stores, population of the city where the dealership is located and the federal spending on infrastructure. Having established the effect of environmental variables on efficiency scores the next analysis was carried out to find the change in efficiency over time using both window analysis and Malmquist productivity index. To find the efficiency of the departments such as sales, service, and parts within the branch, NDEA model was used. The last analysis was to find if there are outliers in the DMUs. Having done all the above analysis reliability and validity test was done on the scores to establish if the findings were consistent. The production process model was thus analyzed in detail.

In the Branch profit maximization model to the basic CCR and BCC models were run. The basic models do not pose restrictions on the weights and therefore weight restrictions were imposed on inputs and outputs based on the industry trends and this helped in bringing the number of efficient units and thereby increasing the discriminatory power of DEA. In case of the other two models Branch, asset maximization model and Branch expense minimization model only the basic CCR and BCC models were run to find efficiency.

The researcher is advised to try different models if the researcher cannot identify the characteristics of the production frontier in the preliminary survey (Cooper et.al., 2007). As per Cooper et al.,2007, it may be risky to rely only on one model and it is wise to try different models and methods and compare the results to arrive at a definitive conclusion if the application has important consequences.

This is the first study in measuring the performance of heavy equipment retailing organizations using DEA. Therefore, there is going to be pushback in accepting this new methodology as an effective tool to measure performance by the industry that is currently using many different methods to measure performance as outlined in chapter 1. Therefore a comprehensive analysis is made as per Cooper et al.2007, by using the basic models and other extended models as described above in the architecture in the study to make it more acceptable with least resistance. In other words, multiple models are used in the analysis to present the versatility of the new method using DEA.

7.3. Key Findings of the Branch PPM- Model Research:

7.3.1: Selection of factors:

It was found using the stepwise approach that the number of efficient DMUs drops down considerably when the number of inputs and outputs are reduced in relation to the number of DMUs. It was found that when eight inputs and five outputs were used in the CCR model, all the thirty-three DMUs were found efficient.However the number of efficient units dropped to two when the number of inputs and outputs used were one and two and when the number of inputs and outputs were two each (Please refer table 5.2).

7.3.2: Returns to Scale:

A K-S test was done to determine the returns to scale to be used in the research. The K-S test statistic showed that constant returns to scale should be used for the research. However variable returns to scale is also used to calculate scale efficiency.

7.3.3: Technical Efficiency (CRS Model):

It was found using the CRS model that 6,11,9,7 and 8 branches were found efficient during the period 2010-2014.Of these nine branches 8,10,12,13,19,23,27,31 and 32 were found consistently efficient in all the periods. All other branches were found inefficient (refer Table 5.10).

7.3.4: Pure technical Efficiency (BCC Model):

Using the BCC model, it was found that 13,16,12,16 and 13 branches were found efficient during the period 2010-2014. Of these eleven branches 7,8,10,12,13,19,23,27,28,32, and 33were found consistently during all the period and all other branches were found inefficient (refer Table 5.10).

7.3.5: Scale Efficiency:

Scale efficiency was calculated by dividing the technical efficiency score with pure technical efficiency score. It was found that 6,11,9,7 and 8 branches were found to be operating under constant scale efficiency during the period 2010-2014,8,9,10,13 and 5 branches were operating under IRS (increasing returns to scale) during the period 2010-2014 and 16,11,13,12 and 20 branches were found operating under DRS (decreasing returns to scale) during the same period. For details of the branches working under different returns to scale please refer to Table 5.6.

7.3.6: Super-Efficiency Model:

The super efficiency model was run using both CRS and VRS models and it was found that under both these models the inefficient branches retained the same score as was in CRS and VRS models whereas the efficient branches under these two had a score more than 1. The super-efficiency score was useful in ranking the efficient in the order of highly efficient to lowly efficient. It was found that branches 22,12,7,21,1 and 9 were ranked in the descending order in 2010, branches 22,26,18,13,29,8,7,6,10,12 and 30 were ranked in the descending order in 2011, branches 22,26,29,13,7,30,12,18 and 10 were ranked in the descending order in 2012, branches 23,32,13,10,6,30 and 12 were ranked in the descending order in 2013 and branches 20,29,2,9,23,13,27 and 7 were ranked in the descending order in 2014. It is found that branches 22 and 26 had a high super efficiency score of more than 2.0 whereas all other branches had a super-efficiency score of less than 2.0.

Under the VRS Super efficiency model it was found that some branches had an infeasible solution to the LP problem. The branches 6,2,25 and 27 had infeasible solution in 2010,branches 5,8,22,23,26 and 27 had infeasible solution in 2011,branches 8,22,23,26,27 and 28 had infeasible solution in 2012, branches 8,15,27 and 28 had infeasible solution in 2013 and branches 8,28 and 29 had infeasible solution in 2014.To solve the infeasibility in the VRS super efficiency another model called FPA model was used to solve the infeasibility. FPA model was used to solve the infeasibility only for the period 2014 and the branches 8,20,19,32,23,28,10,12,27,33,13 and 29 were ranked in the descending order.

7.3.7: Cross Efficiency Model:

The cross-efficiency score was calculated for the period 2010-2014 and it was found that when the branch efficiency score is evaluated by peers, the cross-efficiency score is less than the original CRS and VRS scores. The difference between original VRS scores and cross efficiency scores results in calculating what is called the maverick index. The maverick index is a measure that identifies how differently a branch is performing as against branches that are all round performers.

It is found that in 2014, DMU 29 has a high maverick index of 4.91.

7.3.8: Bootstrap DEA:

Bootstrap-DEA was done by replicating the scores 2000 times the CRS and VRS scores for the period 2010-2014 and it was found that for the CRS scores for the period 2014 the bias ranges from a minimum of 0.65% to a maximum of 4.55%, in 2014,0.65%-5.59% in 2013, 0.58%-4.22% in 2012,0.84% -5.94% in 2011 and 0.95%-6.22% in 2010.In the VRS scores the bias ranges from 0.65% -4.55%, in 2014,0.95%-4.35% in 2013,0.54%-3.37% in 2012,0.83%-3.35% in 2011 and 0.65% -4.03% in 2010. The mean bias in CRS scores is from a minimum of 0.72% to a maximum of 5.30% and the mean bias of VRS scores is from a minimum of 0.73% to a maximum of 3.93%. The bias helped in correcting the original CRS and VRS scores.

7.3.9: Envelopment Model:

The results show the efficiency score, proportional improvement value, slack improvement value and target value (projection), number of efficient DMUs, lambda values and benchmarking with reference set are shown using envelopment form (see tables 5.17,5.18 and 5.19). This model was run to find how to make inefficient branches efficient by using data for the period 2014. The target for the inefficient branch is set by calculating the proportionate movement and slack value with which the inefficient branch can reach the branch in the frontier. Therefore, this model shows how inputs can be decreased and outputs can be increased to make the branch efficient. The envelopment form of DEA gives actionable advice to the organization on improving efficiency that is perceived to be fair and equitable.

On analysis of slacks for DMU1, it was found that it has an excess staff of 7.4563 and excess area of the facility of 63,082. Similarly, on the output side, there is no slack on revenue, but it can be increased by 1,318,131 and the slack on gross margin is 1,071,925. Therefore, DMU1 must make three adjustments to operate on the efficient frontier. First it must reduce all inputs by 5.1%, second, it must reduce staff by 15.86% and reduce the area of the facility by 67.37%. This will result in an improved margin by 1,071,925 and the sales by1,318,131.

Chen (1997) and Chen and Yeh (1998) used the frequency of reference set to discriminate the branches. It is found that branches 12,23,8 and 10 shows up in the reference set of inefficient branches 17,14,11 and 11 times. This indicates that these branches are operating efficiently, and inefficient branches can emulate these branches.

7.3.10: Effect of Contextual variables on efficiency scores:

Both OLS and Tobit regression was used to find the effect of contextual variables. The five contextual variables used were number of competition stores, square of the number of competition stores, competition index, the population of the city where the branch is located and the federal expenditure on machinery and equipment in the location of the branch. It is found from OLS regression competition index and number of competition stores have a negative coefficient for VRS scores implying that an increase in these values will decrease the efficiency. Similarly, in CRS scores it was found that the square of number of competition stores has a negative coefficient indicating that they have a negative effect on efficiency whereas all the other factors had a positive coefficient.

Similarly, in analysis with Tobit regression coefficients of the population, capital expenditure and the square of competition stores all have a negative sign with both CRS and VRS scores and the coefficients of other two factors competition index and number of competition stores have a positive sign.

7.3.11: Efficiency Change over time:

It was found from Window analysis that the CRS scores were stable between adjacent periods but over a three-year window seems to fluctuate and not stable. Similar observations are made with the VRS scores over a three-year period. It was found that of all the DMUs (297), 27.27% of the DMUs exhibited decreasing returns to scale,20.5% DMUs exhibited constant returns to scale and 45.45% indicated increasing returns to scale. This indicates that production scale is a source of inefficiency in the heavy equipment retailing organization and predominantly most of the DMUs operate under a decreasing return to scale.

. It was found that Malmquist Index for the year 2011 is less than 1 indicating that the efficiency has decreased as compared to 2010, in 2012 it is less than 1 indicating that the efficiency has decreased as compared to 2011, in 2013 it is less than 1 and hence efficiency has decreased from 2012 and in 2014 it is greater than 1 indicating that efficiency has increased from 2013. Similarly the frontier shift is calculated for the period 2010-2014. The product of technical change and frontier shift indicates the Malmquist total factor productivity index. If this index is greater than 1 as is observed for DMU1 then it means that there has been a productivity gain and productivity loss if less than 1.

7.3.12: Network DEA:

The research was done with two approaches to Network DEA. In the first approach the departments of the branch were considered as operating in parallel and in the second approach the departments were assumed to be operating in series with the first stage, intermediate stage and second stage all happening in series.

It was found using the network parallel structure that the efficiency of the branch was different from the efficiency of the individual departments. In the case of DMU1, it was found that the efficiency of branch 1 was lower than the efficiency of sales and parts division but greater than the efficiency of the service division.

In the series structure, the intermediate stage is selling, and the final stage is generating profit. It was found for DMU1 that the efficiency of the intermediate stage was high but generating gross margin was less thereby bringing down the overall efficiency of the DMU1.

7.3.13: Detection of outliers:

Two methods were used to find if there are outliers among the DMUs. The first method was for Banker and Chang using super efficiency and the second method was of Tran that uses the lambda value of the efficient units to identify Outliers. It was found using Banker and Chang method that DMU20 was an outlier using VRS model whereas with CCR model using Tran's method it was found that DMU23 and DMU20 are outliers. Both methods identify DMU20 as an outlier and DMU 23 is identified as an outlier only by Tran's method.

7.4. Key Findings of the Branch Profitability Model Research:

7.4.1: Introduction

This model was run with the objective of maximizing profit with five inputs number of employees, the area of facility, COGS of sales, COGS of Service and COGS of parts and two outputs total branch sales and total gross margin for the branch. The model was run with the basic models CCR and BCC and to improve the discriminatory power weight restrictions were imposed and weight restriction model was run.

7.4.2. Basic Models:

It was found using CCR and BCC model that there are 13 branches efficient using CCR model and 23 branches using BCC model. It is found that this model shows many branches as efficient and therefore weight restriction model was used to improve the discriminatory power.

7.4.3: Weight Restriction Model:

In this model weight restrictions were imposed on inputs and outputs. Using these weight restrictions individually and collectively altogether five different models were run using 2014 data. It was found that using all the restrictions together with the number of efficient units dropped by 40% in the case of CCR model and 35% in the case of BCC models.

7.5. Key Findings of the Branch Expenses Minimization Model Research:

7.5.1. Introduction:

The inputs used for this model are department expenses, depreciation and amortization, and total COGS for the branch which is the sum of COGS of equipment sales, COGS of rental sales, COGS of parts sales and COGS of Service sales. Similarly, the output used in this study was the sum of revenue from equipment sale, rental sales, service sales termed as total branch sales. In short, the model was run with three inputs and one output. Since the model was about minimization the model was run as an input-oriented model. However, this research was done only with basic models CCR and BCC.

7.5.2: CCR Model:

This model was run with three inputs and one output using data for the period 2010-2014.It was found that the number of efficient branches in minimizing expenses was 6,4,3,6 and 6 during the period 2010-2014. During this period the minimum score of inefficient branches were 76.13%,73.03%,71.66%,84.93% and 87.94% during the period 2010-2014 respectively. This indicates that there is an opportunity to save 23.87%,26.97%,28.34 %, 15.07% and 12.06% of expenses during the period 2010-2014.

7.5.3: BCC Model:

As above this model was run with three inputs and on output and it was found that there were 11,8,8,12 and 15 that were efficient in minimizing expenses. The minimum VRS inefficiency score of the branches were 85.19%,81.07%,85.61%,86.36% and 90.13% during the period 2010-2014 respectively. In other words, this presents an opportunity for saving expenses to a maximum of 14.81%,18.93%,14.39%,13.64% and 9.87% during the year 2010-2014.

7.5.4: Scale Efficiency:

It is found during the period 2010-2014 there is 6,4,3,6 and 6 DMUs scale efficient respectively. Similarly,2,2,13 and 1 DMUs are having increasing returns to scale in the period 2010,2011,2013 and 2014 respectively indicating that the branches can increase their expenses. There was no DMU with increasing returns to scale in the year 2012There are 22,25,28,13 and 26 DMUs in the period 2010-2014 respectively that have decreasing returns to scale indicating that the expenses can be minimized by downsizing the branch operations.

7.6. Key Findings of the Branch Asset Maximization Model Research:

7.6.1: Introduction:

This model was run to maximize assets in each branch and therefore was run with output orientation using only the basic models CRR and BCC Models. The inputs used for this model are

current assets, fixed assets, other assets and total COGS for the branch which is the sum of COGS of equipment sales, COGS of rental sales, COGS of parts sales and COGS of Service sales. The output used is Total branch sales. In other words, three inputs and one out was used for this model.

7.6.2. CCR Model:

Using the CCR model it was found that 4,5,6,4 and 2 branches were efficient during the period 2010-2014 respectively in utilizing the assets of the branch. The minimum value of inefficiency score of the branches was 78.80%,76.00%,85.38%, 70.39% and 81.74 %. This indicates that there is an opportunity to increase asset utilization by 21.2%,24%,14.62%,29.61% and 18.26% during the period 2010-2014 respectively.

7.6.3.BCC Model:

Using BCC model, it was found that 7,8,5,8 and 5 branches were found efficient during the period 2010-2014 with minimum inefficiency score of 82.72%,81.09%,85.38%,77.98% and 85.03% respectively. This presents an opportunity to improve asset utilization by 17.28%,18.91%,14.62%,22.02% and 14.97% respectively.

7.6.4: Scale Efficiency:

The scale efficiency is found by dividing the CCR score by BCC score and by doing so it is found that there is 20,20,17,21 and11 DMUs scale efficient in the period 2010-2014 respectively. This indicates that these branches are utilizing their assets efficiently. Similarly,1,5,1and12 DMUs are having increasing returns to scale in the period 2011,2013 and 2014 respectively indicating that the branches are maximizing utilization of assets. There are 10,10,9,10 and 10 DMUs in the period 2010-2014 respectively that have decreasing returns to scale indicating that the assets can be maximized by downsizing the branch operations.

The above were the findings using the four research models for the thesis. In the next section recommendations will be proposed based on the findings above that can improve the performance of the heavy equipment retailing organization.

7.7. Recommendations:

7.7.1. Branch Production Process Maximization Model:

The results of the analysis of this research were done with four DEA models. The first DEA model of Branch production process model demonstrates a variety of inefficiencies in the branch operations of heavy equipment retailing organization under study. These results can be used by the managers and decision makers to improve the performance of the heavy equipment retailing organization and some of the recommendations are addressed below using the research findings mentioned in the previous section.

The very first analysis using basic models identified that there are branches with poor performance under both CRS and VRS models. Using the scores from these two models the scale efficiency was calculated and identified branches that were working with decreasing returns to scale, increasing returns to scale and constant returns to scale. The management based on these findings can take a decision to scale down operations of branches with decreasing returns to scale and expand the operations of the branch with increasing returns to scale.

The poor performance of the branches also indicates that these resulted from the underutilization of inputs. The extent of underutilization of inputs can be obtained from the envelopment form of the model and using the slacks in the model targets can be set for potential reduction of inputs. As can be seen from the findings of the model it was evident that some branches had excess staff and excess area of the facility. This will assist managers in managing the branch in an effective way.

The envelopment model provides a reference set for the inefficient branches and this can provide valuable insights to the managers in benchmarking the best practice branches. The benchmarking will help managers to ask inefficient branches to emulate and adapt the best practices of the efficient branches.

The branches can be ranked based on the performance using the super-efficiency model and this ranking can be used to motivate inefficient branches to follow the leader in adapting their process.

It was also found that there is a decline in technological change over the period 2010-2014 and this can be partly attributed to the inability of the staff in following the process and new techniques in place. The branches can improve the technology by adequately training the branch staff to improve their skills and knowledge.

It was found that some of the environmental factors affected the efficiency like a number of competition stores and competition index. This will help the branch managers to be aware of the competition in the industry and therefore educate branch staff in improving customer satisfaction and customer experience.

The analysis indicates that the efficiency of the branch as a whole also depends on the efficiency of the individual departments in the branch and therefore adequate measures to be taken to improve the performance of individual departments.

These recommendations are based on the complete production process in the branch including the individual department of sales, service and parts operations. Therefore, these can be of great use to the Chief Operations Manager of the organization who is always on the lookout for ways and means of improving the efficiency of the organization.

7.7.2. Branch Profitability Maximization Model:

In this model, specific emphasis was on using the inputs and outputs that will maximize the profitability of operations. The analysis demonstrated that there are inefficiencies in some branches and identified branches with different scales of operations. The branches with decreasing returns to scale can be downsized and those with increasing returns to scale can be expanded to improve the profitability of the organization. Although envelopment model was not used in the research, this model can be used to set targets for reducing the inputs. This model will be of interest to the CEO of the organization as it will assist in achieving higher ROI (return on investment).

7.7.3. Branch Expense Minimization Model:

This model was used with inputs and outputs that primarily focus on reduction of expenses such as Total COGS, department expenses and depreciation and amortization expenses. This model will be of interest to the Chief Financial Officer as CFO is engaged all the time in controlling expenses that will further improve profitability. The scale efficiency of the model will help in managing expenses more effectively.

7.7.4. Branch Assets Maximization Model:

This model was used with inputs and outputs that primarily focus on maximizing the assets of the organization. The inputs that were used was Total COGS of the branch, current assets, fixed assets and other assets. This model is of interest to the Chief Financial Officer as CFO is engaged all the time in maximizing the return on assets(ROA).

7.7.5. Performance Enhancement Decision-Making System (PEDMAS):

The various recommendations that have been made above from four different models used for the research can be combined to design a performance measurement system that helps in monitoring the performance of different branches on a regular basis.

The research suggests PEDMAS as a system using DEA as a tool that can use multiple inputs and outputs to measure the performance of the branches of equipment dealership and then helps them in enhancing the performance of the inefficient branches (Athanaspoulos,1995).

Performance enhancement decision-making system (PEDMAS) uses DEA as a tool to measure the performance of these branches that have multiple inputs and outputs and helps them in enhancing the performance by taking a timely decision in enhancing the performance of inefficient branches. The following Figure7.1 exhibits the use of PEDMAS as a system to aid in making decisions to improve performance on a regular basis.

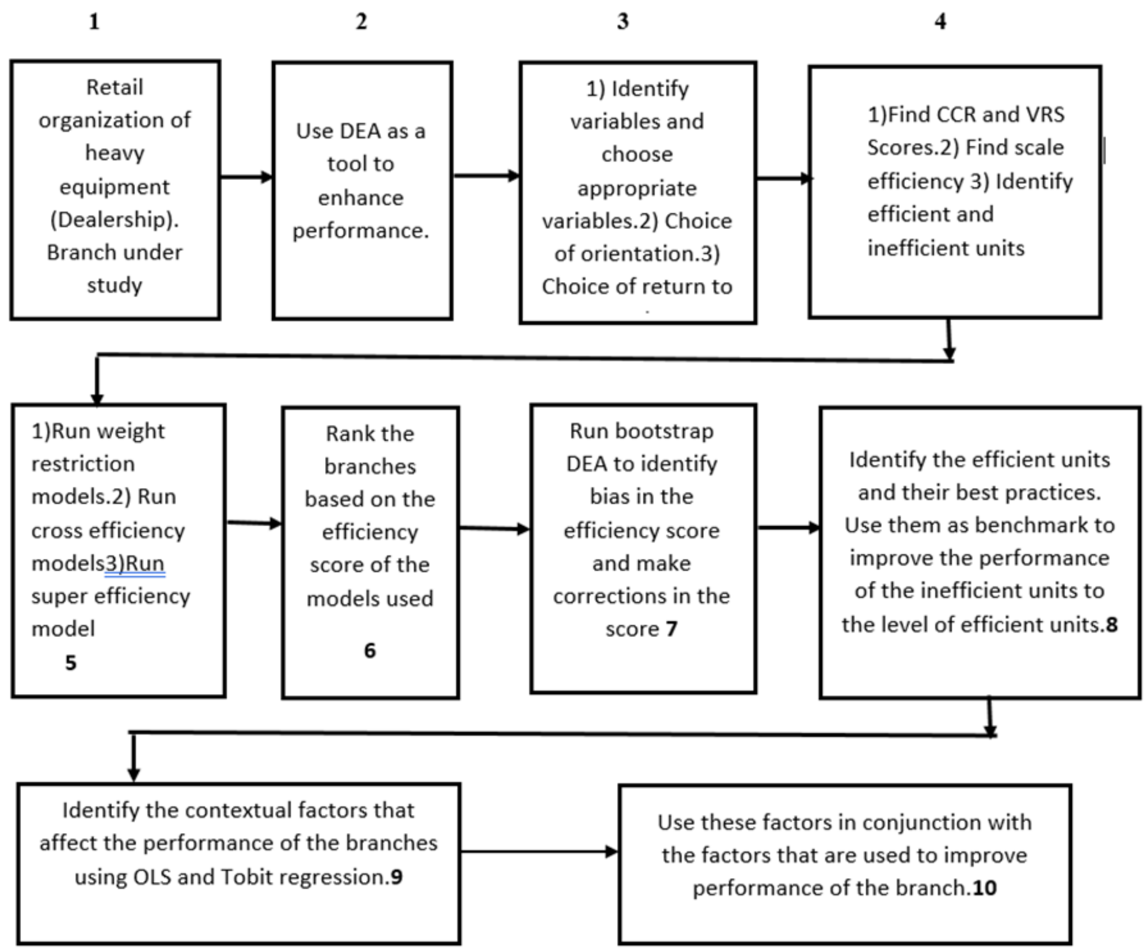

Figure 7.1. Performance Enhancement Decision-Making System (PEDMAS)

7.8: Contributions of the Research:

i) The current research is the first published application of DEA to measure the performance of heavy equipment retailing organization. This was undertaken due to the many shortcomings in the current methods that are used to measure performance in heavy equipment retailing organization. This approach can be generalized to similar industries like automotive and heavy-duty trucks where a single franchise has multiple dealership locations. This approach can also be utilized by manufacturers of equipment and other related industries where the distribution is done through dealerships to measure the relative efficiency of their dealer network.

ii) The research adds to the existing literature on applications of DEA by extending the application of DEA to heavy equipment retailing organization.

iii) The four different models used in the research shows how DEA can be applied to meet different objectives in the organization. The different models can be used by CEO, COO, and CFO. This approach can be extended to study the efficiency of the workshops that services the equipment and improve the profitability of the service operations in heavy equipment retailing. In the service operations of heavy equipment, there are mobile service vans that go to the site of the customer to repair the machines. Each of these mobile service vans can be considered as a DMU and the efficiency of the mobile service van operations can be studied. The application can also be extended to supply chain, human resources, and logistics.

iv) The research has firmly established the consistency of results using various reliability and validity tests and thus establishing DEA as a useful tool to measure performance in heavy equipment retailing organization.

v) While this research is the first publication in the application of DEA to heavy equipment retailing organization, it is also the first publication in using Network DEA to study the efficiency of internal departments of the branch such as sales, service and parts operations. This study also has established a relationship between the efficiency of the branch and its internal divisions.

vi) The research has made a comprehensive analysis of efficiency by using various models (Cooper et al.,2007) to analyze performance so as to enable take a right decision.

vii) The research has the potential to integrate DEA with the ERP (enterprise resource planning) system of the organization and to measure performance on a continuous basis and take corrective action to improve efficiency.

viii) To test the sensitivity of efficiency scores various inputs and outputs combination has been used. Therefore, the sensitivity analysis gives direction in the choice of appropriate inputs and outputs for the study.

ix) Another contribution of the research is to test the environmental factors of the heavy equipment industry and how it affects the efficiency scores using OLS and Tobit regression.

x) The use of CRS and VRS Window analysis and Malmquist index to measure the change in efficiency over time (2010-2014) is a novel method of finding variations in productivity due to technological change.

All these contributions help the decision makers and managers by giving guidelines for improving the performance of heavy equipment retailing organizations.

7.9. Limitations of the Research:

This is a first research paper on measuring the performance of heavy equipment dealership branches using Data Envelopment Analysis. The variable selection is based on expert's opinions and based on details that are available in the literature in the research in automotive dealerships and other retail sectors using DEA. Every effort has been made to get reliable data from the audited financial results of the company and other authorized, authenticate and reliable sources within the organization. However, these data are from the enterprise resource system of the organization and any error inherent in the system would be replicated in the financial statement. Such errors inherent in the system may have influenced efficiency scores. Such errors found in the scores have been corrected by using a bootstrapping technique.

Similarly, some of the data for contextual variables were obtained from Statistics Canada and data related to market share from Trade journals(Statista.com). Some details on market share of equipment manufacturers were not available. Similarly, data on the number of competition stores in each branch had to be compiled in some cases.

The heavy equipment dealership is a service industry and its survival depend on the performance efficiency. This is possible by practicing various strategies that include customer support and service, low-cost innovative products, customer satisfaction index, response time, continuous improvements and service level. These qualitative factors affect the performance of the dealership. Since the qualitative data was not available, these could not be included as variables in the research and the research was limited to the available financial information.

Similarly, data on inventory performance, stock-outs, number of purchase orders, number of vendors, lead time, inventory turns, etc. was not available at branch level and hence not included as variables. This would help in benchmarking the dealerships on inventory performance.

Certain other data related to customer satisfaction like order processing time, physical aspects of the dealership, reliability of service, customer relationship etc. was not available and therefore could not be included as variables in the research.

7.10: Directions for Future Research:

As mentioned in the limitations on the discussions on this research work, the scope of the research is limited to data availability. Therefore, one area for future research is to collect more data and improve the quality of the model by using more variables. Some of the variables could be qualitative like physical appearance of the dealership, customer satisfaction index, quality of service, reliability (is the machine coming to the shop for the same job repeatedly), and response time. Some of the quantitative data could be related to inventory holding, number of turns, lead time, number of purchase orders etc.,

Another area of probable future research is to use Structural Equation Modelling to find the effect of latent factors on efficiency scores in addition to the current methods of Tobit regression and Ordinary Least Squares.

The third area of potential future research is to use Slack Based Efficiency models to find efficiency scores. The slack-based method has not been used in current research.

One of the assumptions made in the parallel structure of NDEA model is that the sum of inputs and outputs of sub DMUs in a DMU should be equivalent to the input and output of the DMU. Therefore, the fourth area of likely future research could be to find a model that can do away with this assumption and this could be a mathematical challenge.

Kao (2014), noted that the most valuable research direction would be in the application of DEA models to real world problems and therefore future research should be directed towards finding models that can improve heavy equipment dealership performance.

7.11: Summary of Findings:

In the current research the four DEA models that are developed to measure the performance of heavy equipment retailing organization are: branch production process model, branch Profitability maximization model, branch expense minimization model and branch asset maximization model. Various branch-level data of inputs and outputs were incorporated in to the above models to capture many efficiency measures using various DEA models. These efficiency measures found by above four different DEA models can be used by various functions in an organization as described below.

These four models will serve different perspectives in an organization looking to improve its efficiency. The production process model will enhance the overall efficiency of production process and will be of interest to the Chief of Operations, the CEO who is striving to improve profitability will be well served by the profit maximization model and the Finance chief who wants to control expenses and improve asset utilization will be benefitted by using the expense minimization model and the asset maximization model.

Table 7.1. Summary of findings of PPM - Model

DMU	**Score**	**Target Staff Reduction**	**Target Area of Facility Reduction**	**Target for expenses Reduction**	**Target for COGS of the Branch**	**Target Branch Sales Improvement**	**Target Gross Margin Improvement**
DMU1	0.94975	16%	67%	3.32%	0	5%	28%
DMU2	0.98002	23%	0%	24%	0	2%	9%
DMU3	0.97509	16%	26%	2%	0	3%	14%
DMU4	0.90682	0%	27%	2%	0	10%	79%
DMU5	0.95082	0%	0%	2%	0	5%	22%
DMU6	0.95184	0%	0%	19%	0	5%	30%
DMU11	0.96706	32%	31%	5%	0	3%	15%
DMU14	0.9172	0%	0%	1%	0	9%	59%
DMU15	0.97383	0%	0%	29%	0	3%	14%
DMU16	0.9424	0%	0%	21%	0	6%	34%
DMU17	0.95327	13%	0%	1%	0	5%	28%
DMU18	0.92672	0%	24%	10%	0	8%	43%
DMU21	0.97356	0%	3%	1%	0	3%	15%
DMU22	0.91701	0%	27%	2%	0	9%	54%
DMU24	0.95908	16%	38%	2%	0	4%	23%
DMU25	0.97146	0%	57%	5%	0	3%	12%
DMU26	0.95387	0%	4%	23%	0	5%	31%
DMU30	0.97971	0%	34%	19%	0	2%	14%
DMU31	0.94884	0%	10%	2%	0	5%	27%

The production process model was analyzed and found that there are branches that are efficient and those that are inefficient. The above table 7.1 lists all the branches that are found inefficient in the Production Process model. It is found from the above table that DMU1 has an inefficiency score of 0.94975 indicating that the branch could become technically efficient(pure) if all its inputs are proportionately reduced by 5.02%. However, this will not make the branch efficient as it has non-zero slacks for this branch. On analysis of slacks for DMU1, it was found that it has an excess staff of 7.4563 and excess area of the facility of 63,082. Similarly, on the output side, there is no slack on revenue, but it can be increased by

1,300,351 and the slack on gross margin is 1,057,466. Therefore, DMU1 has to make three adjustments to operate on the efficient frontier. First it has to reduce all inputs proportionately by 5.02%, (weights to be determined by multiplier model) and then subsequently reduce staff by 16%, area of facility by 67% and expenses by 3.32%. This reduction results in increase in sales by 5% and gross margin by 28%. Similar interpretations can be made for all other inefficient branches from the above table.

A summary of findings of the four different models is given in table 7.2below. It is found that branches 12,23,8 and 10 shows up in the reference set of inefficient branches 17,14,11 and 11 times for the production process model. This indicates that these branches are operating efficiently, and inefficient branches can emulate these branches in the production process model.

From the table 7.2 below it is found that DMU 20 and DMU 23 are efficient consistently under all the above four models under both constant returns to scale and variable return to scale except the expense minimization model for CRS. This means that both these DMUs are operating under the most productive scale size. It is also found that DMU 23 is appearing fourteen times under the reference set of inefficient branches. This is the conclusion from using the basic CRS and BCC models.

Table 7.2. Summary of Four models used in the research

Model	Scale of the Model	Efficient DMUs
Production Process Maximization Model	CRS Model	DMU 7,8,2,13,20,23,27,29
	VRS Model	DMU 7,810,12,13,19,20,23,27,28,29,32,33
Profit Maximization Model	CRS Model	DMU2,6,8,9,10,12,13,20,21,22,23,25,28,32,33
	VRS Model	DMU2,4,5,6,7,8,9,10,11,12,13,20,21,22,23,25,27,28,29,32,33
Expense Minimization Model	CRS Model	DMU12,13,15,23,27,29
	VRS Model	DMU7,8,10,12,13,15,19,20,21,23,27,29,30,32,33
Asset Maximization Model	CRS Model	DMU20,23
	VRS Model	DMU19,20,23,29,32

However, by using the envelopment form of the other three models namely profit maximization model, expense minimization model and asset maximization model, target value of improvement can be found for each of these models.

It is also found from the above table that there are very few branches that are efficient under expense minimization model and asset maximization model as compared to production process model and profit maximization model under both CRS and VRS. Branches that are efficient under production process model are not necessarily efficient under asset maximization and expense minimization model. Therefore, this indicates that one model cannot serve all purpose and therefore based on the objective, efficiency must be modelled, and appropriate factors have to be chosen for the specific model to meet the needs of the model.

The current methods used in measuring performance in dealership branches does not precisely say by how much percentage multiple inputs and outputs can be altered to increase the profit margin and thereby make the operation efficient whereas DEA is able to specifically point out precisely the reductions in input needed to improve efficiency.

With the available data of the equipment organization there was constraint in using only the above four models to analyze the efficiency. However, with the availability of other data on performance, DEA can be effectively used to measure efficiency in other functions of the organization. One can analyze the efficiency of the service repair shops in the organization if we have data on number of technicians, number of bays, number of manhours used in repair, labor rate, number of work orders opened and sales revenue from the workshop. DEA model using these factors can find out the efficiency of workshops. The heavy equipment retailing organizations have several mobile service vans (may be as high as 200 in large organizations) to give onsite service to customers in the field. If we have data on kilometers covered by the van, fuel consumed by the van, maintenance expenses on the van, number of workorders opened and the profit generated by the van, the efficiency of the technicians can be estimated. A large organization has many sales personnel and all of them are given a vehicle to contact customers to generate sales. The efficiency of sales personnel can be calculated if there is data on number of customer calls made by them, fuel consumed by the vehicle and number of sales orders generated by the sales person.

DEA can also be used to formulate strategies. The objective of the management decides the type of model. As described the four models have different objectives and therefore the four models address these objectives. It is possible that some DMUs may be efficient in some of these models and inefficient in other models. This is because the factors chosen for the analysis are different for each model. The DMUs are found efficient or inefficient based on these models. The strategies can be chosen based on the findings of the model and the findings will differ from each model. As the findings will be different for each model, it is not possible to devise a single strategy that will address all models.

However, using the results of a single model strategies can be devised both for long term and short term based on the objective. The model indicates different types of return to scale Constant Returns to scale, Increasing, returns to scale and decreasing returns to scale. If the DMU shows constant returns to scale the outputs are proportional to inputs. On the contrary if DMUs exhibit decreasing returns to scale the strategy should be to downsize branches and increase the size of the branch when the DMU shows increasing returns to scale.

In real world it is very difficult to immediately take such action of downsizing or increasing the size of the branch as some facilities may be under lease and other constraints. However, the returns to scale results of the branch will aid in taking strategic decisions either to downsize or increase the size of operations. In each of the model improvement targets can be found using the envelopment form as discussed in section 5.9.6 in Chapter 5.

Similar such strategies can also be drawn based on the other objectives of the organization. The efficiency/profitability matrix has been used by Boussofiane et al., (1991) to classify performers as stars, sleeper, dog and question mark and formulate strategies for an organization. Sarrico and Dyson (2000) used the framework of BCG (Boston Consulting Group) for formulating strategies in U.K. Universities. Lin and Lin (2017) used BCG matrix and DEA to formulate strategies for securities industry in Taiwan. Pham, Choi and Park (2018) used DEA results and BCG matrix to formulate strategies for major ports in Korea and China. A detailed discussion of using DEA using BCG and profit -efficiency matrix is beyond the scope of the thesis but can conclude that DEA can be used for formulating strategies in an organization.

DEA also has the potential to become a predictive tool if one can interface the ERP system of the organization with a DEA software so that real time efficiency data is available for review monthly.

In summary DEA provides a promising alternative to measure performance in heavy equipment retailing organization. Although the first paper on DEA, the productivity management tool for service organizations appeared in 1978, this is the first research applying DEA to heavy equipment retailing organization. DEA analysis establishes the best practice group of branches, identifies inefficient branches compared to the best group of branches and quantifies the amount of potential improvement possible for each inefficient branch. In other words, DEA indicates the level of resource savings and service improvement possible for each inefficient branch if it is to be on par with the efficiency level of the best branch. The current performance measures used in the heavy equipment retailing organizations does not identify the above parameters identified by DEA, that are needed to improve efficiency. Therefore, DEA is a more powerful tool to measure performance and makes it a suitable candidate than the contemporary tools that are used to measure performance of heavy equipment retailing organizations due its ability to use multiple inputs and multiple outputs.

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References

Abdullahi, I., Zainal, M. A., & Terano, R. (2015). Data Envelopment Analysis Models and Software Packages for Academic Purposes [Abstract]. *Pertanika Journal of Scholarly Research Reviews*, *1*(1), 27-32. Retrieved from www.pjsrr.upm.edu.my/.*Accounting Research, 25*, 147-156. Retrieved from

Achabal, D. D., Heineke, J. M., & McIntyre, S. (1984). Issues and Perspectives on Retail Productivity. *Journal of Retailing, 60*(3). Retrieved from https://www.researchgate.net/publication/228217129_Issues_and_Perspectives_on_Retail_Productivity.

Adler, N., Friedman, L., & Sinuany-Stern, Z. (n.d.).2002 Review of ranking methods in the data envelopment analysis context. *European Journal of Operational Research,* 249-265.

Adler, N., Raveh, A., & Yazhemsky, E. (n.d.). DEA Presented Graphically Using Multi-Dimensional Scaling. *Modeling Data Irregularities and Structural Complexities in Data Envelopment Analysis,* 171-187. doi:10.1007/978-0-387-71607-7_10

Ahmad, M. F., Ishtiaq, M., Hamid, K., Khurram, M. U., & Nawaz, A. (2017). Data Envelopment Analysis and Tobit Analysis for Firm Efficiency in Perspective of Working Capital Management in Manufacturing Sector of Pakistan. *SSRN Electronic Journal*. doi:10.2139/ssrn.2911577

Aikaterini, A. (2012). An Industry and Country Analysis of Technical Efficiency in the European Union, 1980-2005. 1-431. Retrieved from http://theses.gla.ac.uk/3660/1/2012kokkinouphd.pdf

Alex, M., & Jen, S. C. (2013). Performance Analysis of Automobile Industry in Taiwan with Data Envelopment Analysis [Abstract]. *International Research Journal of Applied*

Mathematics and Statistics, 38(8), 84-95.

Allen, R., Athanassopoulos, A., Dyson, R. G., & Thanassoulis, E. (1997). Weight restrictions and value judgements in Data Envelopment Analysis. Evolution development and Future directions. *Annals of Operations Research, 73*, 13-34. doi: Chapter 2

Al-Rashidi, A. N. (2015). DEA for measuring the efficiency of Head Trauma Care in England and Wales. *Ph. D Thesis of University of Salford*.

Al-Refaie, A., Wu, C., & Sawalheh, M. (2018). DEA window analysis for assessing efficiency of blistering process in a pharmaceutical industry. *Neural Computing and Applications*. doi:10.1007/s00521-017-3303-2

Alvandi, M., Masoumi, A., & Rezaei, M. (2012). The evaluation of Automotive and Spare Parts Companies by Balanced Score Card Approach and Data Envelopment Analysis [Abstract]. *International Research Journal of Applied and Basic Sciences, 3*, 2772-2779. Retrieved from Www. irjabs.com.

Amin, G. R., & Sharma, S. K. (2012). Cricket team selection using data envelopment analysis [Abstract]. *European Journal of Sport Science, 14*(Sup1). doi:10.1080/17461391.2012.705333

Amirteimoori, A., & Kordrostami, S. (2005). DEA-like models for multi-component performance measurement. *Applied Mathematics and Computation, 163*, 735-743.

Andersen, P., & Petersen, N. C. (1993). A Procedure for Ranking Efficient Units in Data Envelopment Analysis. *Management Science*, *39*(10), 1261-1264. doi:10.1287/mnsc.39.10.1261

Anderson, D., Sweeney, D., Williams, T., & Martin, K. (2011). *An introduction to Management Science, Quantitative approaches to decision making* (13th edn.). Toronto: Cengage Learning.

Anderson, T., & Uslu, A. (n.d.). Selecting the "best" using data envelopment analysis. *Innovation in Technology Management. The Key to Global Leadership. PICMET 97*. doi:10.1109/picmet.1997.653639

Annual Report. (n.d.). Retrieved December 11, 2015, from http://www.finning.ca/

Arfa, C., Herve, L., Goaied, M., & Mosseveld, C. V. (2016). Measuring the capacity utilization of Public District Hospitals in Tunisia Using Dual Data Envelopment Analysis Approach [Abstract]. *International Journal of Health Policy and Management, 5*(X), 1-10.

Asandului, L., Roman, M., & Fatulescu, P. (2014). The Efficiency of Healthcare Systems in Europe: A Data Envelopment Analysis Approach [Abstract]. *Procedia Economics and Finance, 10*, 261-268. doi:10.1016/s2212-5671(14)00301-3

Asmild, M., Paradi, J. C., Agarwall, V., & Schaffnit, C. (2004). Combining DEA Window Analysis with the Malmquist Index Approach in a Study of the Canadian Banking Industry. *Journal of Productivity Analysis, 21*(1), 67-89. doi:10.1023/b: prod.0000012453.91326.ec

Molem. C. Sama, Beri Parfait. Assessing the Technical Efficiency of Public Hospitals in Cameroon: An Application of Data Envelopment Analysis (DEA) [Abstract]. (2016). *International Journal of Humanities, Social Sciences and Education, 3*(3). doi:10.20431/2349-0381.0303005

Associated Equipment Distributors. Retrieved June 11, 2016, from http://aednet.org/about-aed/

Athanassopoulos, A. D. (1995). Performance improvement decision aid systems (PIDAS) in retailing organizations using data envelopment analysis. *Journal of Productivity Analysis, 6*(2), 153-170. doi:10.1007/bf01073409

Atkinson, M. (2015). Canadian Construction Association. Retrieved November 11, 2015, from http://www.cca-acc.com/

Avkiran, N. (1999). An application reference for data envelopment analysis in branch banking: helping the novice researcher. *International Journal of Bank Marketing, 17/5,* 206-220.

Avkiran, N. K. (2006). *Productivity Analysis in the Service Sector with Data Envelopment Analysis.* (ISBN 0-9580550-1-7). Retrieved from www.uq.edu.au/financesite/

Bahari, A. R., & Emrouznejad, A. (2014). Influential DMUs and outlier detection in data envelopment analysis with an application to health care. *Annals of Operations Research, 223*(1), 95-108. doi:10.1007/s10479-014-1604-8

Baier, S. L., Dwyer, G. P., & Tamura, R. (2002). How Important Are Capital and Total Factor Productivity for Economic Growth? *Federal Reserve Bank of Atlanta, Working Paper Series.* 2002-2a, 2-51.

Banker, R. D. (1984). Estimating most productive scale size using data envelopment analysis [Abstract]. *European Journal of Operational Research, 17*(1), 35-44. doi:10.1016/0377-2217(84)90006-7

Banker, R. D., & Chang, H. (2006). The super-efficiency procedure for outlier identification, not for ranking efficient units. *European Journal of Operational Research, 175*(2), 1311-1320. doi:10.1016/j. ejor.2005.06.028

Banker, R. D., & Morey, R. C. (1986). Efficiency Analysis for Exogenously Fixed Inputs and Outputs. *Operations Research, 34*(4), 513-521. doi:10.1287/opre.34.4.513

Banker, R. D., & Natarajan, R. (2008). Evaluating Contextual Variables Affecting Productivity Using Data Envelopment Analysis. *Operations Research, 56*(1), 48-58. doi:10.1287/opre.1070.0460

Banker, R. D., Conrad, R. F., & Strauss, R. P. (1986). A Comparative Application of Data Envelopment Analysis and Translog Methods: An Illustrative Study of Hospital Production. *Management Science, 32*(1), 30-44. doi:10.1287/mnsc.32.1.30

Banker, R. D., Zheng, Z. (., & Natarajan, R. (2010). DEA-based hypothesis tests for comparing two groups of decision making units. *European Journal of Operational Research, 206*(1), 231-238. doi:10.1016 /j.ejor.2010.01.027

Banker, R., Charnes, A., & Cooper, W. (1984). Some models for estimating technical and scale efficiencies in Data Envelopment Analysis. *Journal of Management Science, 30*, 1078-1092.

Banker, R., Charnes, A., Cooper, W., D, S., & D, T. (1989). An introduction to data envelopment analysis with some of its models and their uses. Researches in governmental and non-profit accounting. [Abstract]. *European Journal of Operations Research, 1*, 125-163.

Banker, R., Zheng, E., & Natarajan, R. (2010). DEA-based hypothesis test for comparing two groups of decision making units. *European Journal of Operational Research, 101-8*, 1-8.

Barros, C. P., & Alves, C. A. (2003). Hypermarket retail store efficiency in Portugal. *International Journal of Retail & Distribution Management, 31*(11), 549-560. doi:10.1108/09590550310503285

Barth, J. (. (2007). Customer engagement and the operational efficiency of wine retail stores. *International Journal of Wine Business Research, 19*(3), 207-215. doi:10.1108/17511060710817230

Beasley, J. (1995). Determining Teaching and Research Efficiencies. *Journal of Operational Research Society, 46*, 441-452.

Becker, M., Bottcher, M., & Klinger, S. (2013). Systemizing Service Classifications. *University of Leipzig.*, 1-17. Bench marking and continuous improvement. *10th Annual Effective Management*

Behr, A. (2016). *Production and efficiency analysis with R*. New York, NY: Springer Berlin Heidelberg.

Berg, S. A., Førsund, F. R., Jansen, E. S., Berg, S. A., & Førsund, F. R. (1992). Malmquist Indices of Productivity Growth during the Deregulation of Norwegian Banking, 1980-89. *The Scandinavian Journal of Economics, 94*. doi:10.2307/3440261

Bhanot, N., Singh, H., & Bhatti, R. S. (n.d.). Benchmarking of Indian Rail Freight by DEA. *Encyclopedia of Business Analytics and Optimization,* 273-291. doi:10.4018/978-1-4666-5202-6.ch026

Biondi, S., Calabrese, A., Capece, G., Costa, R., & Di Pillo, F. (2013). A New Approach for Assessing Dealership Performance: An Application for the Automotive Industry. *International Journal of Engineering Business Management*, 1-8. Doi: 10.5772/56662

Bititci, U. S., Bourne, M., Cross, J. F., Nudurpati, S. S., & Sang, K. (2015). Towards a theoretical foundation for Performance Measurement and Management. *International Journal of Management reviews*, *1*, 1-4.

Bititci, U., Mendibil, K., Turner, T., & Garengo, P. (n.d.). Measuring Business Excellence. Retrieved.November25,2013, from http://www.emeraldinsight.com/journals.htm?articleid

Bogetoft, P. (2012). *Performance benchmarking: Measuring and managing performance*. New York: Springer.

Bogetoft, P., & Otto, L. (2011). *Benchmarking with DEA, SFA, and R*. New York, NY: Springer.

Boussofiane, A., Dyson, R., & Thanassoulis, E. (1991). Applied data envelopment analysis. *European Journal of Operational Research, 52*(1), 1-15. doi:10.1016/0377-2217(91)90331-0

Bowlin, W. F. (1998). Measuring Performance: An Introduction to Data Envelopment Analysis (DEA). *The Journal of Cost Analysis, 15*(2), 3-27. doi:10.1080/08823871.1998.10462318

Broman. M. (2004). Assessing Productivity in Assembly Systems. *Thesis Department of Production Engg. Royal Institute of Technology. Stockholm*.

Büschken, J. (2007). Determinants of Brand Advertising Efficiency: Evidence from the German Car Market. *Journal of Advertising, 36*(3), 51-73. doi:10.2753/joa0091-3367360304

C, J., Havas, & Aliber, M. (1993). Analysis of economic efficiency in Agriculture: A non - parametric approach [Abstract]. *Journal of Agriculture and Resource Economics, 18,*1-16

Canadian Infrastructure. (2013). Retrieved January 11, 2016, from http://www.chamber.ca/

Carter, B. R. (2015, September 22). An introduction to Automobile Dealerships. *Appraisal Journal*.

Castelli, L., Pesenti, R., & Ukovich, W. (2001). DEA-like models for efficiency evaluations of specialized and interdependent units. *European Journal of Operational Research, 132*(2), 274-286. doi:10.1016/s0377-2217(00)00151-x

Castelli, L., Pesenti, R., & Ukovich, W. (2008). A classification of DEA models when the internal structure of the Decision-Making Units is considered. *Annals of Operations Research Annals of Operations Research, 173*(1), 207-235. doi:10.1007/s10479-008-0414-2

Castelli, L., Pesenti, R., & Ukovich, W. (2010). A classification of DEA models when the internal structure of the Decision-Making Units is considered. *Annals of Operations Research Annals of Operations Research, 173*(1), 207-235. doi:10.1007/s10479-008-0414-2

Caves, D. W., Christensen, L. R., & Diewert, W. E. (1982). The Economic Theory of Index Numbers and the Measurement of Input, Output, and Productivity. *Econometrica*, *50*(6), 1393. doi:10.2307/1913388

Cervus Equipment - Canada's Largest Authorized John Deere and Peterbilt Dealer Group. (n.d.). Retrieved December 11, 2015, from http://www.cervusequipment.com/

Charles, V., & Kumar, M. (2012). *Data envelopment analysis and its applications to management*. Newcastle upon Tyne: Cambridge Scholars Publishing.

Charnes, A. (. (2010). *Data envelopment analysis: Theory, methodology, and application*. Milton

Keynes UK: Lightning Source UK.

Charnes, A., Cooper, W. W., & Thrall, R. M. (1986). A Structure for Classifying and Characterizing Efficiencies and Inefficiencies in Data Envelopment Analysis. doi:10.21236/ada176141

Charnes, A., Cooper, W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operations Research,2*,429-444.Doi:10.1016/0377-2217 (79)90229-7

Charnes, A., Cooper, W., & Rhodes, E. (1979). Measuring the efficiency of decision-making units. *European Journal of Operational Research, 3*(4), 339. Doi:10.1016/0377-2217(79)90229-7

Charnes, A., Cooper, W., Golany, B., Seiford, L., & Stutz, J. (1985). Foundations of data envelopment analysis for Pareto-Koopmans efficient empirical production functions. *Journal of Econometrics, 30*(1-2), 91-107. doi:10.1016/0304-4076(85)90133-2

Charnes, A., Cooper, W., Seiford, L., & Stutz, J. (1982). A multiplicative model for efficiency analysis. *Socio-Economic Planning Sciences, 16*(5), 223-224. doi:10.1016/0038-0121(82)90029-5

Charnes, A., Haag, S., Jaska, P., & Semple, J. (1992). Sensitivity of efficiency classifications in the additive model of data envelopment analysis. *International Journal of Systems Science, 23*(5), 789-798. doi:10.1080/00207729208949248

Chen, W., & Johnson, A. L. (2010). A unified model for detecting efficient and inefficient outliers in data envelopment analysis. *Computers & Operations Research, 37*(2), 417-425. doi: 10.1016/j.cor.2009.06.010

Chen, Y. (2005). Measuring super-efficiency in DEA in the presence of infeasibility. *European Journal of Operational Research, 161*(2), 545-551. doi: 10.1016/j.ejor.2003.08.060

Chen, Y. (2011). Productivity of Automobile Industries using the Malmquist Index: Evidence from the Last Economic Recession. *JCC: The Business and Economics Research Journal of CENTRUM Cathedra: The Business and Economics Research Journal, 4*(2), 165-181. doi:10.7835/jcc-berj-2011-0056

Chen, Y., Liang, L., & Yang, F. (2006). A DEA game model approach to supply chain efficiency. *Annals of Operations Research Annals of Operations Res, 145*(1), 5-13. doi:10.1007/s10479-006-0022-y

Cheng, G., & Zerovopoulos, P. (2012). *Munich Personal a RePEc Archive,* 1-20. Retrieved from http://mpra.ub.uni-muenchen.de/42064

Choong, K. K. (2014). A systematic review for research and applications. *International Journal of Production Research, 52*(14), 4174-4197. doi:10.1080/00207543.2013.866285

Coelli, T. J. (1995). Recent Developments in Frontier Modelling and Efficiency Measurement [Abstract]. *Australian Journal of Agricultural Economics, 39*(3), 219-245. doi:10.1111/j.1467-8489. 1995.tb00552.x

Coelli, T. J., Battese, G. E., O'Donnell, C. J., & Rao, D. P. (2005). *An Introduction to Efficiency and Productivity Analysis*. Boston (MA): Springer.

Cook, W. D., & Seiford, L. M. (2009). Data envelopment analysis (DEA) – Thirty years on. *European Journal of Operational Research, 192*(1), 1-17. doi: 10.1016/j.ejor.2008.01.032

Cook, W. D., & Green, R. H. (2005). Evaluating power plant efficiency: A hierarchical model. *Computers & Operations Research, 32*(4), 813-823. doi: 10.1016/j.cor.2003.08.019

Cook, W. D., & Seiford, L. M. (2009). Data envelopment analysis (DEA) – Thirty years on. *European Journal of Operational Research, 192*(1), 1-17. doi: 10.1016/j.ejor.2008.01.032

Cook, W. D., & Zhu, J. (2005). *Modeling performance measurement: Applications and* *implementation issues in DEA*. New York: Springer Science Business media.

Cook, W. D., Liang, L., & Zhu, J. (2010). Measuring performance of two-stage network structures by DEA: A review and future perspective. *Omega*, *38*(6), 423-430. doi: 10.1016/j.omega.2009.12.001

Cook, W. D., Tone, K., & Zhu, J. (2014). Data envelopment analysis: Prior to choosing a model. *Omega*, *44*, 1-4. doi:10.1016/ j. omega.2013.09.004

Cook, W., & Green, R. (2004). Multicomponent efficiency measurement and core business identification in multi plant firms: A DEA model. *European Journal of Operational Research, 157*, 540-551.

Cook, W., & Green, R. (2004). Multicomponent efficiency measurement and core business identification in multi plant firms: A DEA model. *European Journal of Operational Research*, *157*, 540-551.

Cook, W., & Roll, Y. (1993). Partial Efficiencies in Data Envelopment Analysis. *Socio-Econ Planning Sciences, 27*(3), 171-179.

Cook, W., & Zhu, J. (2005). *Modeling performance measurement applications and implementation issues in DEA*. New York: Springer.

Cook, W., Hababou, M., & Teunter, H. (2005). Multicomponent Efficiency Measurement and Shared Inputs in Data Envelopment Analysis: An application to sales and service Performance in Bank Branches. *CIBC Project#(192551),* 1-20.

Cooper, W. W. (2014). Origin and Development of Data Envelopment Analysis: Challenges and Opportunities. *Data Envelopment Analysis Journal, 1*(1), 3-10. doi:10.1561/103.00000002

Cooper, W. W., Seiford, L. M., & Tone, K. (2007). *Data envelopment analysis: A comprehensive text with models, applications, references and DEA-solver software*. New York: Springer.

Cron, W. L., & Sobol, M. G. (1983). The relationship between computerization and performance: A strategy for maximizing the economic benefits of computerization. *Information & Management*, *6*(3), 171-181. Doi :10.1016/0378-7206(83)90034-4

Cullinane, K., Song, D., Ji, P., & Wang, T. (2004). An Application of DEA Windows Analysis to Container Port Production Efficiency. *Review of Network Economics, 3*(2). doi:10.2202/1446 9022.1050

Cummins, J. D., & Weiss, M. A. (2013). Analyzing Firm Performance in the Insurance Industry Using Frontier Efficiency and Productivity Methods. *Handbook of Insurance,* 795-861. doi:10.1007/978-1-4614-0155-1_28

Darijani, M., Darijani, A., & Taboli, H. (2014). Measure and Analyze Total Factor Productivity Index in Automotive Industry: Case Study: Kerman Motor Assembles Saloon. *KCAJBMR Kuwait Chapter of Arabian Journal of Business and Management Review, 3*(8), 188-196. doi:10.12816/0018318

Dasgupta, S., Sarkis, J., & Talluri, S. (1999). Influence of information technology investment on firm productivity: A cross-sectional study. *Logistics Information Management*, *12*(1/2), 120-129. doi:10.1108/09576059910256493

Data Envelopment Analysis. (n.d.). Retrieved November 23, 2014, from http://moezh.tripod.com/DEAhome.html

Data Envelopment Analysis: Ali Emrouznejad 's DEA Homepage, The most comprehensive source of Data Envelopment Analysis!! (n.d.). Retrieved November 23, 2014, from http://www.deazone.com

Davenport, T. H., Harris, J. G., & Morison, R. (2010). Analytics at Work; Smarter Decisions, Better Results. *Harvard Business School Press.*

Dev, N. K., Shankar, R., & Debnath, R. M. (2014). Supply chain efficiency: A simulation cum

DEA approach. *The International Journal of Advanced Manufacturing Technology International Journal of Advanced Manufacturing Technology, 72*(9-12), 1537-1549. doi:10.1007/s00170-014-5779-6

Donthu, N., & Yoo, B. (1998). Retail productivity assessment using data envelopment analysis. *Journal of Retailing, 74*(1), 89-105. doi:10.1016/s0022-4359(99)80089-x

Doyle, J., & Green, R. (1994). Efficiency and Cross-efficiency in DEA: Derivations, Meanings and Uses. *Journal of the Operational Research Society, 45*(5), 567-578. doi:10.1038/sj/jors/0450509

Drost, E. A. (2011). Validity and Reliability in Social Science Research. *Education Research and Perspectives, 38*(1), 105-124. doi: California State University, Los Angeles

Drummond, D. (2015). Progress on the Mystery of Productivity: A Review Article on the OECD Report, The future of Productivity. *International Productivity Monitor, 29*, 83-88.

Duan, S. X., & Deng, H. (2016). Data Envelopment Analysis of the efficiency of Australian Universities: An empirical Study. 1-13. Retrieved from http://www.pacis2016.org/Abstract/ALL/514.pdf

Dubelaar, C., Bhargava, M., & Ferrarin, D. (2002). Measuring retail productivity: What really matters? *Journal of Business Research, 55*(5), 417-426. doi:10.1016/s0148-2963(00)00160-0

Dyson, R. G. (2001). Strategy, Performance and Operational Research. *The Journal of the Operational Research Society, 51*(1), 5-11. doi:10.2307/253942

Dyson, R. G., & Thanassoulis, E. (1988). Reducing Weight Flexibility in Data Envelopment Analysis. *Journal of the Operational Research Society, 39*(6), 563-576. doi:10.1057/ Palgrave. jors.0390606

Efron, B. (1979). Bootstrap Methods: Another Look at the Jackknife. *The Annals of Statistics,* *7*(1), 1-26. doi:10.1214/aos /1176344552

Elahi, M., Afshar, Z. A., & Hooshangi, M. (2014). Classification of Iran Automotive and Parts Manufacturing stocks by DEA. *Data Envelopment Analysis and Decision Science, 2014*, 1-11. doi:10.5899/2014/dea-00047

Eliyasu, A., & Mohamed, Z. A. (2016). Evaluating contextual factors affecting the technical efficiency of freshwater pond culture systems in Peninsular Malaysia: A two-stage DEA approach. *Aquaculture Reports, 3*, 12-17. doi: 10.1016/j.aqrep.2015.11.002

Emrouznejad, A., & Witte, K. D. (2010). COOPER-framework: A unified process for non-parametric projects. *European Journal of Operational Research, 207*(3), 1573-1586. doi: 10.1016/j.ejor.2010.07.025

Emrouznejad, A., & Yang, G. (2017). A survey and analysis of the first 40 years of scholarly literature in DEA: 1978–2016. *Socio-Economic Planning Sciences*. doi: 10.1016/j.seps.2017.01.008

Emrouznejad, A., Parker, B. R., & Tavares, G. (2008). Evaluation of research in efficiency and productivity: A survey and analysis of the first 30 years of scholarly literature in DEA. *Socio-Economic Planning Sciences, 42*(3), 151-157. doi: 10.1016/j.seps.2007.07.002

Envelopment Analysis. *Applied Economics Letters, 17*(4), 313-316.

Equipment & Services: Company Profile. (n.d.). Retrieved December 10, 2015, from http://www.strongco.com/

Eshlaghi, A. T., & Jamalou, F. (2011). A Hybrid Mathematical Model for Product Positioning Based on DEA. (Case Study: Automobile Dealership in UAE-Dubai) [Abstract]. *Applied Mathematical Sciences, 5*(61), 301-3020.

Faraji, O., & Fushazedh, A. B. (2016, January). Measuring the efficiency of Bank Branches Using Data Envelopment Analysis (Case Study: City Bank) [Abstract]. *Global Journal of*

Management Studies and Researches, 3(1), 1-6.

Färe, R., & Grosskopf, S. (2000). Network DEA. *Socio-Economic Planning Sciences, 34*(1), 35-49. doi:10.1016/s0038-0121(99)00012-9

Färe, R., Grabowski, R., & Grosskopf, S. (1985). Technical efficiency of Philippine agriculture [Abstract]. *Applied Economics, 17*(2), 205-214. doi:10.1080/00036848500000018

Färe, R., Grabowski, R., Grosskopf, S., & Kraft, S. (1997). Efficiency of a fixed but allocable input: A non-parametric approach. *Economics Letters, 56*(2), 187-193. doi:10.1016/s0165-1765(97)81899-x

Färe, R., Grosskopf, S., & Whittaker, G. (n.d.). Network DEA. *Modeling Data Irregularities and Structural Complexities in Data Envelopment Analysis,* 209-240. doi:10.1007/978-0-387-71607-7_12

Färe, R., Grosskopf, S., Lindgren, B., & Poullier, J. (1997). Productivity Growth in Health-Care Delivery. *Medical Care, 35*(4), 354-366. doi:10.1097/00005650-199704000-00006

Fare, R., Grosskopf, S., Lindgren, B., & Roos, P. (1992). Productivity changes in Swedish Pharmacies 1980?1989: A non-parametric Malmquist approach. *Journal of Productivity Analysis, 3*(1-2), 85-101. doi:10.1007/bf00158770

Fare, R., Grosskopf, S., Norris, M., & Zhang, Z. (1994). Productivity Growth, Technical Progress, and Efficiency Change in Industrialized Countries. *The American Economic Review, 84*(1), 66-83. Retrieved from http://www.jstor.org/stable/2117971

Farrell, M. (1957). The measurement of productive efficiency. *Journal of the Royal Statistical Society, 120*, 253-281. Retrieved November 2, 2014.

Farrell, M. J., & Fieldhouse, M. (1962). Estimating Efficient Production Functions under Increasing Returns to Scale. *Journal of the Royal Statistical Society. Series A (General), 125*(2), 252. doi:10.2307/2982329

Førsund, F. R., & Kalhagen, K. O. (1999). Efficiency and Productivity of Norwegian Colleges. *Data Envelopment Analysis in the Service Sector,* 269-308. doi:10.1007/978-3-663-08343-6_13

Fox, K. J., Hill, R. J., & Diewert, W. E. (2004). Identifying Outliers in Multi-Output Models. *Journal of Productivity Analysis, 22*(1/2), 73-94. doi:10.1023/b: prod.0000034692. 22162.e8

Fried, H. O., A, L. C., K, S, S. S., & Yaisawarng, S. (2002). Accounting for Environmental Effects and Statistical Noise in Data Envelopment Analysis. *Journal of Productivity Analysis, 17*(1-2), 157-174.

Fried, H. O., Lovell, C. A., & Shelton, S. S. (2008). Efficiency and Productivity. *The Measurement of Productive Efficiency and Productivity Change,* 3-91. doi:10.1093/ /9780195183528.003.0001 from http://www.rockymtn.com/

Fried, H. O., Schmidt, S. S., & Yaisawarng, S. (1999). Incorporating the operating environment in to a non-parametric Measure of Technical Efficiency. *Journal of Productivity Analysis, 12*(3), 249-267.
from *Association of Equipment Distributors* (AED)

Galagedera, D. (., & Edirisuriya, P. (2004). Performance of Indian Commercial Banks (1995-2002): An Application of Data Envelopment Analysis and Malmquist Productivity Index. *SSRN Electronic Journal*. doi:10.2139/ssrn.577922

Gandhi, A., & Shankar, R. (2014). Efficiency measurement of Indian retailers using Data Envelopment Analysis. *International Journal of Retail & Distribution Management*, *42*(6), 500-520. doi:10.1108/ijrdm-10-2012-0094

Gattoufi, S., Oral, M., Kumar, A., & Reisman, A. (2004). Content analysis of data envelopment analysis literature and its comparison with that of other OR/MS fields. *J Oper Res Soc*

Journal of the Operational Research Society, 55(9), 911-935. doi: 10.1057/palgrave.jors.2601767

Ghasemi, V., Lotfi, H., & L, T. (2012). Multicomponent Balanced Scorecard for Perspectives Efficiency Measurement. *Applied Mathematical Sciences, 6*(9), 403-410.

Ghasemi, V., Lotfi, H., & L, T. (2012). Multicomponent Balanced Scorecard for Perspectives Efficiency Measurement. *Applied Mathematical Sciences, 6*(9), 403-410.

Ghobadian, A., & Husband, T. (1990). Measuring total productivity using production functions. *International Journal of Production Research, 28*(8), 1435-1446. doi:10.1080/00207549008942803

Gillen, D., & Lall, A. (1997). Developing measures of airport productivity and performance: an application of data envelopment analysis. *Transportation Research Part E: Logistics and Transportation Review, 33*(4), 261-273. doi:10.1016/s1366-5545(97)00028-8

Girdharry, K., Simonova, E., & Lefebvre, R. (2012). Investment in Machinery and Equipment is Essential to Canada's Future. Retrieved January 11, 2016, from http://www.international.gc.ca/

Gladovic, P., Ralevic, P. V., Mladenovic, S., Denic, N., & Dobrodolac, M. (2012). Quantification of airlines business efficiency using data envelopment analysis (DEA). *Afr. J. Bus. Manage. AFRICAN JOURNAL OF BUSINESS MANAGEMENT, 6*(25). doi:10.5897/ajbm11.1713

Golany, B., & Roll, Y. (1989). An application procedure for DEA. *Omega, 17*(3), 237-250. doi:10.1016/0305-0483(89)90029-7

Goncharuk, A. G., & Figurek, A. (2017). Efficiency of winemaking in developing countries. *International Journal of Wine Business Research, 29*(1), 98-118. doi:10.1108/ijwbr-02-2016-0007

González, E., Cárcaba, A., & Ventura, J. (2015). How car dealers adjust prices to reach the product efficiency frontier in the Spanish automobile market. *Omega, 51*, 38-48. doi: 10.1016/j.omega.2014.08.005

González, E., Ventura, J., & Cárcaba, A. (2013). Product efficiency in the Spanish automobile market. *Investigaciones Europeas De Dirección Y Economía De La Empresa, 19*(1), 1-7. doi: 10.1016/j.iedee.2012.07.003

Graham, A., Wang, M., & Narasimhan, S. (2005). Relative efficiency and Quality of Global Auto Companies. *The Journal of Applied Business Research, 21-4*, 71-82.

Green, R. H., & Doyle, J. R. (1997). Implementing Data Envelopment Analysis: Primal Or Dual? *INFOR: Information Systems and Operational Research, 35*(1), 66-75. doi:10.1080/03155986.1997.11732319

Green, R. H., Doyle, J. R., & Cook, W. D. (1996). Preference voting and project ranking using DEA and cross-evaluation. *European Journal of Operational Research, 90*(3), 461-472. doi:10.1016/0377-2217(95)00039-9

Grewal, D., Levy, M., Mehrotra, A., & Sharma, A. (1999). Planning merchandising decisions to account for regional and product assortment differences. *Journal of Retailing, 75*(3), 405-424. doi:10.1016/s0022-4359(99)00015-9

Grifell-Tatjé, E., & Lovell, C. A. (1999). A generalized Malmquist productivity index. *Top, 7*(1), 81-101. doi:10.1007/bf02564713

Grönroos, C., & Ojasalo, K. (2004). Service productivity. *Journal of Business Research, 57*(4), 414-423. doi:10.1016/s0148-2963(02)00275-8

Gupta, A., & Mittal, S. (2010). Measuring retail productivity of food & grocery retail outlets using the DEA technique. *Journal of Strategic Marketing, 18*(4), 277-289. doi:10.1080/09652540903537055

Hair, J. (2010). *Multivariate data analysis* (7th edn.). Upper Saddle River, NJ: Prentice Hall.

Harrison, J. (2010, March). Data Envelopment Analysis: A Method for measuring efficiency,

Harrison, J., & Rouse, P. (2016). DEA and Accounting Performance Measurement. *Handbook of Operations Analytics Using Data Envelopment Analysis International Series in Operations Research & Management Science*, 385-412. doi:10.1007/978-1-4899-7705-2_15

Hawas, Y., Khan, M., & Basu, N. (2012). Evaluating and Enhancing the Operational Performance of Public Bus Systems Using GIS-based Data Envelopment Analysis. *Journal of Public Transportation JPT, 15*(2), 19-44. doi:10.5038/2375-0901.15.2.2

Heale, R., & Twycross, A. (2015). Validity and reliability in quantitative studies. *Evidence Based Nursing, 18*(3), 66-67. doi:10.1136/eb-2015-102129

He, B. (n.d.). Measuring the operation efficiency of a car dealer in Taiwan by using Data Envelopment Analysis. *Recent Advances in Energy and Environment Integrated Systems, ISBN: 976-1-61804-181-4*, 60-65.

Hillier, F., & Lieberman, G. (n.d.). *Introduction to operations research* (Tenth edn.).

Holweg, M., Luo, J., & Oliver, N. (2009). The past, present and future of China's automotive industry: A value chain perspective. *International Journal of Technological Learning, Innovation and Development IJTLID, 2*(1/2), 76. doi:10.1504/ijtlid.2009.021957

Hsiao, J. (2011). DEA for Establishing Performance Evaluation Models: A Case Study of a Ford Car dealer in Taiwan. *Recent Advances in Energy and Environment Integrated Systems, ISBN: 978-960-474-364-3*.

Iliyasu, A., & Mohamed, Z. A. (2016). Evaluating contextual factors affecting the technical efficiency of freshwater pond culture systems in Peninsular Malaysia: A two-stage DEA approach. *Aquaculture Reports, 3*, 12-17. doi:10.1016/ j. aqrep.2015.11.002

Innovation, 2100-2104. doi:10.2991/asei-15.2015.412

International Standards Organization. (n.d.). Retrieved December 12, 2015, from http://www.iso.org/

Iraizoz, B., Rapún, M., & Zabaleta, I. (2003). Assessing the technical efficiency of horticultural production in Navarra, Spain [Abstract]. *Agricultural Systems*, *78*(3), 387-403. doi:10.1016/s0308-521x (03)00039-8

Ishaq, S. (2015). A Novel Application of DEA and Production Trade Offs for Efficiency evaluations of Banking Institutions. *Ph. D Thesis of University of Warwick*.

Izadbaksh, H., Ali, M. H., Montazeri, A., & Saberi, M. (2009). Performance Assessment and Optimization of After Sales Networks. *International Journal of Mechanical, Aerospace, Industrial, Mechatronic, and Manufacturing Engg.*, *3*(1), 101-105.

Jain, R. K., & Natarajan, R. (2015). A DEA study of airlines in India [Abstract]. *Asia Pacific Management Review*, *20*(4), 285-292. doi: 10.1016/j.apmrv.2015.03.004

Jia, T., & Yuan, H. (2017). The application of DEA (Data Envelopment Analysis) window analysis in the assessment of influence on operational efficiencies after the establishment of branched hospitals. *BMC Health Services Research*, *17*(1). doi:10.1186/s12913-017-2203-6

Johnson, A. L., & McGinnis, L. F. (2008). Outlier detection in two-stage semiparametric DEA models. *European Journal of Operational Research*, *187*(2), 629-635. doi: 10.1016/j.ejor.2007.03.041

Joo, S., Nixon, D., & Stoeberl, P. A. (2011). Benchmarking with data envelopment analysis: A return on asset perspective. *Benchmarking: An International Journal*, *18*(4), 529-542. doi:10.1108/14635771111147623

Joo, S., Stoeberl, P. A., & Fitzer, K. (2009). Measuring and benchmarking the performance of coffee stores for retail operations. *Benchmarking: An International Journal*, *16*(6), 741-753. doi:10.1108/14635770911000088

Jorge, D. (2008). Efficiency and regulation in Spanish hypermarket retail trade – Across-section approach. *International Journal of Retail and Distribution Management*, *36*(1), 71-88.

Joseph, J. (2015). Determination of the relative productivity of Automobile Dealers in Kerala through Data Envelopment Analysis with special reference to the smallest district Alappuzha. *Research Journal of Science, Engineering and Technology, 5*(3), 11-19. Retrieved from ISSN 2454-3195.

Jreisat, A., & Hassan, H. (2016, May). Productivity Change of the Egyptian Banking Sector: A Two Stage Non-Parametric Approach [Abstract]. *Topics in Middle Eastern and African Economies*, *18*(1), 145-155.

Kalirajan, K. P., & Shand, R. T. (1999). Frontier Production Functions and Technical Efficiency Measures. *Journal of Economic Surveys J Economic Surveys*, *13*(2), 149-172. doi:10.1111/1467-6419.00080

Kamakura, W. A., Lenartowicz, T., & Ratchfrord, B. T. (1996). Productivity assessment of multiple retail outlets. *Journal of Retailing, 72*(4), 333-356. doi:10.1016/s0022-4359(96)90018-4

Kao, C. (2009). Efficiency measurement for parallel production systems. *European Journal of Operational Research, 196*(3), 1107-1112. doi: 10.1016/j.ejor.2008.04.020

Kao, C. (2011). Efficiency decomposition for parallel production systems. *Journal of the Operational Research Society, 63*(1), 64-71. doi:10.1057/jors.2011.16

Kao, C. (2014). Network data envelopment analysis: A review. *European Journal of Operational Research*, *239*(1), 1-16. doi: 10.1016/j.ejor.2014.02.039

Kao, C. (2016). *Network data envelopment analysis: Foundations and extensions*. Place of publication not identified: Springer.

Kao, C., & Hwang, S. (2008). Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan. *European Journal of Operational Research, 185*(1), 418-429. doi: 10.1016/j.ejor.2006.11.041

Kao, C., & Hwang, S. (2008). Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan. *European Journal of Operational Research, 185*(1), 418-429. doi: 10.1016/j.ejor.2006.11.041

Kao, C., & Liu, S. (2014). Multi-period efficiency measurement in data envelopment analysis: The case of Taiwanese commercial banks. *Omega, 47*, 90-98. doi: 10.1016/j.omega.2013.09.001

Karim Zadeh, M. (2012, September). Efficiency Analysis by Using Data Envelopment Analysis Model: Evidence from Indian Banks. *International Journal of Latest Trends in Finance and Economics, 2*(2), 228-237. doi: IJLTFES, E-ISSN-2047-0916

Kawaguchi, H., Tone, K., & Tsutsi, M. (2013). Estimation of the efficiency of Japanese hospitals using a dynamic and network data envelopment analysis model. *Health Care Management Science, 17*(2), 101-112. doi:10.1007/s10729-013-9248-9

Keh, H. T., & Chu, S. (2003). Retail productivity and scale economies at the firm level: A DEA approach. *Omega, 31*(2), 75-82. doi:10.1016/s0305-0483(02)00097-x

Kelly, E., Shalloo, L., Geary, U., Kinsella, A., & Wallace, M. (2012). Application of data envelopment analysis to measure technical efficiency on a sample of Irish dairy farms. *Irish Journal of Agricultural and Food Research, 61*, 63-77.

Kerin, R. A., Hartley, S. W., & Rudelius, W. (2016). *Marketing* (13th ed.). New York, NY: McGraw Hill.

Kimes, S. E., & Singh, S. (2008). Spa Revenue Management. *Cornell Hospitality Quarterly,* *50*(1), 82-95. Doi :10.1177/1938965508324868

Kirjavainen, T., & Loikkanen, H. (n.d.). Efficiency differences of Finnish senior secondary schools: an application of DEA Tobit-analysis. *Innovation in Technology Management. The Key to Global Leadership. PICMET 97*. doi:10.1109/picmet.1997.653646

Ko, K., Chang, M., Bae, E., & Kim, D. (2017). Efficiency Analysis of Retail Chain Stores in Korea. *Sustainability*, *9*(9), 1629. doi:10.3390/su9091629

Kotler, P., & Keller, K. L. (2015). *Marketing management* (15th edn.). New York, NY.

Kumar, S. (2011). State road transport undertakings in India: Technical efficiency and its determinants. *Benchmarking: An International Journal, 18*(5), 616-643. doi:10.1108/14635771111166794

Kumar, S., & Gulati, R. (2009). Income-Generating Efficiency of Public Sector Banks in India: An Application of Data Envelopment Analysis. *Artha Vijnana : Journal of The Gokhale Institute of Politics and Economics, 51*(2), 103. doi:10.21648/arthavij/2009/v51/i2/115392

Kumar, S., & Gulati, R. (2013). Relevance of Non-traditional Activities on the Efficiency of Indian Banks. *India Studies in Business and Economics Deregulation and Efficiency of Indian Banks,* 167-200. doi:10.1007/978-81-322-1545-5_5

Lafontaine, F., & Morton, F. S. (2010). Markets: State Franchise Laws, Dealer Terminations, and the Auto Crisis. *Journal of Economic Perspectives, 24*(3), 233-250. doi:10.1257/jep.24.3.233

Lei, X., Li, Y., Xie, Q., & Liang, L. (2014). Measuring Olympics achievements based on a parallel DEA approach. *Annals of Operations Research 226*(1), 379-396. doi:10.1007/s10479-014-1708-1

Liang, L., Yang, F., Cook, W. D., & Zhu, J. (2006). DEA models for supply chain efficiency evaluation. *Annals of Operations Research, 145*(1), 35-49. doi:10.1007/s10479-006-0026-7

Lin, T. T., Lee, C., & Chang, F. (2010). A performance management on automobile dealers with applying data envelopment analysis. *2010 IEEE International Conference on Industrial Engineering and Engineering Management*, 1-5. doi:10.1109/ieem.2010.5674607

Linna, M., & Häkkinen, U. (1998). A Comparative Application of Econometric Frontier and DEA Methods for Assessing Cost Efficiency of Finnish Hospitals. *Developments in Health Economics and Public Policy Health, the Medical Profession, and Regulation*, 169-187. doi:10.1007/978-1-4615-5681-7_9

Liu, J. S., Lu, L. Y., Lu, W., & Lin, B. J. (2013). Data envelopment analysis 1978–2010: A citation-based literature survey. *Omega, 41*(1), 3-15. doi: 10.1016/j.omega.2010.12.006

Liu, J., Lu, L., Lu, W., & Lin, B. (2008). A survey of DEA Applications. *ELSEVIER, 41*, 893-902.

Liu, W. B., Wongchai, A., & Peng, K. (2012). Adopting Super-Efficiency and Tobit Model on Analyzing the Efficiency of Teacher's colleges in Thailand. *International Journal on new trends in education and their applications, 3*(3), 17th ser., 1-13. doi: ISSN 1309-6249

Lorcu, F., & Bolat, B. A. (2015, December). Comparison of secondary education PISA results in European member states and Turkey via DEA and SEM. [Abstract]. *Journal of WEI Business and Economics, 4*(3), 7-17.

Lothgren, M., & Tambour, M. (1999). Bootstrapping the data envelopment analysis Malmquist productivity index. *Applied Economics, 31*(4), 417-425. doi:10.1080/000368499324129

Lovell, C. A., & Rouse, A. P. (2003). Equivalent standard DEA models to provide super-efficiency scores. *Journal of the Operational Research Society, 54*(1), 101-108. doi:10.1057/ palgrave. jors.2601483

Lu, W., Wang, W., & Kweh, Q. L. (2014). Intellectual capital and performance in the Chinese life insurance industry. *Omega, 42*(1), 65-74. doi:10.1016/ j. omega.2013.03.002

Luo, X. (2003). Evaluating the profitability and marketability efficiency of large banks [Abstract]. *Journal of Business Research, 56*(8), 627-635. doi:10.1016/s0148-2963(01)00293-4

Malhotra, R., Malhotra, D., & Lafond, C. A. (2010). Benchmarking large U.S. retailers using a data envelopment analysis model. *Applications in Multicriteria Decision Making, Data Envelopment Analysis, and Finance Applications of Management Science*, 217-235. doi:10.1108/s0276-8976(2010)0000014014

Manasakis, C., Apostolakis, A., & Datseris, G. (2013). Using data envelopment analysis to measure hotel efficiency in Crete [Abstract]. *International Journal of Contemporary Hospitality Management, 25*(4), 510-535. doi:10.1108/09596111311322907

Marchand, M., & Raymond, L. (2008). Researching performance measurement systems. *International Journal of Operations & Production Management, 28*(7), 663-686. doi:10.1108/01443570810881802

Maria Da Conceição Sampaio De Sousa, & Stošić, B. (2005). Technical Efficiency of the Brazilian Municipalities: Correcting Nonparametric Frontier Measurements for Outliers. *Journal of Productivity Analysis, 24*(2), 157-181. doi:10.1007/s11123-005-4702-4

Marschall, P., & Flessa, S. (2011). Efficiency of primary care in rural Burkina Faso. A two-stage DEA analysis. *Health Economics Review, 1*(1), 5. doi:10.1186/2191-1991-1-5

Mateo, F. D., Coelli, T., & O’ Donnell, C. (2006). Optimal Paths and Costs Of Adjustment In Dynamic DEA Models: With Application To Chilean Department Stores. *Annals of Operations Research, 145*(1), 211-227. doi:10.1007/s10479-006-0034-7

Matsumoto, Y. (2011, March 22). Komatsu's business model through the product life cycle. *Discussion Paper Series. RIEB University. Kobe, Japan*.

Matthews, K., Xiao, Z., & Xu, Z. (2011). Rational Cost Inefficiency in Chinese Banks. *SSRN*

Electronic Journal SSRN Journal, 1-25. doi:10.2139/ssrn.1627992

Mcdonald, J. (2009). Using least squares and tobit in second stage DEA efficiency analyses. *European Journal of Operational Research*, *197*(2), 792-798. doi:10.1016/ j.ejor .2008.07.039

Melnyk, S. A., Bititci, U., Platts, K., Tobias, J., & Andersen, B. (2014). Is performance measurement and management fit for the future? *Management Accounting Research, 25*(2), 173-186. doi:10.1016/ j.mar.2013.07.007

Moffat, B., Valadkhani, A., & Harvie, C. (2009). Malmquist indices of productivity change in Botswana’s financial institutions. *Global Business and Economics Review, 11*(1), 28. doi:10.1504/gber.2009.025380

Monafared, M. S., & Safi, M. (2013). Network DEA: An application to analysis of academic performance. *Journal of Industrial Engineering International*. *9*(1), 15. doi:10.1186/2251-712x-9-15

Monafared, M., & Safi, M. (2011). Efficiency Analysis of Public Universities in Iran using DEA Approach: Importance of Stakeholder's perspective. *Journal of Industrial and Systems Engineering, 5*(4), 185-197. doi: ISSN:1735-8272

Monika, D., & Mariana, S. (2015). The Using of Data Envelopment Analysis in Human Resource Controlling [Abstract]. *Procedia Economics and Finance, 26*, 468-475. doi:10.1016/s2212-5671(15)00875-8

Moreno, J. D. (2010). Productivity growth of European retailers: A benchmarking approach. *Journal of Economic Studies, 37*(3), 288-313. doi:10.1108/01443581011061285

Moreno, P., & Lozano, S. (2012). A network DEA assessment of team efficiency in the NBA. *Annals of Operations Research ,214* (1), 99-124. Doi :10.1007/s10479-012-1074-9

Mostafa, M. M. (2009). Benchmarking the US specialty retailers and food consumer stores using data envelopment analysis. *International Journal of Retail & Distribution Management, 37*(8), 661-679. doi:10.1108/09590550910966178

Mujasi, P. N., Asbu, E. Z., & Puig-Junoy, J. (2016). How efficient are referral hospitals in Uganda? A data envelopment analysis and Tobit regression approach. *BMC Health Services Research, 16*(1). doi:10.1186/s12913-016-1472-9

Mutter, R. L., Rosko, M. D., Greene, W. H., & Wilson, P. W. (2010). Translating Frontiers into Practice: Taking the Next Steps toward Improving Hospital Efficiency. *Medical Care Research and Review, 68*(1 Suppl). doi:10.1177/1077558710384878

Naidoo, N. T., S, Y. V., Yadavalli, & G, N. A., Naidoo. (2016). A new approach for extreme values in data envelopment analysis. *Pakistan Journal of Statistics, 32*(4), 279-299.

Narasimhan, S. L., Graham, A. W., & Wang, M. (2011). Relative Efficiency and Quality Of Global Automobile Companies. *JABR Journal of Applied Business Research (JABR), 21*(4), 71-82. doi:10.19030/Jabr. v21i4.1458

Nataraja, N. R., & Johnson, A. L. (2011). Guidelines for using variable selection techniques in data envelopment analysis. *European Journal of Operational Research, 215*(3), 662-669. doi: 10.1016/j.ejor.2011.06.045

Nauenberg, E., Basu, K., & Chand, H. (1997). Hirschman–Herfindahl index determination under incomplete information. *Applied Economics Letters, 4*(10), 639-642. doi:10.1080/758533291

Navarro, T. M., & Roura, J. C. (2010). Growth and Productivity in the Service Sector: The State of the Art. *Instituto Univeritario De Analisis Económico Y Social,* Universito of Alcalá, Spain, 1139-6148, 4-54.

Nazaro, J., & Chodakowska, E. (2015). Measuring Productivity of Construction Industry in Europe with Data Envelopment Analysis. *Procedia Engineering, 122,* 204-212. doi:10.1016/ j.

proeng.2015.10.026

Neely, A., Marr, B., Adams, C., & Kapashi, N. (n.d.). Measuring eBusiness performance. *Business Performance Measurement*, 343-360. doi:10.1017/cbo9780511753695.022

Neely, A., Mills, J., Platts, K., Richards, H., Gregory, M., Bourne, M., & Kennerly, M. (2000). Performance measurement system design: Developing and testing a process-based approach. *International Journal of Production Management, 20* (10), 1119-1145.

Ng, I. C., & Nudurupati, S. (2010). Outcome -Based Service Contracts in the Defense Industry- Mitigating the challenges. *Journal of Service Management, 21-31*.

Norman, M., & Stoker, B. (1991). *Data envelopment analysis: The assessment of performance*. Chichester: Wiley.

Novaes, A. G., Silveira, S. F., & Medeiros, H. C. (2010). Efficiency and productivity analysis of the interstate bus transportation industry in Brazil. *Pesquisa Operational, 30*(2), 465-485. doi:10.1590/s0101-74382010000200012

Nudurupati, S., Bititci, U., Kumar, V., & Chan, F. (2011). State of the art literature review on performance measurement. *Computers & Industrial Engineering, 60*(2), 279-290. doi:10.1016/ j.cie.2010.11.010

Nunamaker, T. R. (1983). Measuring routine nursing service efficiency: A comparison of cost per patient day and data envelopment analysis models [Abstract]. *Health Services Research, 18*(1), 183-208.

Oral, M., Kettani, O., & Lang, P. (1991). A Methodology for Collective Evaluation and Selection of Industrial R&D Projects. *Management Science, 37*(7), 871-885. doi:10.1287/mnsc.37.7.871

Ozcan, Y. A. (2014). *Health Care Benchmarking and Performance Evaluation: An assessment using Data Envelopment Analysis (DEA)*. New York: Springer.

Paradi, J. C., Sherman, H. D., & Tam, F. K. (2018). *Data envelopment analysis in the financial services industry: A guide for practitioners and analysts working in operations research using DEA*. Cham: Springer.

Parthasarathy, S. (2010). A Computationally efficient procedure for Data Envelopment Analysis. *UMI Number: U615720*. doi: http://etheses.lse.ac.uk/2771/1/U615720.pdf

Parkan, C. (1987). Measuring the efficiency of service operations: An application to bank branches *Engineering Costs and Production Economics, 12*(1-4), 237-242. doi:10.1016/0167-188x(87)90083-8

Parkin, D., & Hollingsworth, B. (1997). Measuring production efficiency of acute hospitals in Scotland, 1991-94: Validity issues in data envelopment analysis. *Applied Economics*, *29*(11), 1425-1433. doi:10.1080/000368497326255

Partovi, F. Y., & Kim, R. J. (2013). Selecting A Fuel -Efficient Vehicle using Data Envelopment Analysis. *The Journal of Business and Economic Studies*, *19*(2), 54-80.

Pastor, J. T., Ruiz, J. L., & Sirvent, I. (1999). A statistical test for detecting influential observations in DEA. *European Journal of Operational Research, 115*(3), 542-554. doi:10.1016/s0377-2217(98)00153-2

Pastor, J. T., Ruiz, J. L., & Sirvent, I. (2002). A Statistical Test for Nested Radial Dea Models. *Operations Research, 50*(4), 728-735. doi:10.1287/opre.50.4.728.2866

Pasupathy, K. S. (2002). *Modelling Undesirable Outputs in Data Envelopment Analysis: Various Approaches.* (Unpublished doctoral dissertation). Virginia Polytechnic and State University.

Pedron, T., Akdeniz, B., & Calantone, R. (2014). Benchmarking sales staffing efficiency in dealership using extended data envelopment analysis. *Journal of Business Research, 67*, 1904-1911.

Performance Measurement in Equipment Distributors. 2012 Cost of doing Business. (n.d.). Retrieved December 11, 2015, from (http://www.aednet.org)

Périco, A. E., Santana, N. B., & Rebelatto, D. A. (2016). Estimating the efficiency from Brazilian banks: A bootstrapped Data Envelopment Analysis (DEA). *Production, 26*(3), 551-561. doi:10.1590/0103-6513.185714

Perrigot, R., & Barros, C. P. (2008). Technical efficiency of French retailers. *Journal of Retailing and Consumer Services, 15*(4), 296-305. doi:10.1016 /j.jretconser.2007.06.003

Piran, F. A., Lacerda, D. P., Camargo, L. F., Viero, C. F., Dresch, A., & Cauchick-Miguel, P. A. (2016, August 15). Product modularization and effects on efficiency: An analysis of a bus manufacturer using data envelopment analysis (DEA). *International Journal of Production Economics, 182*, 1-13. doi:10.1016/ j. ijpe.2016.08.008

Pjevčević, D., Radonjić, A., Hele, Z., & Čolić, V. (2012). DEA Window Analysis for Measuring Port Efficiencies in Serbia. *PROMET - Traffic& amp; Transportation PROMET, 24*(1). doi:10.7307/ptt. v24i1.269

Podinovski, V. V. (2015). DEA Models with Production Trade-offs and Weight Restrictions. *International Series in Operations Research & Management Science Data Envelopment Analysis*, 105-144. doi:10.1007/978-1-4899-7553-9_5

Podinovski, V. V., & Bouzdine-Chameeva, T. (2013). Weight Restrictions and Free Production in Data Envelopment Analysis. *Operations Research, 61*(2), 426-437. doi:10.1287/opre.1120.1122

Porcelli, F. (2009). Measurement of Technical efficiency. A brief survey of parametric and non-parametric-techniques. *Http://www2.warwick.ac.uk/fac/soc/economics/staff/fporcelli/porcelli_dea_sfm.pdf,*1-27.

Premachandra, I., Chen, Y., & Watson, J. (2011). DEA as a tool for predicting corporate failure and success: A case of bankruptcy assessment. *Omega, 39*(6), 620-626. doi:10.1016/j .omega .2011.01.002

Pulina, M., Detotto, C., & Paba, A. (2010). An investigation into the relationship between size and efficiency of the Italian hospitality sector: A window DEA approach. *European Journal of Operational Research, 204*(3), 613-620. doi: 10.1016/j.ejor.2009.11.006

Ramadan, I. Z. (2016). Data Envelopment Analysis (DEA) Approach for the Jordanian Banking Sector's Performance [Abstract]. *Modern Applied Science MAS, 10*(5), 170. doi:10.5539/mas. v 10n5p170

Ramanathan, R. (2003). *An introduction to data envelopment analysis: A tool for performance measurement*. Thousand Oaks (Calif.): Sage.

Rangan, N., Grabowski, R., Aly, H. Y., & Pasurka, C. (1988). The technical efficiency of US banks [Abstract]. *Economics Letters, 28*(2), 169-175. doi:10.1016/0165-1765(88)90109-

Rauer, H. P. (2014). Measuring Service Productivity: The Case of a German Mobile Service Provider. *2014 47th Hawaii International Conference on System Sciences*. doi:10.1109/hicss.2014.163

Ray, S. C., & Bhadra, D. (1993). Nonparametric Tests of Cost Minimizing Behavior: A Study of Indian Farms [Abstract]. *American Journal of Agricultural Economics, 75*(4), 990. doi:10.2307/1243986

Rayeni, M. M., & Saljooghi, F. H. (2010). Network Data Envelopment Analysis Model for Estimating Efficiency and Productivity in Universities. *Journal of Computer Science, 6*(11), 1252-1257. doi:10.3844/jcssp.2010.1252.1257

Repková, I. (2013). Estimation of Banking Efficiency In The Czech Republic: Dynamic Data Envelopment Analysis. *Danube, 4*(4). doi:10.2478/danb-2013-0014

Řepková, I. (2014). Efficiency of the Czech Banking Sector Employing the DEA Window

Analysis Approach. *Procedia Economics and Finance*, *12*, 587-596. doi:10.1016/s2212-5671(14)00383-9

Retrieved from http://www.chamber.ca/

Retrieved from https://www.statista.com/2015.

Retrieved from www. wajax.com/2016.

Retrieved from www.caterpillar.com/2016.

Retrieved from www.cca.acc.com/2016.

Retrieved from www.finning.ca/2016.

Retrieved from www.finning.ca/2016.

Retrieved from www.heavyequipment.com/2016.

Retrieved from www.hewitt.ca/2016.

Retrieved from www.rbauction.com/2016.

Retrieved from www.rockymtn.com/2016.

Retrieved from www.toromontcat.com/2016.

Review of heavy equipment industry. (n.d.). 2016, from (http://www.heavyequipment.com)

Review of heavy equipment industry. (n.d.). 2016, from (http://www.heavyequipment.com)

Rocky Mountain Equipment | Dependable Is What We Do. (n.d.). Retrieved December 10, 2015,

Rouse, P., Harrison, J., & Chen, L. (2010). Data Envelopment Analysis: A Practical Tool to Measure Performance. *Australian Accounting Review*, *20*(2), 165-177

Sale, S. R., & Sale, M. L. (2009). Data Envelopment Analysis: A Primer for Novice Users and Students at all Levels. [Abstract]. *OC09050-Instructional Pedagogies,* 1-14. Retrieved from www.aabri.com/OC09manuscripts/OC09050.pdf.

Saranga, H. (2009). The Indian auto component industry – Estimation of operational efficiency and its determinants using DEA. *European Journal of Operational Research, 196*(2), 707-

718. doi:10.1016/ j. ejor.2008.03.045

Sarkis, J. (2002). Preparing Your Data for DEA. *Modeling Data Irregularities and Structural Complexities in Data Envelopment Analysis,* 305-320. doi:10.1007/978-0-387-71607-7_17

Sastry, T., & Mukherji, A. (1998). Identification of Best Practices: A study of passenger car dealers in India using DEA. *Journal of Indian Institute of Management, Ahmedabad*.

Saxena, P., & Saxena, R. R. (2011). Measuring efficiencies in Indian public road transit: A data envelopment analysis approach. *Operations Research, 47*(3), 195-204. doi:10.1007/s12597-011-0034-5

Seiford, L. M., & Thrall, R. M. (1990). Recent developments in DEA. *Journal of Econometrics, 46*(1-2), 7-38. doi:10.1016/0304-4076(90)90045-u

Seiford, L. M., & Zhu, J. (1999). Profitability and Marketability of the Top 55 U.S. Commercial Banks [Abstract]. *Management Science, 45*(9), 1270-1288. doi:10.1287/mnsc.45.9.1270

Selim, S., & Bursalioglu, S. A. (2013). Analysis of the Determinants of Universities Efficiency in Turkey: Application of the Data Envelopment Analysis and Panel Tobit Model. *Procedia Social and Behavioral Sciences, 89*, 895-900. doi:10.1016/ j. sbspro.2013.08.952

Sellers-Rubio, R., & Mas-Ruiz, F. (2006). Economic efficiency in supermarkets: Evidences in Spain. *International Journal of Retail & Distribution Management, 34*(2), 155-171. doi:10.1108/09590550610649803

Sengupta, J. K. (1995). Frontiers of Efficiency Research. *Dynamics of Data Envelopment Analysis,* 277-284. doi:10.1007/978-94-015-8506-4_8

Sexton, T. R., & Communale, C. L. (2010). An efficiency analysis of U.S. business schools [Abstract]. *Journal of Case Studies in Accreditation and Assessment,* 1-19.

Sexton, T. R., Silkman, R. H., & Hogan, A. J. (1986). Data envelopment analysis: Critique and extensions. *New Directions for Program Evaluation, 1986*(32), 73-105. doi:10.1002/ev.1441

Sharma, K., Leung, P., & Zaeski, H. (1997, November). Productive Efficiency of the Swine Industry in Hawaii: Stochastic Frontier vs. Data Envelopment Analysis [Abstract]. *Journal of Productivity Analysis, 8*(4), 447-459. doi:10.1023/A:1007744327504

Shayea, A. M. (2011, December). Measuring hospital unit's efficiency: A DEA Approach. *International Journal of Engineering and Technology, 11*(06), 7-19. doi:110106-8787-IJET

Sherman, D. (1982, March). Measurement of Hospital efficiency using Data Envelopment Analysis. [Abstract]. *Working Paper Alfred Sloan School of Management.,* 1-130.

Sherman, D. H. (2013). Management of Service Organization Productivity. In J. Zhu (Ed.), *Service Productivity Management.* (pp. 9-17). Boston, MA: Springer.

Sherman, H. (1988). *Service organization productivity management*. Hamilton, Ont.: Society of Management Accountants of Canada.

Sherman, H. D., & Zhu, J. (2006). *Service productivity management: Improving service performance using data envelopment analysis (DEA)*. New York: Springer.

Sherman, H., & Gold, F. (1985). Bank branch operating efficiency. *Journal of Banking & Finance, 9*(2), 297-315. doi:10.1016/0378-4266(85)90025-1

Simar, L., & Wilson, P. W. (1998). Sensitivity Analysis of Efficiency Scores: How to Bootstrap in Nonparametric Frontier Models. *Management Science, 44*(1), 49-61. doi:10.1287/mnsc.44.1.49

Simar, L., & Wilson, P. W. (2010). Inference by the m out of n bootstrap in nonparametric frontier models. *Journal of Productivity Analysis, 36*(1), 33-53. doi:10.1007/s11123-010-0200-4

Simar, L., & Wilson, P. W. (2011). Performance of the Bootstrap for DEA Estimators and

Iterating the Principle. *International Series in Operations Research & Management Science Handbook on Data Envelopment Analysis,* 241-271. doi:10.1007/978-1-4419-6151-8_10

Singh, H., Motwani, J., & Kumar, A. (2000). A review and analysis of the state-of-the-art research on productivity measurement. *Industrial Management& Data Systems, 100*(5), 234-241. doi:10.1108/02635570010335271

Sink, D. S., & Tuttle, T. C. (1989). *Planning and measurement in your organization of the future*. Norcross, GA: Industrial Engineering and Management Press.

Society. Series A (General), 120(3), 253. doi:10.2307/2343100

Sowlati, T., & Paradi, J. (2004). Establishing the "Practical Frontier" in Data Envelopment Analysis. *International Journal of Management Science, Omega 32*, 261-272.

Spaho, A. (2015). Determinants of Bank's Efficiency in Albania-Data Envelopment Analysis and Tobit Regression Analysis. *International Conference on Social Sciences, IV,* 1-11. doi: ISBN978880916236

Staat, M., & Hammerschmidt, M. (2005). Product performance evaluation: A super-efficiency model. *International Journal of Business Performance Management IJBPM, 7*(3), 304-319. doi:10.1504/ijbpm.2005.006722

Statistics Canada: Canada's national statistical agency / Statistic Canada: Organisme statistique national du Canada. (n.d.). Retrieved December 11, 2015, from http://www.statcan.gc.ca/

Stoica, M. I., & Ivan, M. L. (2016, April). Using Data Envelopment Analysis in Health Care for estimating Birth Rate Efficiency. *International Journal of Scientific and Technology Research, 5*(4), 225-229. doi: ISSN 2277-8616

Sueyoshi, T., & Goto, M. (2012). Data envelopment analysis for environmental assessment: Comparison between public and private ownership in petroleum industry. *European*

Journal of Operational Research, 216(3), 668-678. doi:10. 1016/j.ejor.2011.07.046

Sueyoshi, T., & Goto, M. (2018). *Environmental assessment on energy and sustainability by data envelopment analysis*. Hoboken, NJ: John Wiley & Sons.

Sun, X., Lv, X., & Li, L. (2015). Sufficient and Comprehensive Measurement of Automobile Manufacturing Industry Performance Applying Bi-objective Super-efficiency DEA. *Proceedings of the 2015 International Conference on Applied Science and Engineering Innovation,* 2100-2104. doi:10.2991/asei-15.2015.412

Systems. *Procedia Technology, 16*, 518-524.

Syverson, C. (2011). What Determines Productivity? *Journal of Economic Literature, 49*(2), 326-365. doi:10.1257/jel.49.2.326

Tan, Y., Zhang, Y., & Khodaverdi, R. (2016, April 29). Service performance evaluation using data envelopment analysis and balance scorecard approach: An application to automotive industry. *Annals of Operations Research.* doi:10.1007/s10479-016-2196-2

Tangen, S. (2004). Performance measurement: From philosophy to practice. *International Journal of Productivity and Performance Management, 53*(8), 726-737. doi:10.1108/17410400410569134

Tauer, L. W. (1998). Productivity of New York Dairy Farms Measured by Nonparametric Malmquist Indices. *Journal of Agricultural Economics, 49*(2), 234-249. doi:10.1111/j.1477-9552. 1998.tb01266.x

Thanassoulis, E. (1999). Data Envelopment Analysis and Its Use in Banking [Abstract]. *Interfaces, 29*(3), 1-13. doi:10.1287/inte.29.3.1

Thanassoulis, E. (2001). *Introduction to the theory and application of data envelopment analysis: A foundation text with integrated software*. Norwell, MA: Kluwer Academic.

Thanassoulis, E., Dyson, R. G., & Foster, M. J. (1987). Relative Efficiency Assessments Using

Data Envelopment Analysis: An Application to Data on Rates Departments. *The Journal of the Operational Research Society, 38*(5), 397. doi:10.2307/2582729

Thomas, R. R., Barr, R. S., Cron, W. L., & Slocum, J. W. (1998). A process for evaluating retail store efficiency: A restricted DEA approach. *International Journal of Research in Marketing, 15*(5), 487-503. doi:10.1016/s0167-8116(98)00021-4

Thompson, R. G., Brinkmann, E. J., Dharmapala, P., Gonzalez-Lima, M., & Thrall, R. M. (1997). DEA/AR profit ratios and sensitivity of 100 large U.S. banks [Abstract]. *European Journal of Operational Research*, *98*(2), 213-229. doi:10.1016/s0377-2217(96)00343-8

Thompson, R. G., Singleton, F. D., Thrall, R. M., & Smith, B. A. (1986). Comparative Site Evaluations for Locating a High-Energy Physics Lab in Texas. *Interfaces, 16*(6), 35-49. doi:10.1287/inte.16.6.35

Toloo, M., & Ertay, T. (2014, March 4). The most cost efficient automotive vendor with price uncertainty: A new DEA approach. *Measurement, 52*, 135-144. doi:10.1016/j.measurement.2014.03.002

Tone, K. (2001). A slacks-based measure of efficiency in data envelopment analysis. *European Journal of Operational Research, 130*(3), 498-509. doi:10.1016/s0377-2217(99)00407-5

Tone, K. (2017). *Advances in DEA theory and applications with extensions to forecasting models*. Hoboken, NJ: Wiley.

Tone, K., & Tsutsui, M. (2009). Network DEA: A slacks-based measure approach. *European Journal of Operational Research*, *197*(1), 243-252. doi: 10.1016/j.ejor.2008.05.027

Toromont Cat: Caterpillar Heavy Equipment Dealer in Canada. (n.d.). Retrieved December 10, 2015, from http://www.toromontcat.com/

Torrisi, G. (2009, January). Public infrastructure: Definition, classification and measurement issues. *Https://mpra.ub.uni-muenchen.de/12990/1/MPRA_paper_12990.pdf, 10*, 95039th ser. University of Catania, Faculty of Economics, DEMQ.

Tran, D. H., & Ngo, D. T. (2014, June). Performance of Vietnamese automobile industry; A measurement using DEA. *Asian Journal of Business and Management*, *2*(3), 184-191.

Tran, N. A., Shively, G., & Preckel, P. (2010). A new method for detecting outliers in Data Envelopment Analysis. *Applied Economics Letters*, *17*(4), 313-316. doi:10.1080/13504850701765119

Tripathy, I. G., Yadav, S. S., & Sharma, S. (2010). Measuring the efficiency of Pharmaceutical firms in India: An application of Data Envelopment Analysis and Tobit Estimation. *Cite Seer*.

Tripathy, I. G., Yadav, S. S., & Sharma, S. (n.d.). Measuring the efficiency of Pharmaceutical firms in India. An application of Data Envelopment Analysis and Tobit Estimation. *Cite Seer*, 1-24. doi: http:// citeseerx.ist.psu.edu/view doc / summary.doi=10.1.1.507.8033

Urso, A., Timpanaro, G., Caracciolo, F., & Cembalo, L. (2018). Efficiency analysis of Italian wine producers. *Wine Economics and Policy*, *7*(1), 3-12. doi: 10.1016/j.wep.2017.11.003

Used Heavy Equipment for Sale | Heavy Equipment Auctions | Ritchie Bros. Auctioneers. (n.d.). Retrieved December 11, 2015, from http://www.rbauction.com/

Vaz, C. B., Camanho, A. S., & Guimarães, R. C. (2008). The assessment of retailing efficiency using Network Data Envelopment Analysis. *Annals of Operations Research*, *173*(1), 5-24. doi:10.1007/s10479-008-0397-z

Wagner, J. M., & Shimshak, D. G. (2007). Stepwise selection of variables in data envelopment analysis: Procedures and managerial perspectives. *European Journal of Operational Research*, *180*(1), 57-67. doi: 10.1016/j.ejor.2006.02.048

Wajax | Home: Company Profile. (n.d.). Retrieved December 10, 2015, from

Wang, C. H., Gopal, R. D., & Zionts, S. (1997). Use of DEA in assessing information technology impact on firm performance. [Abstract]. *Annals of Operations Reasearch, 73*, 191-213.

Wang, C., Nguyen, X., & Wang, Y. (2016). Automobile Industry Strategic Alliance Partner Selection: The Application of a Hybrid DEA and Grey Theory Model. *Sustainability, 8*(2), 1-18. doi:10.3390/su8020173

Wang, J. (2017). Benchmarking Building Energy in the multifamility industry: A Data Envelopment Analysis(DEA)Model. *Ph D. Thesis of Georgia Institute of Technology*.

Webb, R. (2003). Levels of efficiency in UK retail banks: a DEA window analysis. *International Journal of the Economics of Business, 10*(3), 305-322. doi:10.1080/1357151032000126256

Wilson, L. S. (2008). Statistical Inference in Nonparametric Frontier Models: Recent Developments and Perspectives. *The Measurement of Productive Efficiency and Productivity Change,* 421-521. doi:10.1093/

Wilson, P. W. (1993). Detecting Outliers in Deterministic Nonparametric Frontier Models with Multiple Outputs. *Journal of Business & Economic Statistics, 11*(3), 319. doi:10.2307/1391956

Wilson, P. W. (1993). Detecting Outliers in Deterministic Nonparametric Frontier Models With Multiple Outputs. *Journal of Business & Economic Statistics, 11*(3), 319-323. doi:10.1080/07350015.1993.10509959

Wilson, P. W. (1995). Detecting influential observations in data envelopment analysis. *Journal of Productivity Analysis, 6*(1), 27-45. doi:10.1007/bf01073493

Winston, W. (1994). *Operations research: Applications and algorithms* (3rd ed.). Belmont, Calif.: Duxbury Press.

Wong, Y. B., & Beasley, J. E. (1990). Restricting Weight Flexibility in Data Envelopment Analysis. *Journal of the Operational Research Society, 41*(9), 829-835. doi:10.1038/ sj jors/0410905

Xue, M., & Harker, P. T. (2002). Note: Ranking DMUs with Infeasible Super-Efficiency DEA Models. *Management Science, 48*(5), 705-710. doi:10.1287/mnsc.48.5.705.7805

Yang, H., & Chang, C. (2009). Using DEA window analysis to measure efficiencies of Taiwan's integrated telecommunication firms. *Telecommunications Policy, 33*(1-2), 98-108. doi:10.1016/ j. telpol.2008.11.001

Yang, Z. (2013). Performance Analysis of Canadian Power Industry Using Data Envelopment Analysis. *IJCEE International Journal of Computer and Electrical Engineering*, 169-172. doi:10.7763/ijcee. 2013.v5.688

Yang, Z., Wang, X., & Sun, D. (2009). Using the bootstrap method to detect influential DMUs in data envelopment analysis. *Annals of Operations Research, 173*(1), 89-103. doi:10.1007/s10479-009-0520-9

Yasin, M. M., & Gomes, C. F. (2010). Performance management in service operational settings: A selective literature examination. *Benchmarking: An International Journal, 17*(2), 214-231. doi:10.1108/14635771011036311

Yu, M. (2010). Assessment of airport performance using the SBM-NDEA model. *Omega, 38*(6), 440-452. doi:10.1016/ j. omega.2009.11.003

Yu, W., & Ramanathan, R. (2008). An assessment of operational efficiencies in the UK retail sector. *International Journal of Retail & Distribution Management, 36*(11), 861-882. doi:10.1108/09590550810911656

Zadmirzaei, M., Limaei, S. M., & Amirteimoori, A. (2015). Efficiency Analysis of Paper Mill using Data Envelopment Analysis Models (Case Study: Mazandaran Wood and Paper company in Iran) [Abstract]. *Journal of Agricultural Science and Technology, 17*, 1381-1391.

Zbranek, P. (2013). Data Envelopment Analysis as a Tool for Evaluation of Employees Performance [Abstract]. *RAAE Review of Agricultural and Applied Economics, 16*(01), 12-21. doi:10.15414/raae.2013.16.01.12-21

Zhou, G., Chung, W., & Zhang, Y. (2014). Measuring energy efficiency performance of China's transport sector: A data envelopment analysis approach. *Expert Systems with Applications*, *41*(2), 709-722. doi: 10.1016/j.eswa.2013.07.095

Zhu, J. (1996). Robustness of the efficient DMUs in data envelopment analysis. *European Journal of Operational Research*, *90*(3), 451-460. doi:10.1016/0377-2217(95)00054-2

Zhu, J. (2004). *Quantitative models for performance evaluation and benchmarking: Data envelopment analysis with spreadsheets and DEA Excel Solver*. Boston: Kluwer Acad

Zhu, J. (2014). *Data envelopment analysis: Let the data speak for themselves*.

Zofio, J. L. (2007). Malmquist productivity index decompositions: a unifying framework. *Applied Economics*, *39*(18), 2371-2387. doi:10.1080/00036840600606260

2013 Cost of Doing Business Report (PDF version). (n.d.). Retrieved November 23, 2014,

25(2), 173-186. doi: 10.1016/j.mar.2013.07.00749. doi:10.1016/s0038-0121(99)00012

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